

Reconstruction of Surface of Revolution from Two Uncalibrated Views 二枚の写真から回転体を復元する方法

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Abstract : In the process of reconstruction of textured 3D models of potteries from images, the precise 3D shape of the pottery must be obtained in advance. This can be done using computer vision technique and from symmetry property of potteries. This paper addresses the problem of recovering the 3D shape of a surface of revolution from two uncalibrated perspective views. The algorithm makes use of the symmetry properties of a surface of revolution and its silhouette to rectify the image such that the resulting silhouette exhibits bilateral symmetry. The rectifications of the two images lead to a simple equation for the computation of orientation of the revolution axis of the surface of revolution. The rectified image is then used to recover the contour generator and the object is reconstructed up to an unknown scale. Experimental results on real images are presented, which demonstrate the effectiveness of the approach.

Keywords: Surface of revolution; Homography; Silhouette;

概要 : 豊かな歴史を有する京都を核とする日本の文化や芸能などを、最先端のデジタル技術を利用して保存し、その情報をWEBで世界に向けて発信することで世界中の人々は京都の歴史と文化がより広く深く理解することができる。京都の数多くの有形無形の文化財の中で、地中に埋もれた文化財は考古学者の発掘調査により姿を現してくる。このような発掘の成果をデジタルデータとして記録および保存し、そしてホームページでの公開に関する研究を21世紀COEプログラムの「京都アート・エンタテインメント創成研究」の一環として行われている[1]。陶磁器のような回転体のテクスチャ付き三次元復元を行うために、まず陶磁器の三次元形状を復元する必要がある。コンピュータビジョン技術を用いることで今まで使った手作業で求められた陶磁器の形状を高精度で効率的に求めることができる。本研究は二枚の写真から回転体の三次元形状を復元する方法を提案する。回転体の対称性を利用して二枚の画像を左右対称に校正し、回転体の回転軸を求める。校正した画像から回転体の輪郭生成線を計算して回転体の三次元形状を復元する。実画像を用いて実験を行い、本方法の有効性を示した。

1. Introduction

The traditional culture and art entertainment of Kyoto over a long history can be reserved using the latest digital information technology. The results of such digital archiving can be published through the internet to the world.

As a part of material and immaterial cultural

resources of Kyoto, a large number of buried cultural properties are appearing to the public by archaeological excavation. The researches of the digital archiving and internet publication of such excavated objects are carried out in the project of Kyoto Art Entertainment Innovation Research [1]. In order to get a precise textured 3D model of the excavated objects such as potteries, the 3D shape reconstruction must be performed in advance.

This can be performed using computer vision techniques and from the symmetry property of potteries. Compared with the traditional methods, this approach can provide more accurate 3D shapes easily and efficiently.

Computer vision techniques like stereo vision and structure from motion can be employed to recover the 3D structure of imaged objects with multiple images from different viewpoints. For a surface of revolution (SOR), the symmetry properties exhibited in its image can be used to determine the contour generator, which can then be used to recover the SOR up to two degrees of freedom [2]. In this paper, we propose a method to recover the 3D shape of a SOR using two perspective images from different viewpoints. First the motion between the two images is calculated using corresponding points, and then from the symmetry properties of the SOR and its silhouettes, we rectify the two images by planar homography such that each resultant silhouette exhibits bilateral symmetry. The motion between the two rectified imaged revolution axes is exploited and an equation for computing the orientation of the revolution axis is derived. At last the contour generator is determined from the rectified images and the SOR is reconstructed by rotating the contour generator about the revolution axis.

Most of the existing techniques for the reconstruction of SOR made use of a single monocular image to infer geometric information from the silhouette [2] [3] [4]. In [2], an algorithm was proposed which makes use of the invariant properties of a SOR to calibrate the camera and rectify the image, surface normal along the contour generator are then determined from the rectified silhouette, a parameterization formula of the contour generator is derived using a coplanarity constraint between the surface normal and the revolution axis and the 3D shape of the SOR is

recovered. However, the ambiguity in the orientation of the revolution axis cannot be correctly resolved. These methods only work with normalized camera with zero skew and known aspect ratio. Besides, they require the presence of at least one perspective image of cross section of the object. The method introduced here works with general uncalibrated camera, and it does not need the existing of visible cross sections. Therefore it also can be used for those objects without visible cross sections and a SOR can be recovered up to an unknown scale by this method.

In Section 2, we give an introduction of the theoretical background for reconstructing a SOR from a single uncalibrated image. In Section 3, we present a method for reconstructing a SOR from two uncalibrated images. The algorithm and implementation are described in Section 4 and results of real data experiments are presented in Section 5. Finally, conclusions are given in Section 6.

2. Properties of surface of revolution

2.1 Symmetry property of SOR

A surface of revolution S is a surface created by rotating a plane curve C about an axis in the same plane. Through any point r of S there is a meridian curve which is the curve obtained by rotating C to this particular position and a latitude circle which is a circle through r with its center on the axis. The meridian curve and latitude circle are orthogonal [5].

The contour generator Γ is the set of points X on S at which rays are tangent to the surface. The corresponding image apparent contour γ is the set of points x which are the image of X , i.e. γ is the image of Γ . The contour generator Γ only depends on the relative position of the camera centre and the SOR. The apparent contour γ is defined by the intersection of the image plane

with the rays to the contour generator, and so depend on the position of the image plane. In general, under perspective projection Γ is space curve.

Under perspective projection, if the camera is pointing towards the revolution axis of a SOR and if the camera also has unit aspect ratio, the silhouette of the SOR will become bilaterally symmetric. This property of the image of SOR can be used to rectify the images such that the resultant silhouette is bilaterally symmetric. In [2], a harmonic homology defined by the image of the revolution axis and a vanishing point is used to rectify the image [6] [7]. In this paper, we rectify the image by calculating a planar homography from the symmetry property of the image of SOR.

2.2 Parameterization of contour generator

Let $\mathbf{S}_r(s, \theta) = [X(s) \cos \theta Y(s) X(s) \sin \theta]^T$ be a SOR generated by rotating a meridian curve $C_r = [X(s) Y(s) 0]^T$ about the y -axis and a pinhole camera $\hat{\mathbf{P}} = [\mathbf{I}_3 - \mathbf{c}]$ centered at $\mathbf{c} = [0 0 -d_z]^T$ with $d_z > 0$ (Fig. 1).

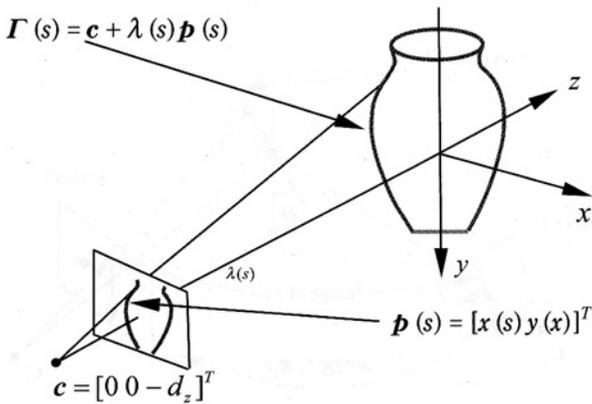


Fig. 1. A surface of revolution, whose axis of revolution coincides with the y -axis, being viewed by a pinhole camera centered at \mathbf{c}

The contour generator $\Gamma(s)$ can be parameterized as [2]

$$\begin{aligned} \Gamma(s) &= \mathbf{c} + \lambda(s) \mathbf{p}(s), \text{ where} \\ \mathbf{p}(s) \cdot \mathbf{n}(s) &= 0, \end{aligned} \quad (1)$$

where $\mathbf{p}(s) = [x(s) y(s) 1]^T$ is the viewing vector from \mathbf{c} to the focal plane at unit distance for the point $\Gamma(s)$, and $\lambda(s)$ is the depth of $\Gamma(s)$ from \mathbf{C} along the z direction. $\mathbf{n}(s)$ is the unit surface normal at $\Gamma(s)$ which can be determined as the cross product between $\mathbf{p}(s)$ and the tangent along silhouette at $\mathbf{p}(s)$. [2]

It can be shown that the surface normal at point $\mathbf{S}_r(s_0, \theta_0)$ is normal to the meridian curve through $\mathbf{S}_r(s_0, \theta_0)$ and lies on the plane containing the y -axis and point $\mathbf{S}_r(s_0, \theta_0)$. This coplanarity constraint can be expressed by [7] [8]

$$\mathbf{n}(s)^T [\mathbf{n}_y]_{\times} \Gamma(s) = 0, \quad (2)$$

where $\mathbf{n}_y = [0 0 1]^T$. This can be expanded to recover the depth

$$\lambda(s) = \frac{d_z n_1(s)}{n_1(s) - n_3(s) x(s)}, \quad (3)$$

and the contour generator can then be recovered in homogeneous coordinates as

$$\tilde{\Gamma}(s) = \begin{bmatrix} d_z \dot{y}(s) x(s) \\ d_z \dot{y}(s) y(s) \\ d_z \alpha_{\tilde{\Gamma}}(s) \\ \dot{y}(s) - \alpha_{\tilde{\Gamma}}(s) \end{bmatrix}, \quad (4)$$

where $\alpha_{\tilde{\Gamma}}(s) = (\dot{x}(s) y(s) - x(s) \dot{y}(s)) x(s)$. Such reconstruction is determined up to a similarity transformation since the distance d_z cannot be recovered. In general case, if the SOR is rotated about x -axis by an angle ψ , the equation (4) of the contour generator becomes

$$\tilde{\Gamma}^{\psi}(s) = \begin{bmatrix} d_z \dot{y}(s) x(s) \\ d_z \dot{y}(s) (y(s) \cos \psi - \sin \psi) \\ d_z \alpha_{\tilde{\Gamma}}^{\psi}(s) \\ \dot{y}(s) (y(s) \sin \psi + \cos \psi) - \alpha_{\tilde{\Gamma}}^{\psi}(s) \end{bmatrix}, \quad (5)$$

where

$$\alpha_{\tilde{\Gamma}}^{\psi}(s) = \{(\dot{x}(s) y(s) - x(s) \dot{y}(s)) \cos \psi - \dot{x}(s) \sin \psi\} x(s).$$

3. Reconstruction from two images

3.1 Image rectification

When a camera is rotated about its center with no change in the internal parameters, and if \tilde{x} and \tilde{x}' are the images of a X point before and after the rotation, then

$$\tilde{x}' \cong KRK^{-1}\tilde{x}, \tag{6}$$

where K is the camera calibration matrix and R is the rotation matrix.

From the symmetry properties of SOR and its silhouette discussed in Section 2.1, given K , it is possible to rectify the image of a SOR by a planar homography H induced by a rotation such that the resultant silhouette is bilaterally symmetric about the y -axis. This corresponds to rotate the camera about its optical center until the revolution axis of the SOR lies on the y - z plane at the camera coordinate system. Note that the homography is not unique, since any homography H' , given by $H' = R_x(\psi)H$ where $R_x(\psi)$ is a rotation about the x -axis by an angle ψ , will also yield a silhouette which will be bilaterally symmetric about y -axis.

3.2 Computation of the revolution axis

In order to recover the shape of SOR using Eq. (5), the orientation of the revolution axis ψ must be determined first. This can be estimated using perspective images of one or two cross sections of the SOR [2] [9]. But for those that have no visible cross sections, these methods do not work. In this section, we will show that this angle can be calculated using two images of SOR and no cross section is needed.

For two images of a SOR from two different viewpoints, we first compute the motion between the two images and the camera intrinsic parameters of the two cameras. Let R and t be the rotation matrix and translation vector, respectively.

Then

$$X_2 = RX_1 + t,$$

where X_1 and X_2 are the camera coordinates of a 3D point X in image 1 and 2, respectively.

Next we rotate the camera 1 about its optical center to rectify the image 1 such that the resultant silhouette is bilaterally symmetric about the y -axis. We have

$$\begin{aligned} X'_1 &= R_1 X_1, \\ X_2 &= RX_1 + t = RR_1^{-1} X'_1 + t, \end{aligned} \tag{7}$$

where X'_1 is the coordinate of the point X in camera coordinate system of camera 1 after rotation, R_1 is the rotation matrix of camera 1. We also rectify the image 2 by rotating the camera 2 and we have

$$\begin{aligned} X'_2 &= R_2 X_2, \\ R_2^{-1} X'_2 &= RR_1^{-1} X'_1 + t. \end{aligned} \tag{8}$$

where X'_2 is the coordinate of the point X in camera coordinate system of camera 2 after rotation, R_2 is the rotation matrix of camera 2. Then

$$\begin{aligned} X'_2 &= R_2 RR_1^{-1} X'_1 + R_2 t \\ &= R_N X'_1 + t_N \end{aligned} \tag{9}$$

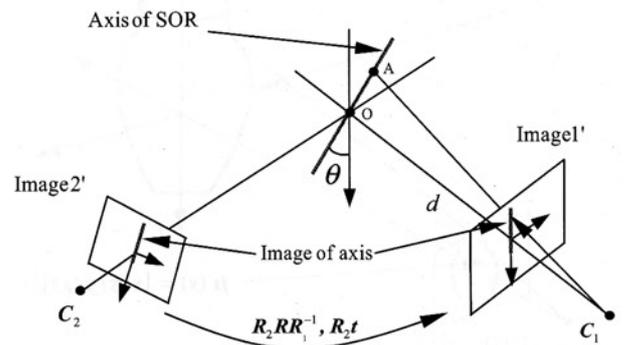


Fig.2. Motion between two imaged axes of SOR after rectification of two images

where $R_N = R_2 RR_1^{-1}$, $t_N = R_2 t$.

Let O and A be two points lie on the revolution axis of the SOR with 3D coordinates $X_{O1} = [0 \ 0 \ d]^T$ and $X_{A1} = [0 \ -\alpha \cos \theta \ d - \alpha \sin \theta]^T$ under camera

coordinate system of image 1 (Fig. 2), respectively. Where θ is the orientation of revolution axis of SOR in y - z plane of camera coordinate system, α is a scale factor which represents the distance for O to A along the revolution axis, d is the distance from the center of camera 1 to O . Then the cameras coordinate of O and A under camera coordinate system of image 2 can be calculated from Eq. (9) as

$$\begin{aligned} \mathbf{X}_{O2} &= \mathbf{R}_N \mathbf{X}_{O1} + \mathbf{t}_N, \\ \mathbf{X}_{A2} &= \mathbf{R}_N \mathbf{X}_{A1} + \mathbf{t}_N. \end{aligned} \quad (10)$$

Since O and A lie in the y - z plane of camera 2, the x coordinates of O and A are equal to zero, we have

$$\begin{aligned} r_{13}d + t_1 &= 0, \\ -r_{12}\alpha \cos\theta + r_{13}d - r_{13}\alpha \sin\theta + t_1 &= 0. \end{aligned} \quad (11)$$

where r_{ij} denotes the ij element of \mathbf{R}_N , t_i denotes the i th element of \mathbf{t}_N . Since $\mathbf{R}, \mathbf{R}_1, \mathbf{R}_2$ and \mathbf{t} are known, \mathbf{R}_N and \mathbf{t}_N can be calculated. Solving Equation (11), we have

$$\tan\theta = -r_{12}/r_{13}. \quad (12)$$

4. Algorithm and implementation

4.1 Computation of homography

Let $\mathbf{H} = \mathbf{K}\mathbf{R}\mathbf{K}^{-1}$ be the homography matrix where \mathbf{K} is the camera calibration matrix and \mathbf{R} is the rotation matrix. There are several ways to represent a rotation, here we use Euler angles which are rotational angles around the three Cartesian coordinate axes.

$$\mathbf{R}(\alpha, \beta, \gamma) = \mathbf{R}(z, \alpha) \mathbf{R}(y, \beta) \mathbf{R}(z, \gamma).$$

Let \mathbf{m}, \mathbf{m}' be the images of a point \mathbf{X} before and after the rotation, and \mathbf{m} lies on the left side of the silhouette. Then

$$\mathbf{m}' = \mathbf{H}\mathbf{m}.$$

Next we perform a transformation which maps \mathbf{m}' to a point \mathbf{m}'' such that \mathbf{m}' and \mathbf{m}'' are symmetric about the axis which pass through the center of the image plane and points down, i.e.

$$\mathbf{m}'' = \mathbf{H}_1 \mathbf{m}' + [w \ 0 \ 0]^T = \mathbf{H}_1 \mathbf{H}\mathbf{m} + [w \ 0 \ 0]^T,$$

where w is the width of the image and \mathbf{H}_1 has the form

$$\mathbf{H}_1 = \begin{bmatrix} -1 & & \\ & 1 & \\ & & 1 \end{bmatrix}.$$

Let \mathbf{H}_2 be a homography which performs a reverse transformation of \mathbf{H} and has the form

$$\begin{aligned} \mathbf{H}_2 &= \mathbf{K}\mathbf{R}'\mathbf{K}^{-1}, \quad \text{where} \\ \mathbf{R}' &= \mathbf{R}^{-1} = \mathbf{R}(z, -\alpha) \mathbf{R}(y, -\beta) \mathbf{R}(z, -\gamma). \end{aligned}$$

Let \mathbf{m}'' be the image after transformation of \mathbf{m}' , then

$$\mathbf{m}'' = \mathbf{H}_2 \mathbf{m}' = \mathbf{H}_2 \mathbf{H}_1 \mathbf{H}\mathbf{m} + \mathbf{H}_2 [w \ 0 \ 0]^T.$$

Since \mathbf{H} and \mathbf{H}_2 are functions of α, β, γ and \mathbf{H}_1, \mathbf{K} are known, the rotation matrix \mathbf{R} related to the homography \mathbf{H} can then be calculated by optimizing the cost function

$$\text{Cost}(\mathbf{R}(\alpha, \beta, \gamma)) = \sum \text{dist}(\mathbf{m}'', \rho)^2,$$

where $\text{dist}(\mathbf{m}'', \rho)$ is the distance from the transformed point $\mathbf{m}'' = \mathbf{H}_2 \mathbf{H}_1 \mathbf{H}\mathbf{m} + \mathbf{H}_2 [w \ 0 \ 0]^T$ to the original silhouette ρ . This can be solved by the Levenberg-Marquardt method [10].

4.2 Depth recovery

Since the rectified silhouette is bilaterally symmetric about the y -axis, only one side of it needs to be considered during the reconstruction of the SOR. The apparent contour is first segmented manually from the rectified silhouette. Points are then sampled from the apparent contour and the tangent vector at each sample point is estimated by fitting a polynomial to the neighboring points. Finally, the depth of each sample point is recovered using Eq. (5), and the contour generator and the SOR follow.

5. Experiments and results

Experiments are carried out to demonstrate the effectiveness of our approach. The silhouette

of a SOR is extracted from the image by applying a Canny edge detector [11]. The initial motion between the two images is calculated by 3DM-Modeler, software developed by 3DMedia Co., Ltd. Fig. 3 shows the reconstruction of a bowl. The rectification of the silhouette (Fig.3 (b)) was done using the algorithm described in Section 4. Fig. 4 and Fig. 5 show other examples of a vase and a

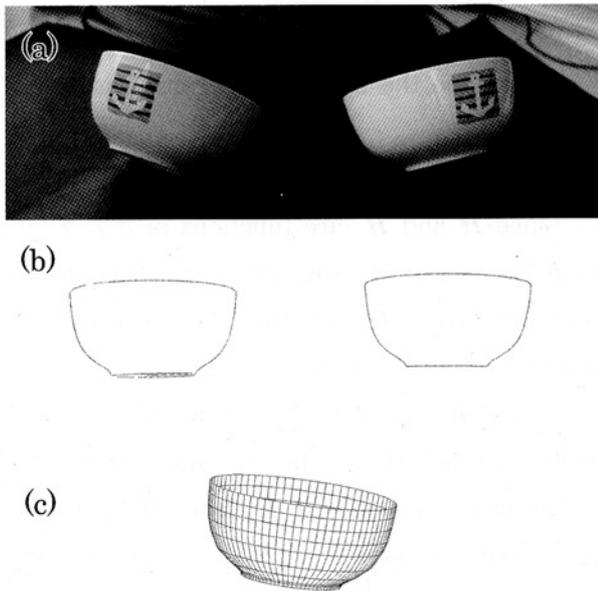


Fig.3. (a) Image of a bowl. (b) Rectified silhouette of the bowl. (c) Reconstructed model.

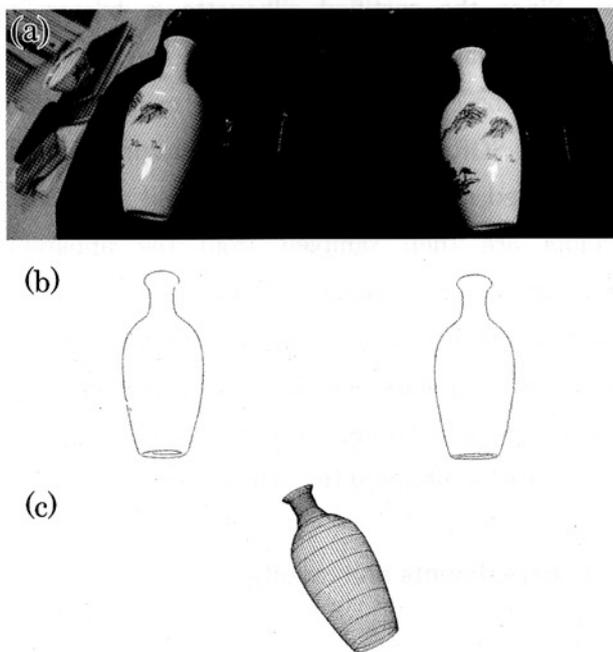


Fig.4. (a) Image of a vase. (b) Rectified silhouette of the vase. (c) Reconstructed model.

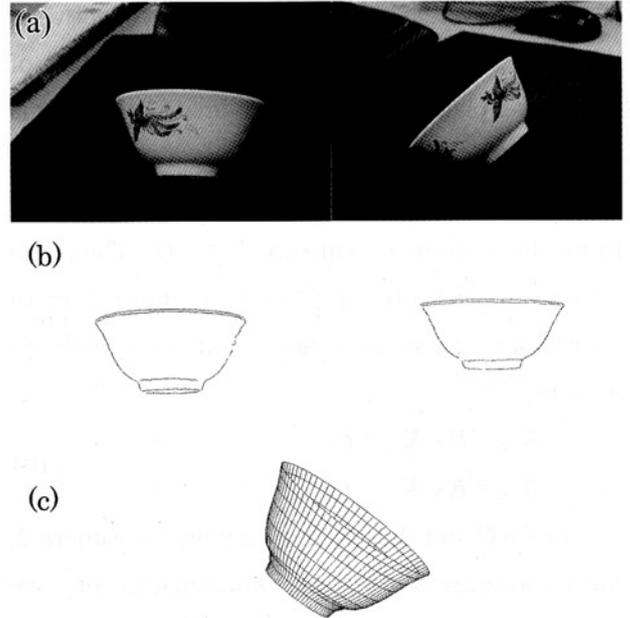


Fig.5. (a) Image of a bowl. (b) Rectified silhouette of the bowl. (c) Reconstructed model.

Table 1. Orientation and Ratio.

	$\theta(^{\circ})$	Ratio(height/diameter)	
		Measure	Reconstruction
Bowl 1	-9.83	0.50	0.48
Vase	-26.9	0.20	0.20
Bowl 3	-25.9	0.51	0.52

bowl. The orientations of the revolution axis of these examples are shown in Table 1. The ratio of the diameter of the topmost circle and the height of the bowl that are measured manually and calculated from the reconstructed model are also shown in Table 1.

6. Conclusions

In this paper, a novel approach is proposed to reconstruct a surface of revolution from two uncalibrated views. The motion between two images is calculated using corresponding points, and then the images are rectified such that the resulting images show symmetry about y -axis. A computation of the motion between the two

rectified images is performed and the result is used to compute the orientation of revolution axis. The depth of contour generator is then calculated and the SOR is reconstructed up to a scale. The test results demonstrated the effectiveness of the approach.

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