論　説

A Simplified Method of Cost Structure Analysis

Takashi Yagi †

1. Introduction

The aim of this paper is to explain a simplified method to calculate the cost structure of industries which can be applied to non competitive import type input-output tables. The approach of this paper uses Sraffa's standard system, which is defined by the eigenvalue and the eigenvector of the input coefficient matrix. The Sraffa system enables us to transform the actual relationship among output, net output and produced means of production (intermediate input) into a hypothetical relationship among them by the standard system. To define the standard system, we need only the data of the input coefficient matrix, the labour input coefficient vector and the standard labour. This is the merit of the Sraffa system. But a simple Sraffa system has no fixed capital and is characterized only by the input coefficient matrix, labour input coefficient vector and actual total labour. Yagi (2016) introduced a new method to extend a simple Sraffa system with no fixed capital to a system with fixed capital. Yagi (forthcoming) applied the method of Yagi (2016) to a price system corresponding to a non competitive import type input-output table. The calculation with input-output table by using our method needs the sectoral data of fixed capital. However, it is usual that the sectoral data of fixed capital corresponding to the input-output tables is not available. In this paper, we will explain how to simplify the calculation of the cost structure of industries without using sectoral data of fixed capital. As regards this, we get through with total amount of fixed capital and adopt an ad hoc assumption, or a proxy, in order to simplify the calculation with input-output table. We will assume that the uniform rate of wages and the uniform rate of profits throughout industries are prevailing. In Section 2, we will explain how to apply the Sraffian standard system to a non competitive input-output table. Section 3 will take up the price equation system corresponding to a non competitive input-output table and explain the determination of distributive variables. We will use two different notions of the rate of profits: one is the rate of profits to the ordinary capital, another is the rate of profits to

† Meiji University　yagi8@meiji.ac.jp

the wage as capital. The rate of profit will be given in our model. Section 4 will discuss a simplified method without using sectoral data of fixed capital.

2. The Cost Structure based on the Total Labour

In a simple Sraffa system, commodities are produced by means of domestic labour and produced means of production (intermediate input). On the other hand, Input-Output (IO) tables of the real world have intermediate inputs, imports, domestic labour and fixed capital as input. In addition, compensation for employees (wages), operating surplus (profits), capital depreciation and net indirect taxes (taxes less subsidies on production) appear in the items of value added. In order to apply the Sraffian standard system to the price system corresponding to a non competitive import type IO table, we should normalize the standard labour by using the actual total labour. In this section, we will explain how to normalize the standard labour in order to apply the standard system to a non competitive import type IO table, and then show price vectors to measure the cost structure of industries.

The standard system of Sraffa (1960) is defined by the eigenvalue and eigenvector of the input coefficient matrix. Let us denote the transposed matrix of the input coefficient matrix by $\mathbf{A}$, and its eigenvalue by λ. We assume that $\mathbf{A}$ is a positive matrix. The maximum rate of physical surplus (or the maximum rate of profit) can be given by

$$R=1/\lambda-1 \tag{1}$$

If we denote the eigenvector of $\mathbf{A}$ by $\mathbf{q}$, the standard system can be represented as

$$\mathbf{q}=(1+R)\mathbf{qA} \tag{2}$$
$$\mathbf{s}=R\mathbf{qA}=\mathbf{q}[\mathbf{I}-\mathbf{A}]>0 \tag{3}$$
$$\mathbf{z}_S=\mathbf{qA}>0 \tag{4}$$

where $\mathbf{q}$ is the output vector of the (hypothetical) standard system, $\mathbf{s}$ is the standard net product vector and $\mathbf{z}_S$ is the vector of produced means of production of the standard system. If we denote the labour input coefficient vector by $\mathbf{l}_A$, the total labour of the standard system is given by $\mathbf{ql}_A$. We assume that $\mathbf{l}_A$ is a positive vector.

Now we turn to Input-Output (IO) table (Yagi (forthcoming)). Let us denote the diagonal matrix of output of an IO table by $\mathbf{X}$, the matrix of domestic intermediate input by $\mathbf{Z}$, the vector of imports from Country I by $\mathbf{M}_M^I$, the vector of imports from the Rest of the World (ROW) by $\mathbf{M}_M^{ROW}$, the vector of compensation for employees by $\mathbf{W}_M$, the vector operating surplus by $\mathbf{\Pi}_M$, the vector of depreciation by $\mathbf{D}_M$, the vector of net taxes by $\mathbf{T}_M$. The data of $\mathbf{X}$, $\mathbf{Z}$, $\mathbf{M}_M^I$, $\mathbf{M}_M^{ROW}$, $\mathbf{W}_M$, $\mathbf{\Pi}_M$, $\mathbf{D}_M$ and $\mathbf{T}_M$ can be obtained from a non-competitive import type IO table and therefore the above matrixes and vectors are given exogenously. The vectors $\mathbf{M}_M^I$, $\mathbf{M}_M^{ROW}$, $\mathbf{W}_M$, $\mathbf{\Pi}_M$, $\mathbf{D}_M$ and $\mathbf{T}_M$ are measured in terms of money. The total profit Π and the total wage W are given as

$$\Pi=\mathbf{e}\mathbf{\Pi}_M \qquad W=\mathbf{e}\mathbf{W}_M \tag{5}$$

where $\mathbf{e}=[\mathbf{1}, \cdots, \mathbf{1}]$. The uniform rate of money wage is denoted by w_M, which is assumed to be equal throughout industries. If we denote the labour amount vector by $\mathbf{L}_A$, the labour amount vector can be obtained by

$$\mathbf{L}_A=\mathbf{W}/w_M \tag{6}$$

This means that the components of labour amount vector are measured in terms of labour commanded. By dividing $\mathbf{M}_M^I$, $\mathbf{M}_M^{ROW}$, $\mathbf{D}_M$ and $\mathbf{T}_M$ by the wage rate w_M, we can obtain the following vectors

$$\begin{aligned}&\mathbf{M}_w^I=\mathbf{M}_M^I/w_M,\ \mathbf{M}_w^{ROW}=\mathbf{M}_M^{ROW}/w_M\\&\mathbf{D}_w=\mathbf{D}_M/w_M,\ \mathbf{T}_w=\mathbf{T}_M/w_M\end{aligned} \tag{7}$$

By multiplying the inverse matrix of $\mathbf{X}$ to the vectors $\mathbf{Z}$, $\mathbf{L}_A$, $\mathbf{M}_w^I$, $\mathbf{M}_w^{ROW}$, $\mathbf{D}_w$, $\mathbf{T}_w$ from left-hand side, we can obtain the following input coefficient matrixes and the coefficient vectors.

$$\begin{aligned}&\mathbf{A}=\mathbf{X}^{-1}\mathbf{Z},\ \mathbf{l}_A=\mathbf{X}^{-1}\mathbf{L}_A,\\&\mathbf{m}_w^I=\mathbf{X}^{-1}\mathbf{M}_w^I,\ \mathbf{m}_w^{ROW}=\mathbf{X}^{-1}\mathbf{M}_w^{ROW}\\&\mathbf{d}_w=\mathbf{X}^{-1}\mathbf{D}_w,\ \mathbf{t}_w=\mathbf{X}^{-1}\mathbf{T}_w,\end{aligned} \tag{8}$$

If we denote the vector of output of IO table by $\mathbf{x}$, the actual total labour can be expressed by $L_A=\mathbf{x}\mathbf{l}_A$ and the total wage by $W=w_M\mathbf{x}\mathbf{l}_A$. In our model, we regard the amounts of imports, depreciation and net taxes as indirect wages. $w_M\mathbf{x}\mathbf{m}_w^I$ is paid for Country I, $w_M\mathbf{x}\mathbf{m}_w^{ROW}$ for ROW. $w_M\mathbf{x}\mathbf{d}_w$ is paid to the capitalists in order to compensate for capital depreciation, a portion of fixed capital which is accumulated in the past. $w_M T_w$ is paid to the government. The total amount of indirect wages can be represented as $\{w_M\mathbf{x}\mathbf{m}_w^I+w_M\mathbf{x}\mathbf{m}_w^{ROW}+w_M\mathbf{x}\mathbf{d}_w+w_M\mathbf{x}\mathbf{t}_w\}$.

In order to apply the standard system to the IO table, let us define the following ratio.

$$\phi=\frac{W+\Pi}{W+\Pi+w_M\mathbf{x}\mathbf{m}_w^I+w_M\mathbf{x}\mathbf{m}_w^{ROW}+w_M\mathbf{x}\mathbf{d}_w+w_M\mathbf{x}\mathbf{t}_w} \tag{9}$$

We consider that a portion of actual total labour, i.e. $\phi\mathbf{x}\mathbf{l}_A$, is used to produce the net output and domestic intermediate input and the other portion of total labour, i.e. $(1-\phi)L_A$, is used to produce the amount of indirect wages. We can define the standard labour as

$$\mathbf{q}\mathbf{l}_A=\phi\mathbf{x}\mathbf{l}_A \tag{10}$$

We can consider $\mathbf{q}$ as the (hypothetical) output vector of the standard system. We call $\mathbf{q}\mathbf{l}_A$ the standard labour.

Let us define the ratios as

$$\delta=\frac{\mathbf{q}\mathbf{d}_w}{\mathbf{q}\mathbf{l}_A},\ \mu^I=\frac{\mathbf{q}\mathbf{m}_w^I}{\mathbf{q}\mathbf{l}_A},\ \mu^{ROW}=\frac{\mathbf{q}\mathbf{m}_w^{ROW}}{\mathbf{q}\mathbf{l}_A},\ \tau=\frac{\mathbf{q}\mathbf{t}_w}{\mathbf{q}\mathbf{l}_A} \tag{11}$$

Then we define the following ratio

$$\psi=1+\mu^I+\mu^{ROW}+\delta+\tau \tag{12}$$

From these notations, we have

$$\begin{aligned}&\mathbf{q}\mathbf{l}_A+\mathbf{q}\mathbf{m}_w^I+\mathbf{q}\mathbf{m}_w^{ROW}+\mathbf{q}\mathbf{d}_w+\mathbf{q}\mathbf{t}_w\\ =&\mathbf{q}\mathbf{l}_A(1+\mu^I+\mu^{ROW}+\delta+\tau)\\ =&\phi\psi\mathbf{x}\mathbf{l}_A\end{aligned} \tag{13}$$

This is the relationship between the labour amount of the standard system and the total labour of the real world.

The relationship between the wage share and the profit share of the standard system can be represented as

$$\omega_v+\pi_v=1 \tag{14}$$

Then the standard net product (net product of the standard system) can be divided into two parts as

$$\mathbf{s}=\omega_v\mathbf{s}+\pi_v\mathbf{s}$$

The standard labour can be rewritten as

$$\mathbf{q}\mathbf{l}_A=\mathbf{q}[\mathbf{I}-(1+r_K)\mathbf{A}][\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{l}_A \tag{15}$$

The term $\mathbf{q}[\mathbf{I}-(1+r_K)\mathbf{A}]$ of the right member of this equation becomes equal to $\omega_v\mathbf{s}$, i.e.

$$\begin{aligned}\mathbf{q}[\mathbf{I}-(1+r_K)\mathbf{A}]&=\mathbf{q}-\mathbf{q}\mathbf{A}-r_K\mathbf{q}\mathbf{A}\\ &=\mathbf{s}-r_K\mathbf{q}\mathbf{A}\\ &=\omega_v\mathbf{s}\end{aligned} \tag{16}$$

Let us define a price vector as

$$\mathbf{p}_v=\omega_v[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{l}_A \tag{17}$$

Then, from (15)–(17), we have

$$\mathbf{ql}_A=\omega_v\mathbf{s}[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{l}_A=\mathbf{sp}_v \quad (18)$$

Like the price equation (17), we will define the following price equations

$$\mathbf{p}_m^I=\omega_v[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{m}_w^I \quad (19)$$
$$\mathbf{p}_m^{ROW}=\omega_v[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{m}_w^{ROW} \quad (20)$$
$$\mathbf{p}_d=\omega_v[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{d}_w \quad (21)$$
$$\mathbf{p}_t=\omega_v[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{t}_w \quad (22)$$

Then, like (18), we have

$$\mathbf{qm}_w^I=\mathbf{sp}_m^I \quad (23)$$
$$\mathbf{qm}_w^{ROW}=\mathbf{sp}_m^{ROW} \quad (24)$$
$$\mathbf{qd}_w=\mathbf{sp}_d \quad (25)$$
$$\mathbf{qt}_w=\mathbf{sp}_t \quad (26)$$

Therefore from (18) and (23)–(26), the equation (13) can be rewritten as

$$\begin{aligned}\psi\mathbf{ql}_A&=\phi\psi\mathbf{xl}_A\\&=\mathbf{ql}_A+\mathbf{qm}_w^I+\mathbf{qm}_w^{ROW}+\mathbf{qd}_w+\mathbf{qt}_w\\&=\mathbf{sp}_v+\mathbf{sp}_m^I+\mathbf{sp}_m^{ROW}+\mathbf{sp}_d+\mathbf{sp}_t\\&=\mathbf{s}(\mathbf{p}_v+\mathbf{p}_m^I+\mathbf{p}_m^{ROW}+\mathbf{p}_d+\mathbf{p}_t)\end{aligned} \quad (27)$$

From the right member of this equation, we can define the following price system.

$$\mathbf{p}_C=\mathbf{p}_v+\mathbf{p}_m^I+\mathbf{p}_m^{ROW}+\mathbf{p}_d+\mathbf{p}_t \quad (28)$$

This equation will express the cost structure of industries when we neglect the profits. The equation (27) can be rewritten as

$$\mathbf{xl}_A=\frac{1}{\phi\psi}\mathbf{sp}_C \quad (29)$$

The value of $\mathbf{sp}_C$ is connected to the total labour of the real-world. This is the important base of our analysis.

3. The IO Price System and Distribution

The Sraffa system has a property that the wage curve, or the relationship between the wage rate (or the wage share) and the rate of profit, becomes linear (Kurz and Salvadori (1995), Pasinetti (1977), Sraffa (1960), Yagi (2000) (2007) (2012)). The rate of profits will take real numbers ranging from 0 to R $(0\leq r_K<R)$. Let us denote the rate of profits by

r_K, which is assumed to be uniform throughout industries. A uniform rate of post factum wage is represented by w_A. When a set of data $(\mathbf{A}, \mathbf{l}_A, \mathbf{q})$ is given, for $0 \leq r_K < R$, we have the following evaluation system

$$\mathbf{s}\mathbf{p}_A = v_L \mathbf{q}\mathbf{l}_A \tag{30}$$

$$\mathbf{p}_A = (1+r_K)\mathbf{A}\mathbf{p}_A + w_A \mathbf{l}_A \tag{31}$$

where v_L is the value of labour. In the system of (30)(31), there are $(n+1)$ independent equations and $(n+3)$ unknowns $(v_L, \mathbf{p}_A, w_A, r_K)$. If the rate of profits or the wage rate is given exogenously and the condition of standard is chosen, the price system will become determinate. From the price system (31), we can obtain

$$\left(1-\frac{r_K}{R}\right)\mathbf{s}\mathbf{p}_A = w_A \mathbf{q}\mathbf{l}_A \tag{32}$$

From this, we have

$$\mathbf{s}\mathbf{p}_A = \mathbf{q}\mathbf{l}_A \Longleftrightarrow r_K = R(1-w_A) \tag{33}$$

Therefore, in the system of (30)(31), the following proposition will hold

$$v_L = 1 \Longleftrightarrow r_K = R(1-w_A) \tag{34}$$

Under the condition of $v_L=1$, the wage measured in terms of the standard labour, or the wage share, can be represented as

$$\omega_v = \frac{w_A}{v_L} \tag{35}$$

And then we have the following price system

$$\mathbf{p}_v = \frac{\mathbf{p}_A}{v_L} = (1-r_K/R)[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{l}_A \tag{36}$$

The prices of this equation are subject to the changes in the rate of profits and the changes in the technological condition of production. They are measured in terms of the standard labour.

We will define the rate of profit to the wage as (Garegnani (1984) (1987), Yagi (2016))

$$r_w = \frac{1-\omega_v}{\omega_v} \tag{37}$$

We call this the surplus rate. From this we have

$$\omega_v = \frac{1}{1+r_w} \tag{38}$$

The relationship between the ordinary notion of rate of profits and the rate of profit to the wage as capital (surplus rate) can be represented as

$$r_K = r_w \omega_v R \tag{39}$$

The introduction of a new notion of surplus rate r_w will be found useful to determine the distributive variables in our model.

Now let us turn to the IO model (Yagi (forthcoming)). In the real world, both the wage rate and the rate of profits will take different values in different sectors. However, we assume that the wage rate is equal throughout industries. We consider that the average wage rate corresponds to this uniform wage rate. We also assume that the rate of profits is equal throughout industries. These are important assumptions of our analysis.

In order to consider the profits to the fixed capital. We will introduce the vector of sectoral fixed capital. We denote the column vector of fixed capital measured in terms of money by $\mathbf{K}_M$. If we divide it by the average wage rate, we have

$$\mathbf{K}_w = \mathbf{K}_M / w_M \tag{40}$$

Multiplying $\mathbf{K}_w$ by the inverse matrix of output from the left-hand side, we have

$$\mathbf{k}_w = \mathbf{X}^{-1}\mathbf{K}_w \tag{41}$$

If we denote the price vector corresponding to an IO table by $\mathbf{p}_M$, we have the following IO price equation

$$\begin{aligned}\mathbf{p}_M = &(1+r_K)\mathbf{A}\mathbf{p}_M + w_M \mathbf{l}_A \\ &+(1+r_K)(w_M \mathbf{m}_w^I + w_M \mathbf{m}_w^{ROW}) \\ &+ r_K w_M \mathbf{k}_w + w_M \mathbf{d}_w + w_M \mathbf{t}_w\end{aligned} \tag{42}$$

This price system can be rewritten as

$$\begin{aligned}(\mathbf{I}-\mathbf{A})\mathbf{p}_M - \frac{r_K}{R} R\mathbf{A}\mathbf{p}_M = w_M \mathbf{l}_A + (1+r_K)(w_M \mathbf{q}\mathbf{m}_w^I + w_M \mathbf{q}\mathbf{m}_w^{ROW}) \\ + r_K w_M \mathbf{k}_w + w_M \mathbf{d}_w + w_M \mathbf{t}_w\end{aligned} \tag{43}$$

By multiplying this equation by the eigenvector $\mathbf{q}$ from the left-hand side, we have

$$\begin{aligned}\mathbf{q}(\mathbf{I}-\mathbf{A})\mathbf{p}_M - \frac{r_K}{R} R\mathbf{q}\mathbf{A}\mathbf{p}_M = w_M \mathbf{q}\mathbf{l}_A + (1+r_K)(w_M \mathbf{m}_w^I + w_M \mathbf{m}_w^{ROW}) \\ + r_K w_M \mathbf{q}\mathbf{k}_w + w_M \mathbf{q}\mathbf{d}_w + w_M \mathbf{q}\mathbf{t}_w\end{aligned} \tag{44}$$

Then, we obtain the following equation:

$$\left(1-\frac{r_K}{R}\right)\mathbf{s}\mathbf{p}_M = w_M \phi \mathbf{q}\mathbf{l}_A \left\{1+\frac{r_K(\mathbf{q}\mathbf{k}_w + \mathbf{q}\mathbf{m}_w^I + \mathbf{q}\mathbf{m}_w^{ROW})}{\phi \mathbf{q}\mathbf{l}_A}\right\} \tag{45}$$

If we denote the ratio between the wage and the standard income, or the wage share in the standard system by ω_S, it can be defined as

$$\omega_S=\frac{w_M\psi\mathbf{q}\mathbf{l}_A}{\mathbf{s}\mathbf{p}_M} \tag{46}$$

This means that ω_S is the real wage measured in terms of the standard net product. Then, from (46), the equation (45) can be written as

$$\begin{aligned}\omega_v&=\omega_S\left\{1+\frac{r_K(\mathbf{q}\mathbf{k}_w+\mathbf{q}\mathbf{m}_w^I+\mathbf{q}\mathbf{m}_w^{ROW})}{\psi\mathbf{q}\mathbf{l}_A}\right\}\\&=\omega_S\left\{1+\frac{r_K}{\omega_v R}\times\frac{\omega_v}{\omega_S}\times\frac{\omega_S R(\mathbf{q}\mathbf{k}_w+\mathbf{q}\mathbf{m}_w^I+\mathbf{q}\mathbf{m}_w^{ROW})}{\psi\mathbf{q}\mathbf{l}_A}\right\}\end{aligned} \tag{47}$$

Now let us define the following ratios.

$$\kappa_Z=\frac{R\mathbf{q}\mathbf{k}_w}{\psi\mathbf{q}\mathbf{l}_A}\qquad \mu_Z^I=\frac{R\mathbf{q}\mathbf{m}_w^I}{\psi\mathbf{q}\mathbf{l}_A},\ \mu_Z^{ROW}=\frac{R\mathbf{q}\mathbf{m}_w^{ROW}}{\psi\mathbf{q}\mathbf{l}_A} \tag{48}$$

We can rewrite the equation (47) as

$$\omega_v=\omega_S\left\{1+r_w\frac{\omega_v}{\omega_S}(\omega_S\kappa_Z+\omega_S\mu_Z^I+\omega_S\mu_Z^{ROW})\right\} \tag{49}$$

From this, we have

$$\omega_v=\frac{\omega_S}{1-r_w(\omega_S\kappa_Z+\omega_S\mu_Z^I+\omega_S\mu_Z^{ROW})} \tag{50}$$

From (38)(50), we have

$$r_w=\frac{1-\omega_S}{\omega_S+\omega_S\kappa_Z+\omega_S\mu_Z^I+\omega_S\mu_Z^{ROW}} \tag{51}$$

From (50)(51), we can obtain the ordinary rate of profits of (39). The rate of profits will be given if ω_S, κ_Z, μ_Z^I, μ_Z^{ROW} is given. ω_S, μ_Z^I, μ_Z^{ROW} are given exogenously by IO table and the average wage rate. To calculate κ_Z needs the sectoral data of fixed capital. By using the rate of profits which is given exogenously, from (39)(46)(50)(51), we can rewrite the equations (19)–(22) and define $\mathbf{p}_k$ as

$$\mathbf{p}_m^I=(1-r_K/R)[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{m}_w^I \tag{52}$$

$$\mathbf{p}_m^{ROW}=(1-r_K/R)[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{m}_w^{ROW} \tag{53}$$

$$\mathbf{p}_d=(1-r_K/R)[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{d}_w \tag{54}$$

$$\mathbf{p}_t=(1-r_K/R)[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{t}_w \tag{55}$$

$$\mathbf{p}_k=(1-r_K/R)[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{k}_w \tag{56}$$

Then we obtain the following price equation system (Yagi (forthcoming)).

$$\mathbf{p}_L=\mathbf{p}_v+\mathbf{p}_m^I+\mathbf{p}_m^{ROW}+\mathbf{p}_d+\mathbf{p}_t+r_K(\mathbf{p}_k+\mathbf{p}_m^I+\mathbf{p}_m^{ROW}) \tag{57}$$

This equation expresses that the price vector measured in terms of labour ($\mathbf{p}_L$) is decomposed into the direct and indirect contributions of domestic labour, imports from Country *I*, imports from *ROW*, capital depreciation, and net taxes and profits to inputs.

4. A Simplyfied Method for Measuring Cost Structure of Industries

It may be usual that there is no sectoral data of fixed capital. In order to calculate the price equations of (52)-(56) in the case of no sectoral data of fixed capital, let us introduce an ad hoc assumption to our analysis. One of ad hoc assumptions, or a proxy, for calculation of κ_Z would be[1)]

$$\kappa_Z=\frac{K_M}{W_M} \tag{58}$$

Where K_M is total amount of fixed capital which is measured in terms of money.

To summarize, let us show our model of cost structure analysis as follows

$$\begin{aligned}
&\omega_S=\frac{w_M\phi\mathbf{q}\mathbf{l}_A}{\mathbf{s}\mathbf{p}_M}\\
&\kappa_Z=\frac{K_M}{W_M}\quad \mu_Z^I=\frac{R\mathbf{q}\mathbf{m}_w^I}{\phi\mathbf{q}\mathbf{l}_A},\ \mu_Z^{ROW}=\frac{R\mathbf{q}\mathbf{m}_w^{ROW}}{\phi\mathbf{q}\mathbf{l}_A}\\
&\omega_v=\frac{\omega_S}{1-r_w(\omega_S\kappa_Z+\omega_S\mu_Z^I+\omega_S\mu_Z^{ROW})}\\
&r_w=\frac{1-\omega_S}{\omega_S+\omega_S\kappa_Z+\omega_S\mu_Z^I+\omega_S\mu_Z^{ROW}}\\
&r_K=r_w\omega_vR\\
&\mathbf{p}_m^I=(1-r_K/R)[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{m}_w^I\\
&\mathbf{p}_m^{ROW}=(1-r_K/R)[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{m}_w^{ROW}\\
&\mathbf{p}_d=(1-r_K/R)[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{d}_w\\
&\mathbf{p}_t=(1-r_K/R)[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{t}_w\\
&\mathbf{p}_k=(1-r_K/R)[\mathbf{I}-(1+r_K)\mathbf{A}]^{-1}\mathbf{k}_w\\
&\mathbf{p}_C=\mathbf{p}_v+\mathbf{p}_m^I+\mathbf{p}_m^{ROW}+\mathbf{p}_d+\mathbf{p}_t
\end{aligned} \tag{59}$$

In our simplified model to calculate cost structure, we neglected the profit parts. As is seen in (29), our price equations can be connected to the actual total labour. In addition to this, our analysis derives a consistent relationship among distributive variables and price vectors, only based on a few data as Input-Output table, the average wage rate (or the actual total labour), and total amount of fixed capital. The results of calculation should satisfy the following equations.

$$\omega_S + r_w\omega_S + r_w\omega_S(\kappa_Z + \mu_Z^I + \mu_Z^{ROW}) = 1 \tag{60}$$

$$\mathbf{sp}_v/\mathbf{ql}_A = 1 \tag{61}$$

$$\frac{1}{R} = \frac{\mathbf{zp}_v}{\mathbf{sp}_v} \tag{62}$$

$$r_w\omega_v R = R(1-\omega_v) \tag{63}$$

$$\frac{(1-\omega_S)}{r_K} = \frac{1}{R} + \frac{\omega_S}{R}(\kappa_Z + \mu_Z^I + \mu_Z^{ROW}) \tag{64}$$

We can check the calculation result of our analysis by calculating both members of the equations (60)-(64).

5. Conclusion

We have shown a simplified method to calculate the rate of profits of ordinary meaning and the price vectors to measure the costs attributed directly and indirectly to the domestic labour inputs, the imports, the depreciation and the taxes respectively. A simplified model was shown in (59). We introduced the assumptions of an uniform wage rate and an uniform rate of profits. We also introduce an ad hoc assumption of (58). However, in our model, the price vectors to measure the cost directly and indirectly entering into the commodity prices and distributive variables such as the rate of profits of ordinary meaning can be derived by a few data. We should like to stress that such a few data as a non competitive import type input-output table, an average wage rate (or the total labour), and the total amount of fixed capital enables us to grasp the cost structure of industries. These are great merits of our model, even if we adopt an ad hoc assumption on fixed capital.

Note

1) Though there are some other ideas to give the value of κ_Z, in this paper, we showed the assumption of (58) because the values of κ_Z calculated by using the sectoral data of fixed capital of Japan and 2000 and 2005 Japan-US International Input-Output Tables are close to the calculation results of (58) with the same data. Since the value of κ_Z is important to our analysis, the estimation of κ_Z should be tested through empirical studies.

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