

論 説

Unsustainable Bond-Financed Deficits in a Monetary Economy*

Noritaka Kudoh

1 Introduction

Many economists have recommended that some fiscal restraint is required for price stability (Sargent, 1999; Sargent and Wallace, 1981; von Thadden, 2004; Woodford, 2001; Bhattacharya and Kudoh, 2002). This paper studies the implications of the sustainability of bond-financed deficits for the conduct of monetary policy in a dynamic model with three assets, money, bonds, and capital. To avoid the Chang-Hamberg-Hirata (1983) critique, this paper builds a model of money as the most liquid asset. It is shown that, if the government rolls over the debt to finance its deficits, then there are infinitely many divergent equilibria, along which the outstanding debt increases, causing a massive crowding out of productive capital. Delayed fiscal reform results in lower output, both at the start of reform and in the long run.

Two new features of this paper are worth emphasizing. One is that the central bank's balance sheet is separated from the government's budget constraint. An important implication of this policy regime is that currency seigniorage does not contribute to the revenue of the government, and government bonds are held entirely by households. The other new contribution of this paper is that it considers the implications of unsustainable bond-financed deficits for disinflation and deflation. The literature has been concerned with inflation as a result of the increasing need for revenue. If currency seigniorage is not part of the fiscal authority's revenue, there is no direct link between fiscal and monetary policies. However, when bond-financed deficits are unsustainable, the economy temporarily experiences a divergent nonstationary equilibrium, along which the real debt outstanding increases over time, causing a massive crowding out.

The crowding out effect of public debt — a permanent increase in government-issued debt reduces the stock of productive capital and thereby reduces output — has long been

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recognized by the profession (Diamond, 1965; Modigliani, 1961; Phelps and Shell, 1969). Chalk (2000) argued that although bond-financed deficits are known to be sustainable if the real interest rate is less than the growth rate of the economy, there is a maximum sustainable level of the deficit. This paper sheds some new light on the crowding out channel of a large and increasing public debt by extending the models of Diamond (1965) and Chalk (2000) to a monetary economy. The key intuition obtained in this paper is that when the bond-financed deficit is unsustainable, the economy temporarily experiences a divergent nonstationary equilibrium. On such a path, the real debt outstanding increases over time, causing the real interest rate to increase. This places a serious restriction on the conduct of monetary policy. An increase in the real interest rate must accompany either an increase in the nominal interest rate or a decrease in the rate of inflation. In other words, the central bank will face a tradeoff between a higher nominal interest rate and a lower inflation rate.

The analytical framework employed in this paper is a monetary growth model with overlapping generations (Aiyagari and Gertler, 1985; Bhattacharya and Kudoh, 2002; Schreft and Smith, 1997; von Thadden, 2002, 2004). There are three means of saving: productive capital, government bonds, and return-dominated fiat money. Money in this economy is valued because it is the only liquid asset that allows agents to consume early (Antonio and Martins, 1980; Diamond and Dybvig, 1983). The key feature of the model is the way money is injected into the economy. The existing monetary growth models with government bonds assume that money is supplied via open market operations. Separating the central bank's balance sheet from the government's budget constraint implies that open market operations are ruled out. This paper alternatively assumes that money is injected into the economy via 'helicopter drops', which is typically employed in monetary growth models without government bonds.

This model reveals that the real interest rate increases over time on a divergent path because the government rolls over the debt aggressively to finance its (unsustainable) deficit. Two interesting behaviors of variables are observed. One is that investment and output decline as a result of massive crowding out. The other is that the economy will be deflationary when the central bank commits to a (low) nominal interest rate. The conventional view regarding unsustainable bond-financed deficits is that they are inflationary because the central bank is assumed to generate large inflation tax revenue when the government's solvency is at stake. If the central bank's balance sheet is not part of the government's revenue, then the sustainability of bond-financed deficits does not influence the rate of inflation directly. Interestingly, however, the central bank's commitment to a (low) nominal interest rate has a serious consequence. In order to maintain the nominal interest rate (or the bond price), the central bank needs to inject more money into the economy via helicopter drops. Because money is a free good (Correia and Teles, 1999), this

helicopter money will create an income effect, through which the real demand for money, as well as the demand for bonds, will expand. This increase in the real money demand implies a greater value of money. This is deflationary.

When the bond-financed deficit is unsustainable, a fiscal reform must take place eventually. This paper characterizes the equilibria after a tax-based reform in which the tax rate is endogenous, while the debt-GDP ratio is held constant. The capital-labor ratio in the last period of the old regime determines the initial condition for the new regime. Thus, a delay in reform results in a low initial output. The analysis also establishes that a larger debt-GDP ratio after the fiscal reform reduces the long-run level of output. Delayed fiscal reform results in lower output, both at the start and in the long run.

2 The Model

2.1 Environment

Consider an economy consisting of an infinite sequence of two-period-lived overlapping generations, an initial old generation, and an infinitely-lived government. Let $t=1, 2, \dots$ index time. At each date t , a new generation is born. The population is normalized to one. Each agent is endowed with one unit of labor when young and is retired when old. In addition, the initial old agents are endowed with $K_1 > 0$ units of capital and M_0 units of fiat money.

There is a single final good produced using the Cobb-Douglas production function $Y_t = AK_t^\alpha [E_t N_t]^{1-\alpha}$ with $A \geq 1$ and $\alpha \in (0, 1)$, where K_t denotes the capital input, N_t denotes the labor input, and E_t denotes the labor-augmented technology, which is assumed to grow exogenously. The gross rate of technical progress is $n \equiv E_{t+1}/E_t$. Let $k_t \equiv K_t/E_t N_t$ denote the effective capital-labor ratio. Then, the intensive production function is $f(k_t) = Ak_t^\alpha$. It is easy to see that $f(0) = 0$, $f' > 0 > f''$, and the Inada conditions hold. The final good can either be consumed in the period it is produced, or stored to yield capital in the next period. For expositional reasons, capital is assumed to depreciate 100% between periods.

2.2 Factor Markets

Factor markets are perfectly competitive. Thus, factors of production receive their marginal products. Let r_t and w_t denote the rental rate of capital and the real wage rate. Each young agent supplies his or her labor endowment inelastically in the labor market. Profit maximization requires $r_t = f'(k_t)$ and $w_t = f(k_t)E_t - k_t f'(k_t)E_t$. For convenience, let $w(k) \equiv f(k) - kf'(k)$ so that $w_t = w(k_t)E_t$. Note that $w'(k) = -kf''(k) > 0$. For the Cobb-

Douglas specification, $r_t = \alpha A k_t^{\alpha-1}$ and $w_t = (1-\alpha) A k_t^{\alpha} E_t$.

2.3 Government

Since Sargent and Wallace's (1981) contribution, a consolidated government budget constraint has become the building block of monetary policy analysis. However, such budget constraints with fiscal deficits imply that the monetary authority must raise revenue by printing money in order to maintain the solvency of the government. The implicit assumption of a weak or subordinate central bank appears inconsistent with the recent state of central banking observed in developed economies.

In order to capture a tough, or independent, central bank, this paper follows Kudoh (2007) to adopt a model which separates the central bank's budget from the fiscal authority's budget constraint. The two budget constraints are separated when (i) the fiscal authority does not receive any seigniorage revenue from the central bank; and (ii) the central bank never purchases government bonds. The requirement (i) is insufficient for separating the monetary authority's budget from that of the fiscal authority because if money is supplied via open market purchases of government bonds, then the two budget constraints are connected and only the consolidated budget constraint matters. Since open market operations are ruled out, government bonds are held entirely by households.

Let G_t denote government spending, T_t denote the amount of tax revenue, $I_t \geq 1$ denote the gross nominal interest rate, and B_t denote the amount of one-period government bonds issued in period t . The fiscal authority's budget constraint under this regime is

$$G_t + I_t B_{t-1} = T_t + B_t \quad (1)$$

for $t \geq 2$ and $G_1 = T_1 + B_1$ for $t=1$. I assume that the government simply consumes G_t and that it does not affect the utility of any generation or the production process at any date. Divide (1) by $p_t E_t$, and use the Fisher equation, $R_{t+1} \equiv I_{t+1} p_t / p_{t+1}$, to obtain

$$g_t = \tau_t + b_t - \frac{R_t}{n} b_{t-1}, \quad (2)$$

where $g_t = G_t / p_t E_t$, $\tau_t = T_t / p_t E_t$, and $b_t = B_t / p_t E_t$. Since bonds and capital are competing financial assets in this economy, the non-arbitrage condition requires the rates of return on these assets be the same in equilibrium. Thus, $R_t = f'(k_t)$.

In order to separate the budgets, I assume money to be supplied via "helicopter drops". The quantity of money injection at time t is denoted by H_t . Thus,

$$H_t = M_t - M_{t-1} \quad (3)$$

for $t \geq 1$, where $M_0 > 0$ because the initial old is endowed with fiat money. From the household's perspective, H_t is a subsidy from the government. Thus, equation (3) implies

that the monetary authority returns the revenue from printing money to the consumers. Divide (3) by $p_t E_t$ to obtain

$$h_t = m_t - \frac{p_{t-1}}{p_t n} m_{t-1}, \quad (4)$$

where $h_t = H_t / p_t E_t$ and $m_t = M_t / p_t E_t$.

2.4 Consumers

In order to focus on agents' portfolio choice, I assume that all individuals save all their income. As a means of saving, agents may hold money and non-monetary assets. In order to motivate the demand for money as a liquid asset, divide each period into two subperiods. The non-monetary assets, denoted by Z_t , are assumed to yield a gross nominal return of $I_{t+1} \geq 1$ in the next period. However, the non-monetary assets cannot be liquidated until the second subperiod. Money, whose nominal interest rate is zero, is assumed to be the only liquid asset in this economy. Thus, the only distinction between money and non-monetary assets is that non-monetary assets must be held a little longer (Antonio and Martins, 1980). This liquidity structure helps resolve the Chang-Hamberg-Hirata (1983) critique on the traditional money demand theory.

Suppose that each individual wishes to consume in both subperiods. Let c_{1t} and c_{2t} denote the consumption of the final good in the first and second subperiods by an old agent born at date t . The consumer's objective function is $\phi u(c_{1t}) + (1-\phi)u(c_{2t})$, where ϕ captures the relative weight of utility between the two subperiods. Throughout, I use the following specification: $u(c) = [1-\rho]^{-1} c^{1-\rho}$ with $\rho \neq 1$ and $\rho > 0$. Since the individual cannot liquidate non-monetary assets in the first subperiod, the agent faces a cash-in-advance constraint:

$$p_{t+1} c_{1t} \leq M_t. \quad (5)$$

The individual's budget constraint when young is

$$M_t + Z_t = p_t w_t + H_t - T_t, \quad (6)$$

where the consumer takes H_t and T_t as given. Similarly, the budget constraint when old is

$$p_{t+1} c_{1t} + p_{t+1} c_{2t} = M_t + I_{t+1} Z_t. \quad (7)$$

The cash-in-advance constraint binds as long as the nominal interest rate is positive (i. e., $I_t > 1$). Under binding cash-in-advance constraint, (7) implies $p_{t+1} c_{2t} = I_{t+1} Z_t = I_{t+1} [p_t w_t + H_t - T_t - M_t]$. Thus, a young individual's maximization problem is:

$$\max_{M_t} \left\{ \phi \frac{[M_t / p_{t+1}]^{1-\rho}}{1-\rho} + (1-\phi) \frac{[(p_t w_t + H_t - T_t - M_t) I_{t+1} / p_{t+1}]^{1-\rho}}{1-\rho} \right\}.$$

Use the first-order condition to obtain the money demand function as

$$M_t = \gamma(I_{t+1})(p_t w_t + H_t - T_t), \quad (8)$$

$$\gamma(I_{t+1}) \equiv \left[1 + \left(\frac{1-\phi}{\phi} \right)^{1/\rho} I_{t+1}^{1/\rho-1} \right]^{-1}. \quad (9)$$

It is easy to establish that (a) $\gamma'(I) < 0$ holds for $\rho \in (0, 1)$; (b) $\lim_{I \rightarrow \infty} \gamma(I) = 0$ for $\rho \in (0, 1)$; and (c) $\lim_{I \rightarrow 1} \gamma(I) = [1 + ((1-\phi)/\phi)^{1/\rho}]^{-1}$. The value of ρ captures the strength of the income effect of a change in I . Throughout, I focus on the case in which $\rho \in (0, 1)$ so that the income effect is relatively weak. There are several other environments which lead to the same money demand function. A leading example is Schreft and Smith (1997), who consider a model with spatially separated markets.

Divide (8) by $p_t E_t$ to obtain

$$m_t = \gamma(I) [w(k_t) + h_t - \tau_t]. \quad (10)$$

The asset market equilibrium requires $Z_t = B_t + p_t K_{t+1}$. Divide it by $p_t E_t$ to obtain

$$b_t + n k_{t+1} = [1 - \gamma(I)] [w(k_t) + h_t - \tau_t]. \quad (11)$$

These two equations imply $b_t + n k_{t+1} = m_t / \Gamma(I)$, where $\Gamma(I) \equiv \gamma(I) [1 - \gamma(I)]^{-1}$.

3 Equilibria under Permanent Deficits

3.1 Characterization

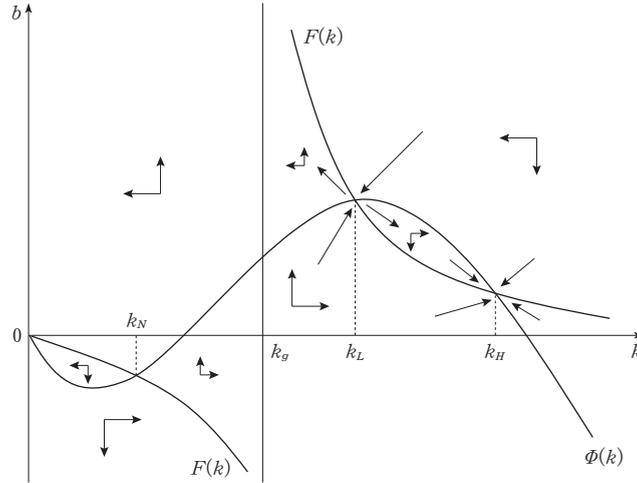
A monetary equilibrium is a set of sequences for real allocations $\{m_t, b_t, k_t\}$ and relative prices $\{R_t, \Pi_t\}$ and initial conditions $M_0 \geq 0, B_0 \geq 0$ such that (a) each generation maximizes utility; (b) asset market clears; (c) factor markets clear; (d) the fiscal authority's flow budget constraint (2) is satisfied for $t \geq 2$ and $g_1 = \tau_1 + b_1$ for $t = 1$; (e) money injection satisfies (4); (f) fiscal policy specifies $g_t = g$ and $\tau_t = \tau$; and (g) monetary policy specifies $I_t = I$. The Fisher equation implies $I = \Pi_t f'(k_t)$, from which it is easy to verify that $\Pi_t f''(k_t) dk_t + f'(k_t) d\Pi_t = 0$. Thus, in any equilibrium, k_t and Π_t are positively related under nominal interest rate pegging.

To simplify the analysis, in what follows I let $\tau = 0$. Then the fiscal authority's budget constraint becomes

$$b_t = \frac{f'(k_t)}{n} b_{t-1} + g. \quad (12)$$

Thus, the real debt in the next period is influenced by the current outstanding debt and

Figure 1: Phase diagram



the real interest rate on the debt. It is important to notice that the government is running a debt Ponzi game — it issues bonds each period in order to finance the deficit and the interest obligation on the outstanding debt. As is well known, a debt Ponzi game is sustainable in a deterministic environment if and only if the real interest rate is less than the growth rate of the economy. This suggests that the government can roll over the debt forever as long as $f'(k_t) < n$.

The evolution of the capital-labor ratio is given by

$$k_{t+1} = \frac{f(k_t) - g}{n} - \mu(I) \frac{f'(k_t)}{n} \left(k_t + \frac{b_{t-1}}{n} \right), \tag{13}$$

where $\mu(I) \equiv 1 + \gamma(I) [1 - \gamma(I)]^{-1} I^{-1}$ and $\mu'(I) < 0$. Difference equations (12) and (13) jointly determine the paths for b_{t-1} and k_t , given the initial conditions, b_0 and k_1 .

From (12) and (13), a steady-state equilibrium is characterized by a pair of b and k that satisfy

$$b = \frac{g}{1 - f'(k)/n} \equiv F(k), \tag{14}$$

$$b = \frac{f(k) - g - nk}{\mu(I) f'(k)/n} - nk \equiv \Phi(k). \tag{15}$$

Let k_g solve $f'(k) = n$. For $k > k_g$, the economy is dynamically inefficient, so the government can run a debt Ponzi game. Function F slopes down for all k and is positive for $k > k_g$. Function Φ is S-shaped. Figure 1 depicts typical configurations of functions F and Φ . The figure suggests that there are four steady-state equilibria to consider (Azariadis, 1993; de la Croix and Michel, 2002). One is the trivial one at $(0, 0)$. Another one, labeled $k = k_N$, satisfies $b < 0$. This steady state is quantitatively unimportant because k_N is negligibly

small. There are at most two steady-state equilibria with $b > 0$. For these steady states to exist, the productivity of the economy must be sufficiently large and the deficit must be sufficiently small.

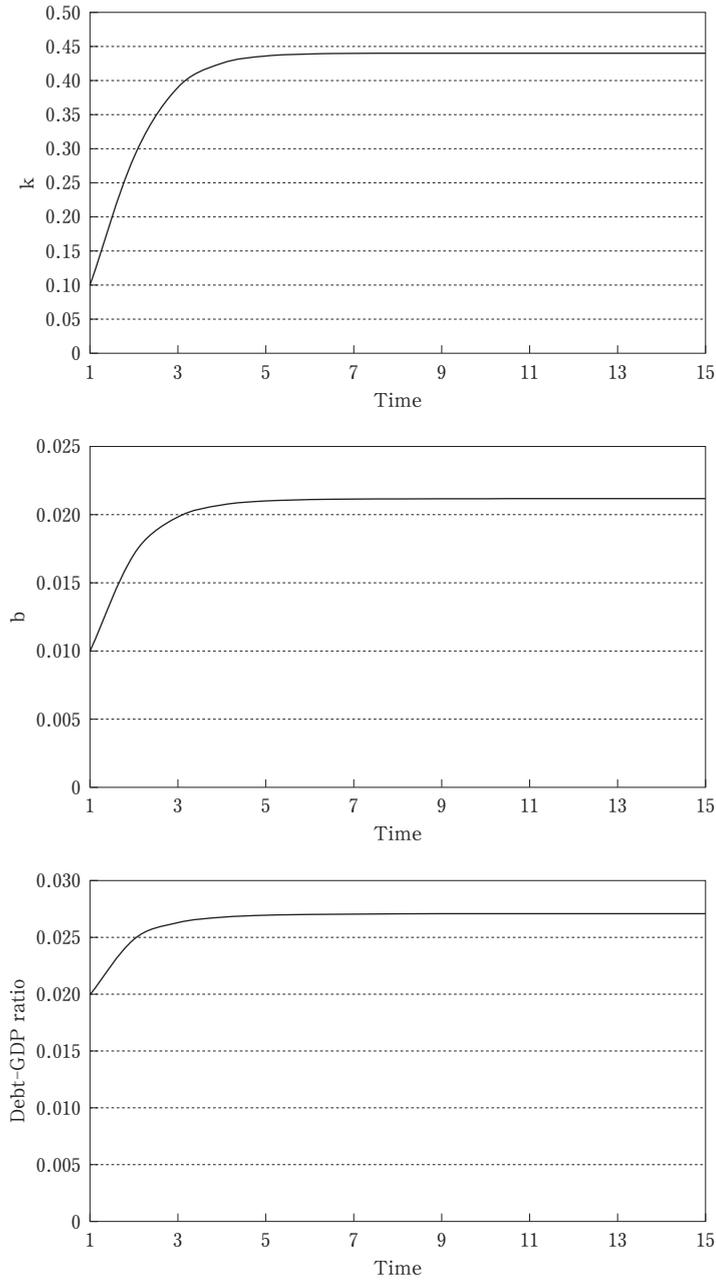
3.2 Dynamic Properties

Consider the stability of each steady state. Subtract b_{t-1} from (12) to obtain $b_t - b_{t-1} = [f'(k_t)/n - 1]b_{t-1} + g$. Thus, $b_t > b_{t-1} \Leftrightarrow b > F(k)$ for $k < k_g$ and $b_t > b_{t-1} \Leftrightarrow b < F(k)$ for $k > k_g$. Subtract k_t from (13) to obtain $k_{t+1} - k_t = [f(k_t) - g]/n - k_t - \mu(I)[k_t + b_{t-1}/n]f'(k_t)/n$, from which it is easy to verify that $k_{t+1} > k_t \Leftrightarrow b < \Phi(k)$. Figure 1 depicts a phase diagram of the model. It is easy to verify that the low- k steady state (k_L) is a saddle while the high- k steady state (k_H) is a sink. The phase diagram suggests that the trivial steady state at $(0, 0)$ and $k = k_N$ are both unstable; for sufficiently small values of k , the outstanding debt increases until the economy violates the equilibrium conditions. Thus, $k = k_H$ is the only stable steady state under this policy regime.

According to the textbook macroeconomic dynamics, the low- k steady-state, which is a saddle, is said to be *locally* determinate, because there is only one convergent path to this steady state. If all other paths were ruled out, then the steady state would be stable and unique. This economy, however, never approaches the low- k steady state unless it happens to be on the saddle path initially. This is because the high- k steady state is a sink: the economy starting at any initial condition below the saddle path approaches the high- k steady state.

In order to understand the working of the model, I present a numerical example which illustrates how the economy approaches the long-run equilibrium from an initial condition. First, compute the two steady states using (14) and (15). I choose the parameter values as $A=1$, $\alpha=0.3$, $\phi=0.45$, $\rho=0.2$, and $n=1.01$. For policy parameters, I choose $g=0.01$ and $I=1.02$. Then the steady-state equilibria are $k_L=0.20$ and $k_H=0.44$. The associated levels of b are 0.02 and 0.13. Equilibrium for $t \geq 2$ is fully described by (12) and (13). Since $b_0=0$, at date $t=1$, $k_2 = [f(k_1) - g]/n - \mu(I)f'(k_1)k_1/n$, from (13), and $b_1 = g$ from (12). Thus, once the initial capital-labor ratio is given, the equilibrium sequences of all endogenous variables are determined. It is important to note here that the amount of fiscal deficit at the beginning of the world determines b_1 . Figure 2 computes the transition path to the stable steady state, starting from $k_1=0.1$. As is evident, the economy approaches $k=0.44$, the high- k steady state, although it started below the low- k steady state. This verifies that the low- k steady state, although a saddle, is indeed unstable. The economy approaches the stable steady state without any jump in variables because there are infinitely many paths leading to the steady state.

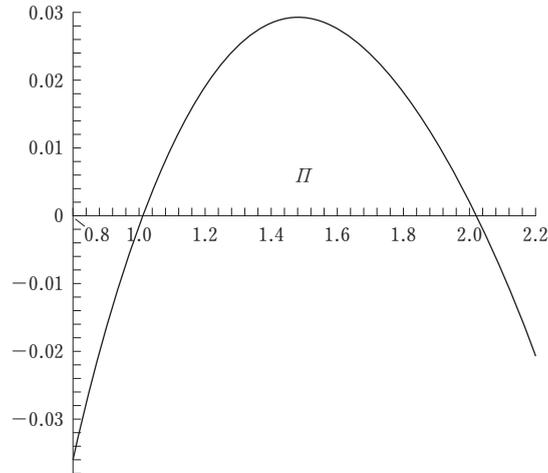
Figure 2: Transition to the stable steady state



3.3 Maximum Sustainable Deficit

It is illustrative to investigate how the steady state revenue of the government is influenced by the rate of inflation in this economy. Focus on a steady state. Expressions (10) and (11) imply $b+nk=m/\Gamma(I)$. Eliminate h from (4) and (10) to obtain $m = \gamma(I)w(k)[1-\gamma(I)(1-1/\Pi n)]^{-1}$. Substitute this into $b+nk=m/\Gamma(I)$ to obtain $w(k)[b+nk]^{-1} = 1+\Gamma(I)/n\Pi$. The Fisher equation, $I=\Pi f'(k)$, implicitly defines $k=\lambda(\Pi)$.

Figure 3: Revenue from Bonds



Solve $w(k)[b+nk]^{-1}=1+\Gamma(I)=n/\Pi$, for b as $b=[1+\Gamma(I)=n/\Pi]^{-1}w(\lambda(\Pi))-n\lambda(\Pi)\equiv\Psi(\Pi)$. Thus, fiscal authority's budget constraint is written in terms of Π as

$$g=\left(1-\frac{R}{n}\right)b=\left(1-\frac{I}{n\Pi}\right)\Psi(\Pi)\equiv L(\Pi). \quad (16)$$

The right-hand-side of equation (16), $L(\Pi)$, is referred to as a Laffer curve, which maps the rate of inflation into the total government net revenue. A Laffer curve tells us mainly two things. One is the maximum possible revenue. The government can never finance the deficit that exceeds the peak of the Laffer curve. The other important information the Laffer curve contains is whether the government should increase or decrease inflation to finance a greater amount of deficit. If the Laffer curve slopes up, then the government can raise more revenue by expansionary monetary policy.

Although $L(\Pi)$ is not very tractable analytically, numerical examples suggest that it is humpshaped, as in Figure 3. The parameters are the same as in the previous example. From the peak of the Laffer curve, the maximum sustainable level of deficit for this economy is about $g=0.029$. It is interesting to note that Laffer curve analysis is still possible, even though the fiscal authority in this economy receives no currency seigniorage. The fiscal authority's revenue is still influenced by inflation because changes in inflation affect the real interest obligation on the outstanding debt. There is another channel, through which changes in inflation affect the fiscal authority's revenue. Changes in inflation affect the demand for bonds and this effect is captured by function $\Psi(\Pi)$.

4 Unsustainable Deficits and Stabilization Policy

4.1 Preliminaries

This section studies the economy in which the fiscal authority plays an unsustainable Ponzi game. For $t \leq J-1$, the fiscal authority runs a permanent deficit of fixed size, $g - \tau$, which is financed entirely by issuing bonds. This section is interested in the case where the deficit is not sustainable so that the real debt b_t increases without bound. If bond-financed deficits are unsustainable, then sooner or later the fiscal authority will have to shift its policy regime. Suppose that a fiscal reform takes place in period $t=J$. The date of stabilization is assumed to be known. Let $\bar{b} \equiv b_{J-1}$ denote the level of real debt outstanding at the beginning of period J (or, the real bonds sold in period $J-1$). It is assumed that for $t \geq J$, the government maintains $b_t = \bar{b}$, and the tax rate is determined so as to maintain solvency. This regime is related to the “passive fiscal policy” (Leeper, 1991; von Thadden, 2004). The initial condition for the capital-labor ratio is given by k_J , which is determined as part of the equilibrium in period $J-1$.

4.2 Stabilization

Consider the equilibria for $t \geq J$. Since government expenditures are not the central issue, I maintain the assumption of g being constant. Since the central bank’s revenue is separated from the fiscal authority’s budget, the fiscal authority in this new regime must raise revenue through tax.

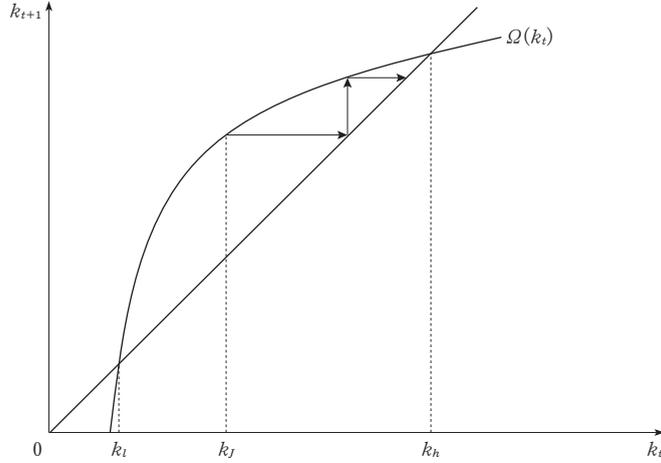
Thus, from (1) the fiscal authority’s budget constraint determines the tax: $\tau_t = g - \bar{b} - f'(k_t)\bar{b}/n$. It is easy to verify that the amount of tax is decreasing in capital. The reason is because as the stock of capital increases, the real interest rate is reduced. Since the real return on capital equals the return on government bonds in equilibrium, the increase in the real interest rate raises the interest payment on the outstanding debt. Since there is no currency seigniorage available for the fiscal authority, the deficit must be cut by raising tax.

The evolution of the capital-labor ratio is given by

$$k_{t+1} = \frac{f(k_t) - g}{n} - \mu(I) \frac{f'(k_t)}{n} \left(k_t + \frac{\bar{b}}{n} \right) \equiv \Omega(k_t). \quad (17)$$

The map $k_{t+1} = \Omega(k_t)$ describes the equilibrium law of motion of the capital-labor ratio for $t \geq J$. The initial condition for k_t is given by k_J . Thus, although the initial condition k_J and \bar{b} are treated as exogenous in the post-stabilization economy, they are determined in the

Figure 4: Economy after the fiscal reform



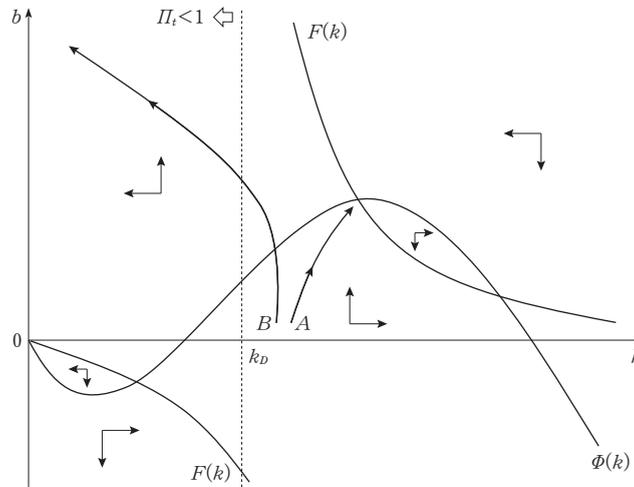
pre-stabilization economy and therefore not free parameters. It is easy to verify that $\Omega(k) > 0$ holds only if $\alpha\mu(I) < 1$. To understand the condition, suppose for the moment that there is no fiscal spending or debt outstanding. Then the economy evolves according to $k_{t+1} = [1 - \alpha\mu(I)]Ak_t^\alpha/n$. Thus, condition $\alpha\mu(I) < 1$ requires a Laissez-faire economy with money and capital to have a nontrivial steady state.

In what follows, I focus on the case in which $\alpha\mu(I) < 1$. For the Cobb-Douglas specification, $\Omega(k) = [1 - \alpha\mu(I)]Ak^\alpha/n - g/n - \mu(I)Ak^{\alpha-1}\bar{b}/n^2$. Then, $\Omega'(k) = [1 - \alpha\mu(I)]\alpha Ak^{\alpha-1}/n + \mu(I)(1 - \alpha)Ak^{\alpha-2}\bar{b}/n^2$. It is then easy to establish that Function Ω satisfies (a) $\Omega'(k) > 0$ for all k , (b) $\lim_{k \rightarrow 0} \Omega(k) = -\infty$, (c) $\lim_{k \rightarrow 0} \Omega'(k) = \infty$, (d) $\lim_{k \rightarrow \infty} \Omega'(k) = 0$, (e) $\partial\Omega/\partial g < 0$, and (f) $\partial\Omega/\partial \bar{b} < 0$.

There are at most two steady-state equilibria, as shown in Figure 4. The existence of a steady state is not guaranteed. In particular, if g or \bar{b} are too high, then there is no steady state. Note that, although g may be chosen to be zero for $t \geq J$, the level of \bar{b} is *predetermined*. Thus, even if $g=0$, a steady state may not exist if \bar{b} is too large. This suggests that J cannot be too large.

Throughout this paper, I focus on the case in which there are two distinct steady states, in other words, the case of successful stabilization. From the figure, it is easy to verify that the high- k steady state is stable and the low- k steady state is unstable. Let k_l and k_h denote respectively the low- k and high- k steady states. Since the high- k steady state is stable, the economy that restarts with $k_J \in (k_l, \infty)$ will eventually reach the stable steady state. Since k_t and Π_t are positively related under the nominal interest rate targeting, the inflation rate increases over time on the transition if $k_J \in (k_l, k_h)$. If the economy restarts with $k_J \in (0, k_l)$, then it will shrink over time and the capital-labor ratio will be zero eventually.

Figure 5: Divergent path and deflation



Proposition 1 *An increase in \bar{b} raises k_l and reduces k_h .*

The result is easily verified by noticing $\partial\Omega/\partial\bar{b} < 0$. The implications of Proposition 1 are quite important. First, an increase in k_l implies that the post-stabilization economy must start with a higher capital-labor ratio. Otherwise the economy cannot reach the steady state and the reform will fail. Second, a reduction in k_h implies a lower output in the long run. To summarize, there are two benefits of fiscal reform taking place sooner: One is that the economy can restart at a high level of initial capital. The other is that it can restart with a low \bar{b} , implying a high long-run output.

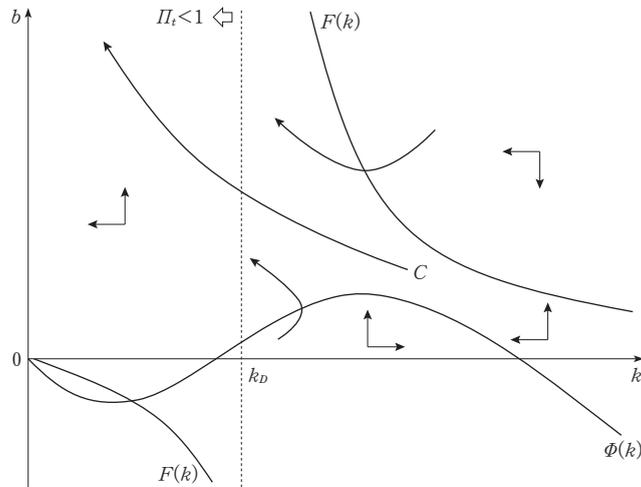
4.3 Unsustainable Ponzi Game, Crowding out, and Deflation

Turn to the first regime, in which the fiscal authority runs a permanent deficit of a fixed size and rolls over the debt to finance the deficit. As shown in Section 3, a debt Ponzi game is sustainable in steady states k_L and k_H . This section explores a scenario in which the economy fails to reach a steady state. Such a scenario arises for two reasons. One is when the economy's initial condition does not lead to a steady state. The other is when there is no steady state in the first place (Chalk, 2000).

Consider Figure 5. All paths below the saddle path are convergent and those above the saddle path are divergent. The trajectory starting at point A is the saddle path, while the trajectory starting at point B is an example of a divergent path. The phase diagram suggests that on a divergent path, such as the trajectory starting at point B, the public debt increases and the capital-labor ratio decreases over time. In other words, the high growth of government bonds crowds out productive capital aggressively and output declines over time as a result.

Since the capital-labor ratio declines, the real interest rate increases over time on a

Figure 6: Nonexistence of steady state

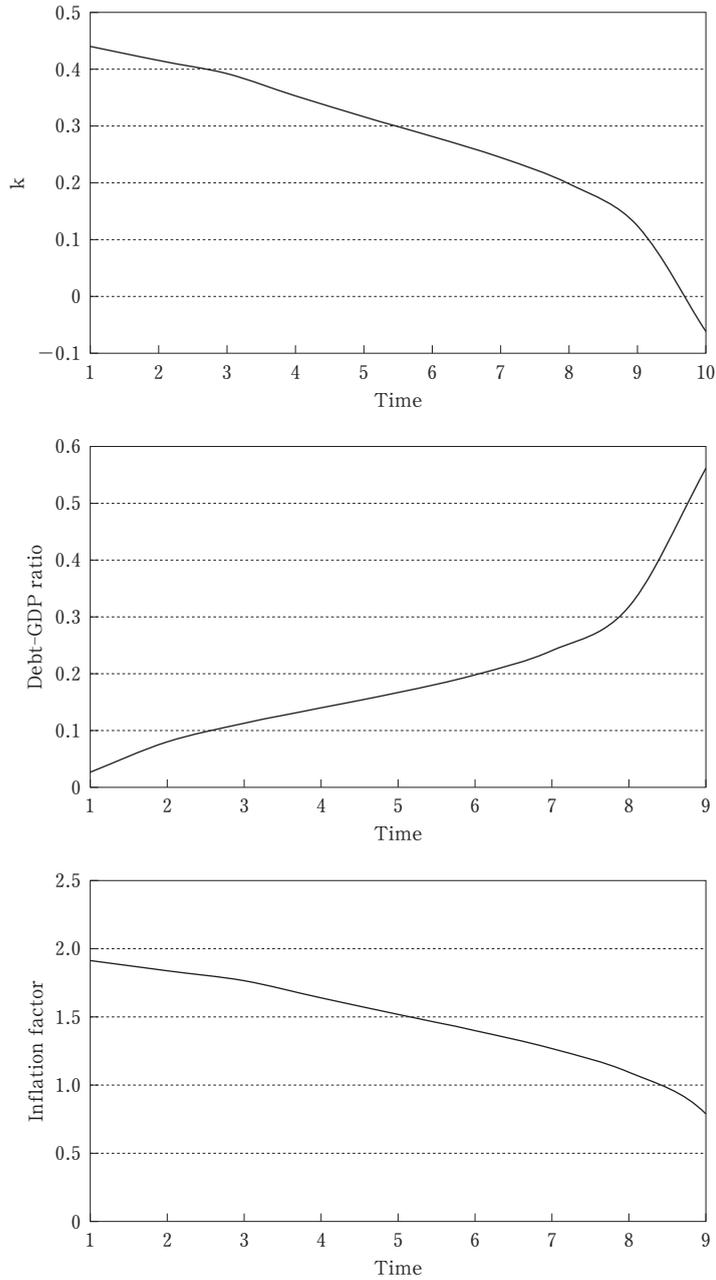


divergent path. Under nominal interest rate targeting, a rise in the real interest rate translates into a decline in the rate of inflation. Thus, the economy on the divergent path is *disinflationary*, although not necessarily deflationary. The Fisher equation gives the condition for deflation: $I < f'(k)$. Let k_D solve $I = f'(k)$. Then the economy becomes deflationary if it enters the region $k_t < k_D$. It is evident that the deflationary region expands as the nominal interest rate gets closer to its lower bound at $I = 1$.

In equilibrium, the demand and supply of government bonds must be balanced. An increase in the supply of bonds must accompany an increase in the demand of the same quantity. If the nominal interest rate were allowed to adjust, then an increase in the demand for bonds would imply an increase in the nominal interest rate. Suppose the central bank commits to a certain level of the nominal interest rate. The central bank can implement its commitment by injecting money into the economy. Since money creation requires no real resource cost, money is a free good (Correia and Teles, 1999). Thus, money injection creates an income effect, and it expands the demand for bonds without raising the nominal interest rate. As a by-product, there is an increase in the real demand for money, and this raises the value of fiat money.

An interesting case arises when there is *no* steady-state equilibrium. Suppose that the economy is originally at the high- k , the stable, steady state. Suppose also that the fiscal authority increases g too much. A possible phase diagram under such a scenario is depicted in Figure 6. Clearly, there is no non-trivial steady state. The economy will approach the northwest of the diagram for any initial condition. On a divergent path, the debt grows over time and as a result of crowding out, the capital-labor ratio, output, and the rate of inflation decrease over time. The possibility of deflation on a divergent path has been pointed out by Buiter (1987), who adopted a version of Sargent and Wallace (1981) to consider the case in which there is no steady state equilibrium as a result of the

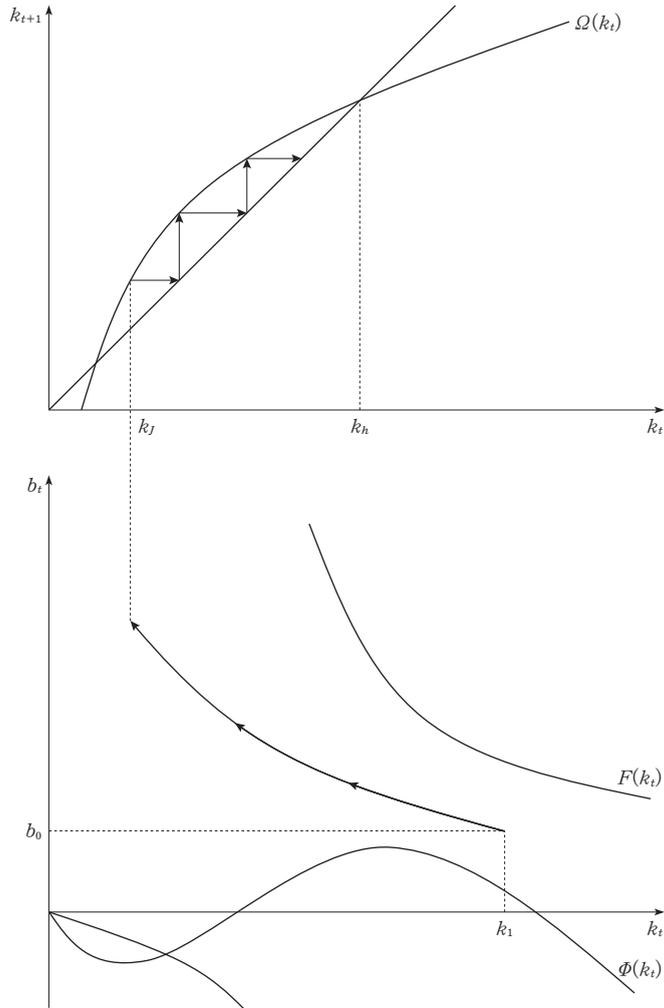
Figure 7: Economy on a divergent path



government running too large a deficit. In Buiter (1987), stabilization policy is not considered even though the deficit is unsustainable.

Figure 7 computes the paths for k_t , b_t , and the inflation rate for the same economy as the one presented previously, starting with the old steady state level at $k_1=0.44$ and $b_1=0.02$. I let $g=0.05$ instead of 0.01. In this case, there is no steady state because the maximum sustainable deficit is 0.029. Other parameter values are the same as before. The

Figure 8: Equilibrium path



debt-GDP ratio surges over time and the rate of inflation declines.

Figure 8 describes an example of the entire equilibrium path of the economy for $t=1, \dots, \infty$. For $t \leq J-1$, there is no steady state, so the government's Ponzi game is unsustainable. Thus, given k_1 and b_0 , the economy moves to the northwest of the diagram. Along the path, the capital-labor ratio, output, and the rate of inflation decline, and the public debt and the real interest rate increase over time. A new policy regime starts in period J with the capital labor ratio k_J . The economy evolves according to $k_{t+1} = \Omega(k_t)$ for $t \geq J$ to approach k_h . Along the path, the capital-labor ratio, output, and the rate of inflation increase over time while the real interest rate decreases.

5 Conclusion

In the literature, the analysis usually stops when no stable steady-state equilibrium is found. The main contribution of this paper involves exploring the behavior of economies on divergent paths. Along a divergent path, the capital declines and the public debt increases over time. Since the bond-financed deficit is unsustainable, a fiscal reform will have to take place. Delayed fiscal reform implies a lower capital and greater public debt. This results in lower output, both at the start and in the long run.

This paper has maintained the assumption that all agents know that a regime shift takes place in period J . It is worthwhile to explore a scenario in which each generation faces a positive probability of a regime change, as considered by Drazen and Helpman (1990). Such an extension would create a direct link between an inevitable future regime shift and the current output and prices. It is also important to investigate a "Ponzi gamble," the notion proposed by Ball *et al.* (1998). In this scenario, the government attempts to run a Ponzi game, but it is not sure if the policy is sustainable. In this sense, Ponzi games become gambles in environments with uncertainty. Ball *et al.* (1998) considered a Ponzi gamble in a real economy. It would be interesting to explore the implications of a Ponzi gamble for the conduct of monetary policy.

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