Strategic Debt/surplus Policy under Vertical Fiscal Competition

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Abstract

Using a two-period model where two levels of government co-occupy the same tax base, this paper analyzes the strategic incentive to debt/surplus policy induced by vertical fiscal competition. Based on the anticipation of the second-period tax-setting game, the governments set their debt/surplus policies in the first period. Within this framework, each government has an incentive to use its debt/surplus policy in order to affect the second-period policy set by the other level of government. As a result of this strategic behavior, vertical fiscal competition creates a tendency towards public surplus. A numerical calculation of the model shows that strategic debt/surplus policy will lead to a lower welfare than when this policy is not available.

JEL Classification : H71 ; H72 ; H74

1. Introduction

In the literature on fiscal federalism, substantial attention has been devoted to the influences of horizontal or vertical fiscal competition on policy-making by governments. The study of horizontal fiscal competition analyzes competition among governments at the same level, which is caused by mobility of tax bases, tax exporting, spillover of public service benefits and so forth.[1] On the other hand, the study of vertical fiscal competition, which is this paper's concern, typically focuses on interactions between different levels of government due to overlapping tax bases. Since the seminal works of Johnson (1982) and Flowers (1988), several papers have clarified the nature of equilibrium public policies under vertical fiscal competition (e. g., Dahlby (1996) ; Keen and Kotsogiannis (1996, 2003) ; Keen (1998) ; Wrede (1996, 2000) ; Flochel and Madies (2002) ; Dahlby and Wilson (2003)). The standard (but not necessarily common) argument in the literature is that this competition will result in inefficiently high tax rates. With tax bases being overlapped, a tax increase made by one level of government will decrease the co-occupied tax bases and create a negative externality on the other level of government.[2]

Most of the studies of vertical fiscal competition have been limited to the analyses of equilibrium public policies in single-period models. This paper intends to gain some insight into vertical fiscal competition when public debt/surplus is available as an additional policy instrument. As for horizontal fiscal competition, there have been some research papers that analyze how allowing for debt/surplus policy influences policy-making by governments. (e. g., Jensen and Toma (1991); Bruce (1995); Schultz and Sjostrom (2001); Krogstrup (2002).) Among these papers, my approach follows that of Jensen and Toma in the sense that it focuses on the strategic use of debt/surplus policy in a subgame perfect equilibrium, rather than debt-financed public policy in a growing economy.[3] For this purpose, I construct a simple two-period model with two levels of government, where there is no capital accumulation in the production sector and neither public debt nor public surplus is needed in the second-best optimum with distortionary taxation. Within this framework, this paper examines the subgame perfect Nash equilibrium in which, based on the anticipation of the second-period tax-setting game, each level of government sets its debt/surplus policy in the first period.

To study the strategic policy-making induced by vertical fiscal competition, assuming that debt/surplus policy is not initially available, the impact of introducing this policy is analyzed in the first place. This analysis makes it clear that each level of government uses debt/surplus policy as a means of decreasing the second-period tax rate set by the other level of government and expanding the common tax bases in that period. Starting from a balanced-budget equilibrium, governments engaging in vertical fiscal competition have an incentive to introduce public surplus (debt) if and only if their tax rates are strategic complements (substitutes). In particular, it is shown that the condition for public surplus, i. e., strategic complements, is met when consumption taxation is co-occupied and when the elasticity of the demand for private goods is constant. This tendency towards public surplus basically carries over to the case where debt/surplus policy is set according to the subgame prefect Nash equilibrium conditions. To demonstrate, this paper conducts a numerical calculation of the model. The outcomes show that both the levels of governments accumulate public surplus if there is no large difference of the welfare weights on the public goods provided by these governments. Even if the difference of the welfare weights is large, one level of government chooses surplus. Also, a welfare comparison is made between the second-best optimum and the equilibrium where debt/surplus policy is prohibited or allowed for. Without debt/surplus policy, as the previous studies of vertical fiscal competition assert, excessive taxation under overlapping tax bases causes a welfare loss relative to the second-best optimum. However, strategic debt/surplus policy is not necessarily beneficial to residents because it may induce a situation similar to prisoner's dilemma between governments. Indeed, in my numerical model, allowing for debt/surplus policy results in a lower welfare than when a balanced-budget requirement is imposed in each period.

This paper is organized as follows. The model and the equilibrium conditions for public policies are, respectively, described in Sections 2 and 3. Section 4 investigates the strategic incentive to debt/surplus policy under vertical fiscal competition, and argues its welfare implication. While my analysis in this paper focuses on consumption taxation, Section 4 also refers to the case of other taxation. Section 5 gives concluding remarks.

2. The Model

Consider an economy with two periods and two levels of governments. For the sake of convenience, these governments are, respectively, called the state and federal governments. The economy is small open in the sense that residents and the governments face a common fixed interest rate. In each period, competitive firms produce an output using labor. One unit of labor produces one unit of the output, which can be transformed into one unit of a private good, a state public good, or a federal public good. While the state and federal governments impose taxes on private good consumption, they can also issue public debt or accumulate public surplus to finance public expenditure in each period. To concentrate on the inefficiency caused by strategic interaction between different levels of government through debt/surplus policy, this paper employs several assumptions that simplify the analysis. There are no intergovernmental transfers in the model. Following most of the studies in the fiscal federalism literature, this paper assumes that there is no money creation by the federal government and that state debt/surplus policy is not subject to federal regulation. In the present model, any source of horizontal fiscal competition such as population mobility, commuting across states and cross-border shopping is excluded. Given this nature of the model, it is assumed that there is only a single state in the economy. Any result in this paper carries over to the case with a finite number of identical states.[4]

I now turn to detailed description of the model. The utility function of a representative resident is quasi-linear:

$$u(\phi(X_i)-L_i+b(g_i)+B(G_i)), \tag{1}$$

where X_i is private good consumption in period i (subscript, 1 or 2, stands for the period.), L_i is labor supply, and g_i and G_i are, respectively, the supply of the state and federal public goods. In what follows, lower-case letters (capital letters) denote the policy variables of the state (federal) government. u, ϕ, b and B are all concave functions: $u'>0>u''$; $\phi'>0>\phi''$; $b'>0>b''$; $B'>0>B''$. For the sake of brevity, it is assumed that the constant interest rate is zero and that there is no discount factor of utility in the second period. (These assumptions are not crucial to this paper's arguments.) The private budget constraint in each period is:

$$Q_1X_1=L_1-S \tag{2a}$$

$$Q_2X_2=L_2+S \tag{2b}$$

where Q_i is the consumer price of X_i and S is private savings. The wage rate is normalized to 1. (L_i is chosen as the numeraire in the model.) The price of the private good equals

$$Q_i=1+t_i+T_i, \tag{3}$$

where t_i and T_i are, respectively, the state and federal consumption tax rates. Using (2a, b), total utility is given by

$$u(\phi(X_1)-Q_1X_1-S+b(g_1)+B(G_1))+u(\phi(X_2)-Q_2X_2-S+b(g_2)+B(G_2)). \tag{4}$$

Taking Q_i, g_i and G_i as given, the representative resident chooses X_i and S to maximize (4). The first-order conditions are

$$\phi'(X_i)=Q_i \tag{5}$$

$$u'_1=u'_2 \tag{6}$$

Note that under the assumed quasi-linear utility function, the tax base in period i depends only on the total tax rate in that period:

$$X_i=X(Q_i);\ X_i'=\frac{1}{\phi_i''}<0. \tag{7}$$

The state and federal budget constraints in the first period are, respectively,

$$t_1X_1+e=g_1; \tag{8a}$$

$$T_1X_1+E=G_1. \tag{8b}$$

where e and E stand for state and federal debt, respectively. (If e and E are negative, the governments are accumulating public surplus.) Similarly, the second-period public budget constraints are:

$$t_2X_2-e=g_2; \tag{9a}$$

$$T_2X_2-E=G_2. \tag{9b}$$

3. Equilibrium Conditions

3.1. Second-best optimum

Before examining the governments' strategic behavior, this subsection presents the second-best conditions for public policies. Suppose that a unified government provides both g_i and G_i by imposing consumption taxation. The unified government sets policy variables to maximize total utility, (4), subject to $g_1+G_1=\tau_1X_1+\varepsilon$ and, $g_2+G_2=\tau_2X_2-\varepsilon$ where τ_i is the unified tax rate ($Q_i=1+\tau_i$) and ε is public debt. In the present model, the second-best optimum involves neither debt nor surplus. To see this, note that the first-order conditions for taxation and public goods are

$$-X_i+b_i'(X_i+\tau_iX_i')=0; \tag{10a}$$

$$b_i'=B_i'. \tag{10b}$$

Eq. (5) was applied to derive (10a). Eq. (10a) corresponds to the familiar tax-financing rule for public expenditure under distortionary taxation. Eq. (10b) implies that the marginal benefits of both the public goods are equalized in the optimum. If the unified government would have public debt/surplus as a policy instrument, it would be set such that $b_1'=b_2'$ (or, equivalently, $B_1'=B_2'$). However, because the tax base in period i depends only on the tax rate in that period, (10a, b) effectively imply that the first-period tax and expenditure policies are the same as those in the second period, showing that ε is a redundant policy instrument.[5]

3.2. Subgame Perfect Nash Equilibrium

In the subgame perfect Nash equilibrium, the state and federal governments strategically set their policy variables. The standard backward induction procedure to obtain the equilibrium conditions starts with examination of the second-period optimization made by these governments. Given the first-period policy variables (t_1, T_1, g_1, G_1, e and E) and S, each government chooses its second-period tax rate and public good supply to maximize the second-period utility subject to (9a) or (9b). In this maximization, the other government's second-period tax rate and public good supply are taken as given. As for the state government, the objective function is. $u(\phi(X_2(Q_2))-Q_2X_2(Q_2)+S+b(t_2X_2(Q_2)-e)+B(G_2))$. Using (5), the first-order condition for t_2 is

$$z(t_2,\ T_2,\ e)\equiv -X_2+b_2'X_2+t_2X_2'=0. \tag{11a}$$

Similarly, the first-order condition for T_2 set by the federal government is

$$Z(t_2,\ T_2,\ E)\equiv -X_2+B_2'(X_2+T_2X_2')=0. \tag{11b}$$

In (11a, b), $\partial z/\partial t_2$ and $\partial Z/\partial T_2$ must be negative to satisfy the second-order condition for optimization. It is assumed that, given e and E, (9a, b) and (11a, b) have a stable solution. The resultant tax rates are given by

$$t_2=t_2(e,\ E); \tag{12a}$$
$$T_2=T_2(e,\ E). \tag{12b}$$

In the first period, the governments take the second-period tax functions, (12a, b), into account in their policy-making. Denoting the maximized second-period utility at a given amount of e, E and S by $V(e,\ E,\ S)$, the utility can be expressed as

$$u(\phi(X_1)-Q_1X_1-S+b(g_1)+B(G_1))+V(e,\ E,\ S). \tag{13}$$

Each government chooses its first-period tax rate, public good supply and the amount of public debt/surplus to maximize (13) subject to (8a) or (8b). Similar to the second-period optimization, the other government's policy variables are taken as given. In choosing the level of e, the state government considers the impact on its own tax revenue in the second period[6]:

$$\partial V/\partial e=-u_2'b_2'(1+X_2\partial T_2/\partial e). \tag{14a}$$

Similarly, in the optimization made by the federal government, the impact of E on V is relevant:

$$\partial V/\partial E=-u_2'B_2'(1+X_2\partial t_2/\partial E); \tag{14b}$$

Note, from (6), that $-u_1'+\partial V/\partial S=0$ since $\partial V/\partial S=u_2'$. This means that although private savings, S, is affected by policy changes, the policy-induced changes in S do not appear in the first-order conditions for policy variables. Using (5), the first-order conditions for t_1 1and T_1 are, respectively,

$$-X_1+b_1'(X_1+t_1X_1')=0; \tag{15a}$$
$$-X_1+B_1'(X_1+T_1X_1')=0. \tag{15b}$$

The first-order conditions for e and E are derived by using (14a, b) and (6):

$$b_1-b_2'-b_2'X_2\partial T_2/\partial e=0 \tag{16a}$$
$$B_1-B_2'-B_2'X_2\partial t_2/\partial E=0. \tag{16b}$$

Throughout this paper, (13) is assumed to be concave with respect to the first-period policy variables, so that the second-order conditions for optimization hold.[7)]

Eqs. (11a, b) and (15a, b) represent the tax-financing rule for public expenditure in each period. Eqs. (16a, b) are the conditions that determine inter-temporal allocation of tax revenue through debt/surplus policy. In the present model where the governments set their first-period policies according to the anticipation of the tax-setting game in the second period, the key determinant of strategic debt/surplus policy is the impact of e and E on the second-period tax rates, $\partial T_2/\partial e$ and. $\partial t_2/\partial E$. In the next section, the incentive to debt/surplus policy and the nature of the second-period tax functions are examined in detail.

4. Public Policy

4.1. The Impact of Introducing Public Debt/surplus

To obtain a theoretical insight into the strategic incentive to debt/surplus policy, it is helpful to investigate the impact of introducing this policy. Suppose that e and E are initially constrained to be zero. In this balanced-budget equilibrium, the state and federal governments will simply set their tax rates according to (11a, b) and (15a, b).[8)] Starting from this equilibrium, consider that the governments are allowed to introduce public debt or surplus by a small amount. If, taking $E=0$ as given, the LHS of (16a) is positive (negative) in this equilibrium, the state government has an incentive to introduce debt (surplus). Similarly, if the LHS of (16b) is positive (negative), the federal government will introduce debt (surplus). In the balanced-budget equilibrium, this incentive to introduce debt or surplus can directly be related to the sign of the derivatives of the second-period tax functions.

Proposition 1.
If $\partial T_2/\partial e$ and $\partial t_2/\partial E$ are positive (negative), the governments have an incentive to introduce public surplus (debt) in the balanced-budget equilibrium.

Proof. Eqs. (11a, b) and (15a, b) imply that, without debt/surplus policy, the second-period equilibrium is a replica of the first-period one ($g_1=g_2$; $G_1=G_2$; $t_1=t_2$; $T_1=T_2$), because X_i depends only on the tax rate in that period. Thus, $b_1'=b_2'$ and $B_1'=B_2'$ hold, implying that the welfare impact of introducing debt/surplus policy is given by the second term on the LHS of (16a, b). Q. E. D.

The intuition behind this proposition is straightforward. Under vertical fiscal competition, a tax increase made by one level of government decreases the common tax base and the tax revenue of the other level of government. Anticipating this game structure, each government strategically introduces debt/surplus policy in the first period to decrease the second-period tax rate set by the other level of government and to expand its own tax revenue. For example, if $\partial t_2/\partial E>0$ then the federal government attempts to induce the state government to decrease t_2 by introducing public surplus. This increases the federal tax revenue, T_2X_2, at a given T_2.[9]

The terms, $\partial T_2/\partial e$ and $\partial t_2/\partial E$, can be derived from comparative statics of the second-period equilibrium conditions, (11a, b):

$$\partial T_2/\partial e=\frac{(\partial Z/\partial t_2)(\partial z/\partial e)}{\Omega}; \tag{17a}$$

$$\partial t_2/\partial E=\frac{(\partial z/\partial T_2)(\partial Z/\partial E)}{\Omega}, \tag{17b}$$

where. $\Omega=(\partial z/\partial t_2)(\partial Z/\partial T_2)-(\partial z/\partial T_2)(\partial Z/\partial t_2)$. Stability of the second-period equilibrium implies that $\Omega>0$. The sign of $\partial z/\partial e$ and $\partial Z/\partial E$ are easily identified:

$$\partial z/\partial e=-b_2''(X_2+t_2X_2')=\frac{-b_2''X_2}{b_2'}>0; \tag{18a}$$

$$\partial Z/\partial E=-B_2''(X_2+T_2X_2')=\frac{-B_2''X_2}{B_2'}>0. \tag{18b}$$

In deriving these equations, the second equality of (11a, b) was used. From (17a, b) and (18a, b), the sign of $\partial t_2/\partial E$ and $\partial K_2/\partial e$ depends on that of $\partial Z/\partial t_2$ and $\partial z/\partial T_2$. This sign, in turn, depends on the slope of the second-period reaction functions of the state and federal governments in the (t_2, T_2) plane, which are, respectively, given by

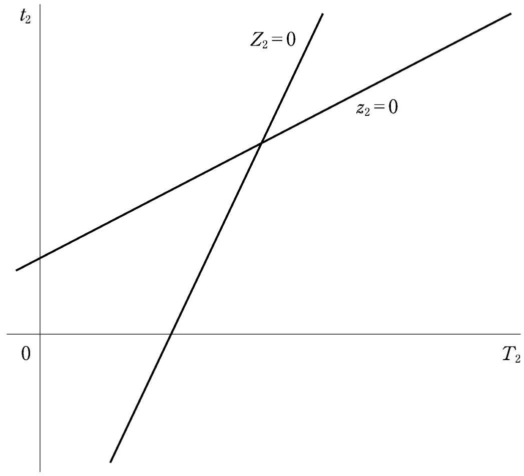
$$dt_2/dT_2=-\frac{\partial Z/\partial T_2}{\partial z/\partial t_2} \tag{19a}$$

$$dT_2/dk_2=-\frac{\partial Z/\partial t_2}{\partial Z/\partial T_2}. \tag{19b}$$

Figure 1 describes the case where the tax rates are strategic complements. In this case, $\partial T_2/\partial e$ and $\partial t_2/\partial E$ are positive because $\partial Z/\partial t_2$ and $\partial z/\partial T_2$ are positive, so that Proposition 1 implies that the governments have an incentive to introduce public surplus in the balanced-budget equilibrium.

To see the relationship between the incentive for debt/surplus policy and the nature of the

Figure 1



second-period tax or reaction functions, it may be useful to make a comparison between the present analysis and Jensen and Toma (1991), where governments at the same level, competing for mobile tax bases, strategically issue public debt or accumulate public surplus. In both of these studies, an increase in a government's debt increases the second-period tax rates set by other governments if the tax rates are strategic complements in the second period. However, Jensen and Toma (see their Propositions 1 and 4) show that, in this case, governments introduce public debt, rather than public surplus, in the balanced-budget equilibrium. This difference occurs because, under horizontal fiscal competition, each government attempts to increase other governments' second-period tax rates in order to attract mobile tax bases. On the contrary, as was discussed with respect to Proposition 1, vertical fiscal competition due to overlapping tax bases gives each government an incentive to decrease the tax rate set by other level of government, thereby generating the opposite incentive to debt/surplus policy.

In the present analysis, the slope of the second-period reaction functions is generally ambiguous even in the neighborhood of the balanced-budget equilibrium. Interestingly, however, a clear result is obtained when the price elasticity of the demand for the private good is constant. In this case, as described in Fig. 1, the tax rates are strategic complements in the balanced-budget equilibrium.

Proposition 2.

If, starting from the balanced-budget equilibrium, a small amount of public debt or surplus is allowed, both the levels of government have an incentive to introduce public surplus when the price elasticity of demand is constant.

Proof. Denote the elasticity by $\eta=-(X_i'Q_i)/X_i$. The first-order conditions for the second-period tax rates, (11a, b), can be rewritten as

$$z(t_2,\ T_2,\ e)=b_2'\left(1-\frac{t_2}{Q_2}\eta\right)-1=0; \tag{20a}$$

$$z(t_2,\ T_2,\ E)=B_2'\left(1-\frac{T_2}{Q_2}\eta\right)-1=0. \tag{20b}$$

If η is constant, one has

$$\partial z/\partial T_2=\frac{t_2}{Q_2}\eta\left(\frac{b_2'}{Q_2}-\frac{b_2''}{b_2'}X_2\right); \tag{21a}$$

$$\partial Z/\partial t_2=\frac{T_2}{Q_2}\eta\left(\frac{B_2'}{Q_2}-\frac{B_2''}{B_2'}X_2\right). \tag{21b}$$

The second equality of (20a, b) was applied to derive (21a, b). Because the parenthesized terms of (21a, b) are positive, the sign of $\partial Z/\partial t_2$ and $\partial z/\partial T_2$ is the same as that of the second-period tax rates. This implies, from (17a, b), that sign $\partial T_2/\partial e$=sign t_2 and sign $\partial t_2/\partial E$ =sign t_2 under the constant elasticity assumption. Note that the tax rates must be positive in the balanced-budget equilibrium as long as the public goods are supplied. Thus, $\partial T_2/\partial e$ and $\partial t_2/\partial E$ must be positive in this equilibrium. This result, together with Proposition 1, completes the proof. Q. E. D.

4.2. Strategic Debt/surplus Policy in the Subgame Perfect Nash Equilibrium

Suppose now that there are no constraints on debt/surplus policy, so that e and E are chosen according to (16a, b). As long as $\partial t_2/\partial E$ and $\partial T_2/\partial e$ are non-zero, debt/surplus policy will be used in the equilibrium, affecting the relative magnitude of the tax rates and expenditure levels in each period. It is easy to see, from (16a, b), that the relative magnitude of public good supply in each period depends on the sign of $\partial T_2/\partial e$ and $\partial t_2/\partial E$. For example, if $\partial T_2/\partial E$ and $\partial t_2/\partial E$ are positive then $b_1'>b_2'$ and, $B_1'>B_2'$, implying that $g_1'<g_2'$ and $G_1'<G_2'$ because the marginal utilities are diminishing. When, as in Proposition 2, the price elasticity of demand is assumed to be constant, the whole structure of tax and expenditure policies can clearly be identified.

Proposition 3.

In the subgame perfect Nash equilibrium, the first-period tax rates are higher than the second-period ones while public expenditures are higher in the second period if $\partial T_2/\partial e$ and $\partial t_2/\partial E$ are positive and if the price elasticity of demand is constant.

Proof. Because the nature of expenditure policy has already been discussed, the proof is made with respect to tax policy. The fact that $b_1'>b_2'$ and $B_1'>B_2'$ when $\partial T_2/\partial E$ and $\partial t_2/\partial E$ are positive means that the marginal cost of public funds is higher in the first period. Under the constant elasticity assumption, the first-order conditions for the tax rates, (20a, b) and similar equations in the first period, imply that

$$1-\frac{t_1}{Q_1}\eta<1-\frac{t_2}{Q_2}\eta; \tag{22a}$$

$$1-\frac{T_1}{Q_1}\eta<1-\frac{T_2}{Q_2}\eta, \tag{22b}$$

which immediately show that $t_1/Q_1>t_2/Q_2$ and $T_1/Q_1>T_2/Q_2$. Summing these inequalities gives $(t_1+T_1)/Q_1(t_2+T_2)/Q_2$, which, in turn, means that $Q_1>Q_2$. Recalling, from the proof of Proposition 2, that the second-period tax rates must be positive when $\partial T_2/\partial e$ and $\partial t_2/\partial E$ are positive under the constant elasticity assumption (that is, sign $\partial T_2/\partial e=$sign T_2 and sign $\partial t_2/\partial E=$sign t_2), it follows that $t_1/t_2>Q_1/Q_2>1$ and $T_1/T_2>Q_1/Q_2>1$. Q. E. D.

In the present model, there are other possible types of equilibrium where either $\partial T_2/\partial e$ or $\partial t_2/\partial E$ is negative: note that $\partial T_2/\partial e$ and $\partial t_2/\partial E$ cannot simultaneously be non-positive under the constant elasticity assumption.[10] In this case, either T_2 or t_2 must be negative, implying that at least one level of government will accumulate public surplus in the subgame perfect Nash equilibrium. Proposition 3 suggests that when all the tax rates are positive, there will also be a tendency towards public surplus because the tax rates are higher and public expenditures are lower in the first period. However, the precise nature of equilibrium debt/surplus policy cannot theoretically be proven in the present model where the policies set by different levels of government interact through an overlapping tax base. Therefore, the rest of my analysis depends on a numerical calculation of the model.

In what follows, the utility function is specified such that $u=\sqrt{\phi(X_i)-L_i+b(g_i)+B(G_i)}$. Suppose that $\phi(X_i)$ takes the form

$$\phi(X_i)=\frac{AX_i^{1-\frac{1}{\eta}}}{1-\dfrac{1}{\eta}}, \tag{23}$$

where A is positive and constant. The functions, $b(g_i)$ and $B(G_i)$, are, respectively, specified as follows:

$$b_i=\sqrt{g_i}\,; \tag{24a}$$
$$B_i=\alpha\sqrt{G_i}, \tag{24b}$$

where α is the relative welfare weight on the federal public good to the state public good.[11] With these specifications of the functions, the subgame perfect Nash equilibrium, consisting of (8a, b), (9a, b), (11a, b), (15a, b) and (16a, b), was calculated. Under the assumptions that $A=3$ and $\eta=1.5$, Table 1 summarizes equilibrium policy policies and residents' welfare within the range of $\alpha\in[0.5, 2]$, which satisfy the usual stability condition for the Nash equilibrium in both the periods.[12]

First of all, see the case with $\alpha=1$, where the state and federal public goods are effectively identical from the viewpoint of residents, so that a symmetric equilibrium occurs where both the levels of governments set the same policy. In this simplest case, the second-period tax rates are positive and public surplus is chosen ($t_2=T_2=0.03312$; $e=E=-0.07093$). Even if $\alpha\neq 1$, these natures of public policy hold as long as the difference of the welfare weights on

Table 1 Subgame Perfect Nash Equilibrium

$A=3$; $\eta=1.5$

α	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
t_1	0.04205	0.04559	0.04967	0.05423	0.05923	0.06463	0.07039	0.07647
T_1	0.0369	0.04092	0.04569	0.05124	0.05755	0.06463	0.07247	0.08107
t_1+T_1	0.07895	0.08651	0.09536	0.10547	0.11678	0.12926	0.14286	0.15754
t_2	0.04939	0.04673	0.04372	0.04042	0.03687	0.03312	0.02922	0.02522
T_2	−0.01029	−0.00343	0.00444	0.01322	0.02281	0.03312	0.04405	0.05552
t_2+T_2	0.0391	0.0433	0.04816	0.05364	0.05968	0.06624	0.07327	0.08074
g_1	0.22163	0.21952	0.21715	0.21456	0.21181	0.20892	0.20594	0.20291
G_1	0.05625	0.08012	0.10765	0.13853	0.1724	0.20892	0.24769	0.28834
g_1+G_1	0.27788	0.29964	0.3248	0.35309	0.38421	0.41784	0.45363	0.49125
g_2	0.21562	0.21753	0.21969	0.2221	0.22459	0.22725	0.23	0.2328
G_2	0.06437	0.09809	0.12095	0.15404	0.18963	0.22725	0.2664	0.30666
g_2+G_2	0.27999	0.31562	0.34064	0.37614	0.41422	0.4545	0.4964	0.53946
e	0.02668	0.01034	−0.00797	−0.02787	−0.04897	−0.07093	−0.09342	−0.11616
E	−0.11483	−0.10762	−0.09947	−0.09054	−0.08098	−0.07093	−0.06053	−0.04989
$\partial T_2/\partial e$	−0.00278	−0.00093	0.00121	0.0036	0.00624	0.0091	0.01215	0.01538
$\partial t_2/\partial E$	0.01421	0.01335	0.01239	0.01134	0.01024	0.0091	0.00793	0.00676
Welfare	6.53916	6.54675	6.55564	6.56577	6.5771	6.58959	6.60317	6.61781

α	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
t_1	0.08284	0.08948	0.09635	0.10343	0.1107	0.11818	0.12581	0.13359
T_1	0.0904	0.10044	0.11119	0.1226	0.13467	0.14735	0.16062	0.17446
t_1+T_1	0.17324	0.18992	0.20754	0.22603	0.24537	0.26553	0.28643	0.30805
t_2	0.02115	0.01704	0.01292	0.00881	0.00474	0.00071	−0.00325	−0.00714
T_2	0.06743	0.07972	0.09231	0.10515	0.11817	0.13133	0.14459	0.15791
t_2+T_2	0.08858	0.09676	0.10523	0.11396	0.12291	0.13204	0.14134	0.15077
g_1	0.19985	0.19678	0.19374	0.19073	0.18777	0.18487	0.18203	0.17927
G_1	0.33049	0.37377	0.41785	0.4624	0.50713	0.55178	0.5961	0.63991
g_1+G_1	0.53034	0.57055	0.61159	0.65313	0.6949	0.73665	0.77813	0.81918
g_2	0.23564	0.23849	0.24131	0.24411	0.24685	0.24953	0.25214	0.25468
G_2	0.34763	0.38897	0.43038	0.4716	0.5124	0.55261	0.59209	0.63071
g_2+G_2	0.58327	0.62746	0.67169	0.71571	0.75925	0.80214	0.84423	0.88539
e	−0.13889	−0.16142	−0.18356	−0.20518	−0.22617	−0.24646	−0.266	−0.28473
E	−0.03913	−0.02832	−0.01755	−0.00688	0.00364	0.01398	0.02409	0.03396
$\partial T_2/\partial e$	0.01877	0.0223	0.0259	0.02971	0.03357	0.0375	0.04152	0.04559
$\partial t_2/\partial E$	0.0056	0.00445	0.00333	0.00223	0.00119	0.00018	−0.00079	−0.00171
Welfare	6.63343	6.65	6.66745	6.68573	6.70479	6.72459	6.74506	6.76617

g_i and G_i is not large; see the cases where $\alpha\in[0.7, 1.6]$.[13] On the other hand, as α rises or falls further, one level of government starts issuing debt while the other still chooses public surplus. For $\alpha=0.5$ and $\alpha=0.6$, in which cases the welfare weight on the federal public good is relatively low, the federal tax is negative in the second period and the state government issues debt. On the contrary, if this welfare weight is high, federal debt is issued; see the cases with $\alpha=1.7\sim2$. Note that, except for the cases with $\alpha=1.7$ and $\alpha=1.8$, the sign of $\partial T_2/\partial e$ and $\partial t_2/\partial E$ is correlated to the choice of debt or surplus in the manner argued in Section 4.1 (see Proposition 1). That is, surplus (debt) is chosen when a marginal increase in debt increases (decreases) the second-period tax rate set by the other level of government.[14)] Note also that, in the present numerical model, there is a monotonous correlation between the marginal impact of debt/surplus on the second-period tax rates and the amount of surplus. As α rises, state surplus and $\partial T_2/\partial e$ increase while federal surplus and $\partial t_2/\partial E$ decrease. These changes in public surplus affect equilibrium tax and expenditure policies. A rise in α, by increasing state surplus, causes t_1 (t_2) to rise (fall) while decreasing g_1 (increasing g_2). On the other hand, T_i and G_i are increased in both the periods as a result of a rise in α, because the evaluation of the federal public good increases.

To examine how public debt/surplus policy influences welfare, Tables 2 and 3 present the outcomes of the numerical calculations of the second-best optimum and the balanced-budget equilibrium under the assumption that $A=3$ and $\eta=1.5$. As will be expected, inefficiently high tax rates and public expenditures due to vertical fiscal competition lead to a lower welfare in the balanced-budget equilibrium than in the second-best optimum. Of particular interest is that, for each α, the welfare is lower in the subgame perfect Nash equilibrium than in the balanced-budget one. At a glance, this goes against the arguments in Section 4.1 where, in the neighborhood of the balanced-budget equilibrium, the governments have an incentive to introduce surplus in order to improve welfare. Although this "local" analysis was helpful to identify the strategic incentive to debt/surplus policy, it does not necessarily give a clear insight into a "discrete" comparison of the welfare obtained from different types of equilibrium. In the subgame perfect Nash equilibrium, debt/surplus policy causes a situation similar to prisoner's dilemma by distorting tax policy significantly. Tables 1-3 show that, regardless of the value of α, the ranking of the total tax rate is that $t_1+T_1>t+T>\tau>t_2+T_2$, where t_i+T_i and $t+T$ are, respectively, the total tax rate in the subgame perfect Nash equilibrium and the balanced-budget one. Thus, debt/surplus policy makes excessive taxation more serious in the first period while the total tax rate is too low in the second period. On the other hand, oversupply of the public goods still persists. Tables 1 and 2 show that, for each α, both the state and federal public good levels are too high in the subgame perfect Nash equilibrium relative to the second-best optimum.[15)] Given that expenditure distortion remains, the tax distortion induced by debt/surplus policy creates a large difference of private consumption and utility between both the periods, requiring that private savings is adjusted to keep the equality of the marginal utility of the first-period and second-period consumption (see (6)). Given that utility is concave and that private consumption will be quite low in the first period, this adjustment will lead to a lower total utility.

The main results based on the numerical calculations are summarized in the following proposition:

Table 2 Second-best optimum

$A=3$; $\eta=1.5$

α	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
τ	0.05535	0.05981	0.06502	0.07093	0.07752	0.08473	0.09253	0.10088
g	0.21221	0.20946	0.20631	0.20279	0.19896	0.19485	0.19051	0.186
G	0.05305	0.07541	0.10109	0.12979	0.16116	0.19485	0.23052	0.26783
$g+G$	0.26526	0.28487	0.3074	0.33258	0.36012	0.3897	0.42103	0.45383
Welfare	6.53971	6.54743	6.55648	6.56682	6.57841	6.59121	6.60518	6.62027

α	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
τ	0.10974	0.11907	0.12882	0.13896	0.14945	0.16025	0.17133	0.18265
g	0.18133	0.17657	0.17173	0.16687	0.16199	0.15714	0.15233	0.14759
G	0.30645	0.34607	0.3864	0.42718	0.46816	0.50914	0.54992	0.59035
$g+G$	0.48778	0.52264	0.55813	0.59405	0.63015	0.66628	0.70225	0.73794
Welfare	6.63643	6.65362	6.67178	6.69087	6.71084	6.73164	6.75323	6.77555

Table 3 Balanced-budget equilibrium

$A=3$; $\eta=1.5$

α	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
t	0.04581	0.04617	0.04659	0.04707	0.0476	0.04819	0.04883	0.04951
T	0.01263	0.01806	0.02438	0.03153	0.03949	0.04819	0.05759	0.06762
$t+T$	0.05844	0.06423	0.07097	0.0786	0.08709	0.09638	0.10642	0.11713
g	0.21859	0.21852	0.21844	0.21834	0.21824	0.21812	0.218	0.21787
G	0.06028	0.08548	0.11428	0.14627	0.18103	0.21812	0.25711	0.29759
$g+G$	0.27887	0.304	0.33272	0.36461	0.39927	0.43624	0.47511	0.51546
Welfare	6.53961	6.54727	6.55625	6.56649	6.57797	6.59063	6.60422	6.6193

α	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
t	0.05023	0.051	0.0518	0.05264	0.05351	0.0544	0.05533	0.05628
T	0.07825	0.08942	0.10108	0.11317	0.12567	0.13852	0.15167	0.16511
$t+T$	0.12848	0.14042	0.15288	0.16581	0.17918	0.19292	0.207	0.22139
g	0.21773	0.21759	0.21744	0.21728	0.21713	0.21697	0.2168	0.21664
G	0.33918	0.38152	0.42428	0.46718	0.50997	0.55241	0.59434	0.63558
$g+G$	0.55691	0.59911	0.64172	0.68446	0.7271	0.76938	0.81114	0.85222
Welfare	6.63522	6.65213	6.66997	6.6887	6.70827	6.72862	6.74972	6.77151

Proposition 4.

If there is no large difference of the welfare weights on the state and federal public goods, both the levels of government choose public surplus in the subgame perfect Nash equilibrium. If this difference is large, one level of government chooses surplus while the other issues debt. As a result of this strategic policy-making, the welfare of residents becomes lower than that obtained when debt/surplus policy is not available.

As for the matter of whether debt or surplus is chosen, Proposition 4 is in contrast to the numerical example of Jensen and Toma (1991), where horizontal fiscal competition induces governments to issue debt in the subgame perfect Nash equilibrium. As was discussed in Section 4.1, this contrast can be explained by nothing that the opposite incentives to debt/surplus policy arise from horizontal and vertical fiscal competition. Thus, the result that there is an incentive to choose public surplus under vertical fiscal competition is interesting but not necessarily surprising. Rather, a more important implication of the present analysis might be that allowing for debt/surplus policy results in a welfare loss, which is consistent with the Jensen-Toma numerical analysis of horizontal fiscal competition. Both of these studies suggest that strategic debt/surplus policy will not mitigate the inefficiency caused by intergovernmental competition. Although the choice of debt or surplus itself may change if the present analysis is extended to take horizontal fiscal competition into account, this extension will not alter the welfare implication as long as either debt or surplus is strategically deployed.

4.3. Other Taxation

This paper has assumed that the co-occupied tax base is private consumption. But, Proposition 1 will apply to the case of other taxation. The strategic incentive to debt/surplus policy generally depends on whether the tax rates are strategic complements or substitutes. On the other hand, in the case of consumption taxation, the slope of the second-period reaction functions is closely related to the sign of the tax rates when the price elasticity of demand is constant. This nature was used to analyze whether debt or surplus is chosen in the balanced-budget equilibrium (Proposition 2) and examine the tax structure in the subgame perfect Nash equilibrium (Proposition 3). Unfortunately, these propositions do not necessarily carry over to the case of other taxation. For example, suppose that different levels of government co-occupy labor taxation. In this case, the slope of the second-period reaction functions is no longer correlated to the sign of the tax rates in that period even if the elasticity of labor supply is assumed to be constant. As a result, the analysis becomes considerably more complex even in the neighborhood of the balanced-budget equilibrium.[16]

Despite this difference between consumption and labor taxation, the nature of equilibrium debt/surplus policy and its welfare implication, which were discussed in Proposition 4, appear to be robust. To verify this, I constructed a numerical model of labor taxation. In that model, like (1), the utility function was assumed to be additively separable:

$$u_i=\sqrt{X_i-\varphi(L_i)+b(g_i)+B(G_i)}\,; \tag{25}$$

$$\varphi(L_i)=\frac{AL_i^{1+\frac{1}{\gamma}}}{1+\dfrac{1}{\gamma}}. \tag{26}$$

(As in Footnote 17, γ stands for the elasticity of labor supply.) The specifications of $b(g_i)$ and $B(G_i)$ functions are the same as (24a, b). Other aspects of the model are quite similar to those in the case of consumption taxation. With these specifications, setting $A=0.2$, my calculations were conducted with respect to $\gamma\in[0.1,\ 0.5]$. (The detailed model and the calculated outcomes are available upon request.) In these calculations, for each γ, the symmetry case ($\alpha \neq 1$) shows that the tax rates are strategic complements in the balanced-budget equilibrium, and that the governments accumulate public surplus in the subgame perfect Nash equilibrium. Even if $\alpha \neq 1$, at least one level of government chooses public surplus in the subgame perfect Nash equilibrium.[17)] Moreover, the welfare is lower in the subgame perfect Nash equilibrium than in the balanced-budget one, implying that strategic debt/surplus policy has a harmful welfare impact.

5. Concluding Remarks

The previous studies of vertical fiscal competition have mainly used single-period models and analyzed how inefficient tax and expenditure policies occur because of overlapping tax bases, or how fiscal arrangements are developed to overcome these inefficiencies. This paper has incorporated strategic debt/surplus policy into a two-period model of vertical fiscal competition, and demonstrated that this competition creates a tendency towards public surplus and causes a welfare loss. Of course, it should be emphasized that these are the results of abstracting other microeconomic or macroeconomic aspects of debt-financed public policy such as capital accumulation and governments' investment on durable public services. Given that the public sector in many countries suffers from persistent deficits, the incentive to use public surplus induced by vertical fiscal competition may have quite different welfare implications once these aspects are taken into account. There will be other important directions for the future research. For example, this paper's analysis depends on the concept of subgame perfect Nash equilibrium under which different governments set their policies simultaneously. But, the previous studies of vertical fiscal competition (with single-period models) include the analyses based on the Stackelberg equilibrium concept, as well as the Nash equilibrium concept. To obtain a further insight into debt/surplus policy in the context of fiscal federalism, the dynamic approaches based on more sophisticated equilibrium concepts will be useful.

Notes

1) See Wilson (1999) and Wellisch (2000) for comprehensive serveys of related studies.

2) This argument has been derived from models with revenue-maximizing governments, as well as benevolent governments. Throughout this paper, in order to examine the welfare implication of strategic debt/surplus policy, it is assumed that all governments set public policies to maximize the welfare of residents.

3) This paper does not address the policy issues related to persistent deficits in the public sector. Wenzel and Wrede (1996) analyze the existence and stability of equilibrium when governments at different levels accumulate public capital and issue public debt in a growing economy. However, they do not consider how governments, engaging in vertical fiscal competition, strategically set their policies.

4) The presence of many small states will not be compatible with the present analysis of the subgame perfect Nash equilibrium, where the governments consider the impact of their first-

period policies on the second-period equilibrium. Except for this case, the absence of horizontal fiscal competition means that the number of states is largely irrelevant to the analysis. Even if there are a finite number of identical states, the qualitative arguments in Section 4.1 (Propositions 1 and 2) do not change at all, but the numerical model in Section 4.2 must slightly be modified (see Footnote 12).

5) It should be noted that this nature of the optimum is not because the interest rate is set equal to zero and discounting is ignored. Rather, it is because the utility function is assumed to be separable. In particular, it can be shown that the outcome involving $\varepsilon=0$ is the sole possible second-best allocation if the price elasticity of the demand for the private good is constant.

6) Derivation of (14a). Noting that the state government ignores its policy's impact on the federal tax revenue in each period, differentiating the second-period utility with respect to e and using (5) and (11a) yields

$$\partial V/\partial e = u_2' \left[-b_2' - X_2(\partial t_2/\partial e + \partial T_2/\partial e) + b_2'(X_2 \partial t_2/\partial e + t_2 X_2'(\partial t_2/\partial e + \partial T_2/\partial e))\right]$$
$$= u_2' \left[-b_2' - (X_2 - b_2' t_2 X_2')(\partial T_2/\partial e)\right]$$

Again, applying (11a) to the second expression gives (14a). A similar procedure yields (14b).

7) It is difficult to derive general conditions under which the concavity of (13) is ensured. However, the numerical model in Section 4.2 gives an example of the subgame perfect Nash equilibrium where the concavity condition is satisfied at least in the neighborhood of the equilibrium.

8) Without debt/surplus policy, comparing these equations and (10a) establishes the standard argument concerning the excessive taxation caused by vertical fiscal competition. To demonstrate, the first-order condition for the state tax policy is rewritten as

$$-X_i + b_i'(X_i + (t_i + T_i)X_i') - b_i' T_i X_i' = 0$$

When this equation is evaluated at the second-best optimum where, $t_i + T_i = \tau_i$, (10a) implies that $-b_i' T_i X_i' > 0$, leading to a contradiction. This argument shows that, starting from the optimum, the state government has an incentive to increase its tax rate. (A similar argument applies to the federal tax rate.)

9) Although the change in E also affects T_2, this change has no welfare impact when tax policy is set according to the first-order conditions in the second period.

10) If $\partial T_2/\partial e$ and $\partial t_2/\partial E$ were both negative, it would be that $t_1/Q_1 \leq t_2/Q_2 \leq 0$ and $T_1/Q_1 \leq T_2/Q_2 \leq 0$. (One can confirm this by using a procedure similar to the proof of Proposition 3.) Thus, as long as Qi is positive, all the tax rates must be negative, leading to a contradiction.

11) In the case with a finite number of identical states, if the federal public good is pure public in the entire economy, its effective supply is equal to NG_i, where N is the number of states and G_i now stands for federal expenditure per state. (E in (8b) and (9b) should be regarded as federal debt per state.) Then, (24b) becomes $B_i(G_i) = \beta\sqrt{NG_i}$, where α is equal to $\beta\sqrt{N}$.

12) The value of A, which represents the welfare weight on the private good, was set not to yield extremely high or low tax rates. With respect to the elasticity, any result argued in this subsection carries over to the cases with higher or lower values of the elasticity. While my numerical calculation was conducted with respect to $\eta \in [1.1, 2]$, this paper presents the "intermediate" case only. (The calculated outcomes when the elasticity is higher or lower are available upon request.)

13) The value of the elasticity appears not to be crucial to the range α of in which both the levels of government accumulate public surplus. As for $\eta \in [1.1, 2]$, the largest range is obtained when $\eta \in 1.4 (\alpha \in [0.6, 1.7])$. When $\eta \in 1.9$ or 1.3, the range is $\alpha \in [0.7, 1.7]$. In other cases, the state and federal governments choose public surplus when$\alpha \in [0.7, 1.6]$.

14) As for $\alpha = 1.7$ and $\alpha = 1.8$, federal debt is issued even if $\partial t_2/\partial E$ is positive (but very low). This

situation arises when federal policy is changing from public surplus to debt. In this case, depending on the relative and absolute values of the state and federal tax rates, the federal government can raise the second-period tax revenue enough to repay the debt issued in the first period while keeping $G_2 > G_1$, even if $T_1 > T_2$. But, as federal debt increases further, the sign of $\partial t_2/\partial E$ changes from positive to negative; see the case with $\alpha = 1.9$.

15) Tables 1 and 3 show that, except for the cases with $\alpha = 1.7$ or 1.8, when a government chooses public surplus in the subgame perfect Nash equilibrium, its first-period (second-period) expenditure is low (high) relative to the balanced-budget equilibrium. (When public debt is chosen, the opposite ranking holds.) On the other hand, the first-period (second-period) total public expenditure is always lower (higher) in the subgame perfect Nash equilibrium than in the balanced-budget one.

16) To understand how this difference arises, one must look into the first-order conditions for the second-period tax rates. In the case of consumption taxation, it can be seen, from (20a), that a rise in T_2 affects $z(t_2, T_1, e)$ in two different manners: First, given that $t_2 > 0$, a rise in T_2 increases the marginal benefit of g_2 (i. e., b_2') by reducing the common tax base in the second period. Second, it increases the reciprocal of the marginal cost of public funds of the state government, $1 - t_2\eta/Q_2$, by increasing Q_2. Both of these two impacts imply that the parenthesized term of (21a) is positive. In the case of labor taxation, however, these impacts take the opposite sign each another. The first-order condition for the second-period tax policy, which corresponds to (20a), is

$$z(t_2 T_2 e) = b_2' \left(1 - \frac{t_2}{w_2 \gamma}\right) - 1 = 0$$

where w_2 is the net wage rate, γ is the elasticity of labor supply and t_2 and T_2 now stand for the labor tax rates. As T_2 rises, b_2' increases because the labor tax base declines, but $1 - t_2\gamma/w_2$ decreases because w_2 declines. As a result, unlike (21a), the sign of $\partial z/\partial T_2$ is not correlated to the sign of t_2.

17) Unlike the case of consumption taxation, the range of α, where both the levels of government choose surplus in the subgame perfect Nash equilibrium, quite varies with the value of the elasticity. The range tends to be wider as γ rises ($\alpha \in [0.6, 1.1]$ if $\gamma = 0.1$; $\alpha \in [0.6, 1.2]$ if $\gamma = 0.2$; $\alpha \in [0.7, 1.3]$ if $\gamma = 0.3$; $\alpha \in [0.7, 1.4]$ if $\gamma = 0.4$; $\alpha \in [0.7, 1.6]$ if $\gamma = 0.5$.) In my numerical model where the parameter, α, is attached to the federal public good, the state government issues debt once α deviates from these ranges. On the other hand, the federal government chooses public surplus regardless of the value of α.

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