論　説

Tax Smoothing and Bangladesh Government Debt

Dilruba BEGUM*
David FLATH**

Abstract

In a previous paper we forecasted Bangladesh GDP and government spending through the end of the current century under three plausible scenarios and constructed estimates of the 'sustainable tax rate' under each — that is the tax rate that, if maintained, would at the end of the century return the government-debt-to-GDP ratio to its initial current level. A latent idea behind the concept of a sustainable tax rate is the tax-smoothing logic elucidated by Barro (1979, 1995). Under certain conditions that he stated — that the marginal tax rate is proportional to the average tax rate and all taxes are fully shifted onto labor, and so on — minimizing the burden of taxes needed to fund a projected future stream of government expenditures entails the setting of an overall tax rate that is stable through time. A single sustainable tax rate to be established and maintained into the future could be understood as minimizing the present value of future tax burden. In other words, the sustainable tax rate is the tax-smoothing tax rate. This suggests a way of evaluating the welfare costs of deviation from the sustainable tax rate or delay in implementing it: Calculate the excess burden of taxes in all years under each scenario and compare each with the optimal case. Here, we do this and find that reduction in cumulative excess burden through the end of the century from initiating a sustainable tax rate immediately, compared to delaying doing so by ten years or twenty years, ranges from about 2 percent to 30 percent of the current Bangladesh GDP. It seems from this finding that concern about primary fiscal deficits in Bangladesh — recently around 5% of GDP — must be based on the perceived perils of a high and rising government-debt-to-GDP ratio, not on the added excess burden caused by deviation from tax-smoothing.

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* Doctoral Student, Economics, Ritsumeikan University
Email: mishu.kafrul@gmail.com
** Professor, Faculty of Economics, Ritsumeikan University
Email: flath@fc.ritsumei.ac.jp

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1. Introduction

In a previous paper we forecasted Bangladesh GDP and government spending through the end of the current century under three plausible scenarios and constructed estimates of the 'sustainable tax rate' under each — that is the tax rate that, if maintained, would at the end of the century return the government-debt-to-GDP ratio to its initial level. A latent idea behind the concept of a sustainable tax rate is the tax-smoothing logic elucidated by Barro (1979, 1995). Under certain conditions that he stated — that the marginal tax rate is proportional to the average tax rate and all taxes are fully shifted onto labor, and so on — minimizing the burden of taxes needed to fund a projected future stream of government expenditures entails the setting of an overall tax rate that is stable through time. A single sustainable tax rate to be established and maintained into the future could be understood as minimizing the present value of future tax burden. In other words, the sustainable tax rate is the tax-smoothing tax rate. This suggests a way of evaluating the welfare costs of deviation from the sustainable tax rate or delay in implementing it: Calculate the excess burden of taxes in all years under each scenario and compare each with the optimal case.

Here, we make this sort of calculation for Bangladesh and find that reduction in the present value of excess burden through the end of the century from initiating a sustainable tax rate immediately, compared to delaying doing so by ten years or twenty years, ranges from about 2 percent to 30 percent of the current Bangladesh GDP. It seems from this finding that concern about primary fiscal deficits in Bangladesh — recently around 5% of GDP — must be based on the perceived perils of a high and rising government-debt-to-GDP ratio, not on the added excess burden caused by deviation from tax-smoothing.

2. Tax smoothing

2.1. Basic framework

The simplest framework for determining the optimal trajectory of government debt is the one first proposed by Barro (1979). Its novelty and simplicity lie in posing the question of government debt as one of determining the optimal trajectory of tax revenue subject to the constraint that its present value is at least as great as that of the given trajectory of government spending. The setting is an Arrow-Debreu economy in competitive equilibrium. All variables are deterministic rather than stochastic. Furthermore, the tax system itself is a simple one in which all taxes are fully shifted onto labor and entail no intertemporal distortions. Taxes do not affect saving or investment, and so do not affect potential output. The future trajectory of potential real GDP, like that of government spending, is predetermined and unaffected by taxes or government debt. The main result from this framework is that 'tax smoothing' is optimal — excess burden is minimized by setting an unchanging tax rate and allowing government debt to evolve according to the rises and falls in government spending relative to GDP. Before developing this argument in more detail, we first elaborate on its key assumptions, drawing heavily on Flath (2014, ch. 11).

All taxes are shifted onto labor

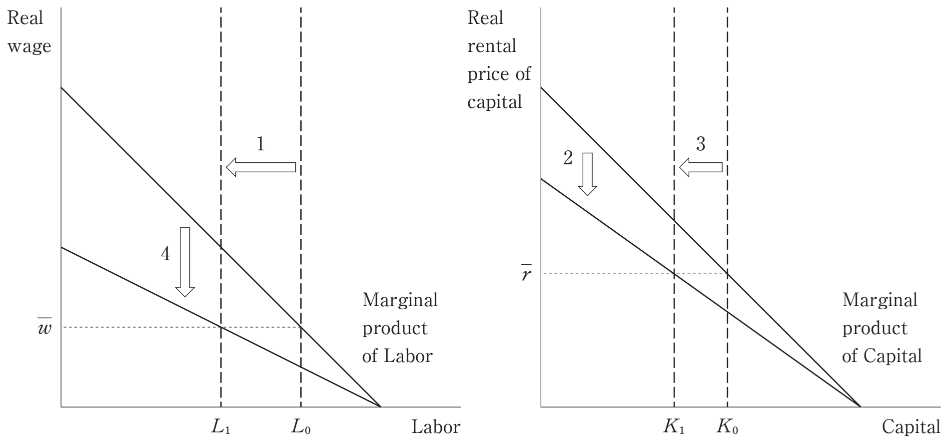
A general consumption tax that applied the same tax rate to all goods and services including leisure, would have no effect on relative prices. It would have the same behavioral effects as a uniform lump-sum head tax and have no excess burden. But a general consumption tax does not tax leisure and it is difficult to see how it could be made to do so. Because leisure is not taxed, its relative price is made lower by the taxes on other goods, and people wastefully substitute leisure for those other goods. They work less, enjoying more leisure but buying fewer consumption goods. This is wasteful because the relative prices that induce these choices differ from the marginal social costs of the corresponding objects of choice.

Corporate income taxes also contribute to excess burden. Some excess burden of the corporate income tax arises from wasteful substitution of labor for the taxed capital inputs of corporations. But more of the excess burden of the corporate income taxes results from tax shifting. To the extent the corporate taxes are shifted onto workers and consumers as lower wages and higher prices, they are effectively further taxes on the consumption of goods other than leisure and contribute to excess burden in the same way as do other taxes on consumption. As a first approximation, one can regard this tax shifting as total; virtually all corporate taxes are shifted onto workers and consumers.

All aspects of a tax system, including personal income taxes, corporate taxes, taxes on property, and taxes on consumption, fundamentally amount to a tax on the reward from work. The distorting effects of a tax system mostly reside in the reduced incentive to work entailed in taxes on goods other than leisure and on the income needed to purchase such goods. The size of these effects and their implied excess tax burden depend on the marginal tax rate on consumption embodied in the tax system and depend also on the responsiveness of people to reduced economic incentives to work.

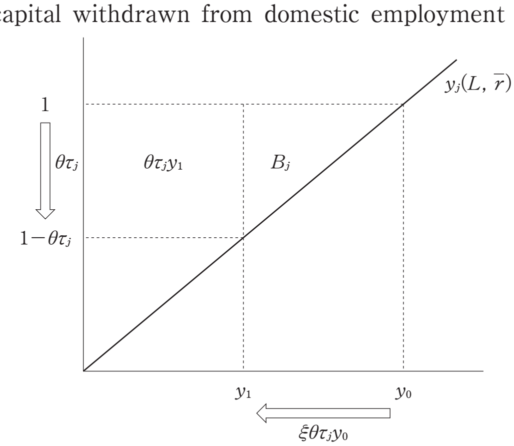
One way to represent these ideas is to posit that real GDP is the value of a Cobb-Douglas production function of labor and capital with constant returns to scale. Taxes of whatever kind being fully shifted onto labor mean that the real wage and real rental price

Figure 1. A tax fully shifted onto labor.



of capital are unaffected by taxes, but the supply of labor is reduced by any rise in the marginal tax rate as workers substitute leisure for commodities to avoid the burden the taxes place upon their consumption of commodities. It is a competitive economy with zero economic profit. All income is realized as wages. The only relative price affected by a change in taxes is the 'price' of leisure — which is nothing more than the real wage net of taxes. Figure 1 depicts the effect of taxes in the Cobb-Douglas economy as just described. In the bottom diagram depicting the excess burden of taxes, the real wage is rescaled to equal one minus the marginal tax rate, which means that labor is rescaled to equal real GDP. The upward-sloping supply of labor curve shows how the supply of labor responds to a changing tax rate, holding the capital-to-labor ratio constant. As drawn, the supply of labor exhibits no income effects (workers have quasi-linear utility functions).

A tax fully shifted onto labor (1) causes the supply of labor to constrict, which (2) reduces the derived demand for capital, and then (3) causes the supply of capital to constrict, finally (4) reducing the derived demand for labor. The real wage and real rental price of capital borne by the employers of labor and capital are unchanged from their original levels, $\overline{w}$ and $\overline{r}$. The capital withdrawn from domestic employment is redeployed in foreign



countries at the same rental price as before. Workers as suppliers of domestic labor bear the full burden of the tax.

Output response to a tax fully shifted onto labor, showing excess burden B_j. The marginal tax rate is $\theta\tau_j$ and ξ is the income-compensated elasticity of labor supply. The excess burden is $B_j=1/2\xi\theta^2\tau_j^2 y_0$.

The excess burden of taxes varies with the square of the marginal tax rate

Excess burden is the most that people would pay to replace the existing tax system with a non-distorting system of head taxes that collected the same tax receipts from each as before. If the income-compensated supply of labor has elasticity ξ measured from the no-tax point (L_0, w_0), then by linear approximation[1)]

$$\begin{aligned} \textit{excess burden} &= 1/2tw_0\Delta L \\ &= 1/2t^2\xi w_0 L_0 \end{aligned}$$

The excess burden varies in proportion to the elasticity of income-compensated supply of labor and in proportion to the square of the tax rate. This famous result was first noted by Dupuit (1844). The distorting effects of taxes depend upon the responsiveness of people to changes in marginal incentives. The more responsive they are, the greater the distortion and the greater the excess burden of an otherwise neutral tax. Furthermore, doubling the tax rate quadruples the excess burden.

If elasticity of the income-compensated supply of labor ξ is measured from the point with taxes $(L_1, w_0(1-t))$, then we reach another useful linear approximation[2)]:

$$\frac{\textit{excess burden}}{\textit{tax receipts}}=\frac{1/2t\xi}{(1-t)}.$$

Excess burden relative to tax receipts varies in proportion to the tax-exclusive tax rate $\dfrac{t}{(1-t)}$.

There have been many estimates of the income-compensated labor supply elasticity of workers in the US and some European countries. In a recent survey of published estimates Keane (2011) shows that a simple average of estimates of elasticity of income-compensated male supply of labor across more than thirty papers published since 1969 is 0.31.

A major difficulty in estimating the elasticity of supply of labor relevant for computing excess tax burden is that there are many ways labor supply can be reduced. For example, one may decide to work shorter hours, take more vacation days each year, work part-time rather than full-time, withdraw from the labor force altogether, or retire at an earlier age. All these margins of choice matter, yet econometric estimates of labor supply typically focus on only one of them. Feldstein (1999) has shown that when all margins of adjustment in labor supply are operative, excess burden can be represented in a formula like the one shown above but with compensated elasticity of labor supply replaced by compensated elasticity of taxable income with respect to change in marginal tax rates. Estimates of this elasticity are imprecise but do not differ much from the estimates of compensated elasticity of supply of labor already mentioned. Economists Kitamura and Miyazaki (2010), using individual income tax data for Japan, estimate the compensated elasticity of taxable income with respect to changes in the marginal tax rate to be in the range 0.2 to 0.28.

The marginal tax rate is proportionate to the average tax rate

Stipulate that national taxes in year j depend on nominal GDP according to a set schedule $T_j(y_j)=a_j y_j^{\theta}$, where y is nominal GDP and a and θ are parameters. Yousuf and Huq (2013), using 1980-2011 data, estimate the tax elasticity for Bangladesh as $\theta=1.14$, which indicates the Bangladesh tax system is progressive. In this formulation, the average tax rate in each year is $\tau_j=a_j y^{\theta-1}$. The marginal tax rate $\theta\tau_j$ is proportionate to the average tax rate. The excess burden of taxes in year j varies with the square of the marginal tax rate, $\theta\tau$, and with the elasticity, ξ, of the income-compensated supply of labor: $B_j=1/2\xi\theta^2\tau_j^2 y_j^0$. And so, $\frac{\partial B_j}{\partial \tau_j}=\frac{2B_j}{\tau_j}=\xi\theta^2\tau_j y_j^0$.

Tax revenue equals government spending. Incipient tax revenue can be postponed but not avoided except by curtailing government spending.

Denote the present value of the infinite future stream of government spending other than for servicing government debt as G. Similarly, denote the present value of the stream of taxes as T, and the present value of the excess tax burden as B. Suppose that, as viewed currently (beginning of period *0*), the present value of government debt outstanding at the beginning of period *1* is D_0^*[3] and consider what average tax rate in each subsequent year would minimize the present value of excess tax burden, taking as given that the present value of taxes must be large enough to pay for the given stream of government spending and also service the government debt. The problem is to choose (τ_1, $\cdots$, τ_j, $\cdots$) to minimize B, subject to the constraint that $T\geq D_0+G$. This way of framing the issue assumes an Arrow-Debreu competitive loan market in which any payment stream can be traded for any other having equal present value. All variables are deterministic — not stochastic. It is the simplest model for exploring government debt.

2.2. Tax rates that minimize excess burden

In this framework, the tax stream that minimizes the burden of funding the given stream of government spending entails the same average tax rate in each year, $\tau_j = \frac{D_0 + G}{Y}$, $\forall j$. This is the famous tax-smoothing result of Barro (1979). To derive this result, form the Lagrangian objective function as follows:

[1] $\min_{\tau 1, \cdots, \tau\infty} \mathcal{L} = B - \lambda (T - D_0 - G)$,

where i_j is the rate of interest in year j and

$$B = \Sigma_{t=1}^{\infty} \frac{1/2 \xi \theta^2 \tau_j^2 y_j^0}{\prod_1^j (1+i_j)}, \quad T = \Sigma_{j=1}^{\infty} \frac{\tau_j y_j^1}{\prod_1^j (1+i_j)}, \quad G = \Sigma_{j=1}^{\infty} \frac{G_j}{\prod_1^j (1+i_j)}.$$

Here, y_j^0 is what the real GDP would be if taxes were zero — in other words the 'potential real GDP' — and y_j^1 is the actual real GDP given that the tax rate is τ_j. Note that y_j^1 depends on τ_j, so it is more convenient to write tax revenue as a function of y_j^0.

$$y_j^1 = y_j^0 (1 - \xi \theta \tau_j)$$

$$\text{Tax revenue}_j = T_j = \tau_j y_j^1 = \tau_j y_j^0 (1 - \xi \theta \tau_j)$$

$$\frac{\partial T_j}{\partial \tau_j} = y_j^0 (1 - 2\xi \theta \tau_j)$$

Also keep in mind the tax revenue, $T_j = \tau_j y_j^1$, depends on the average tax rate, τ_j, while excess burden, $B_j = 1/2 \xi \theta^2 \tau_j^2 y_j^0$, depends on the marginal tax rate, $\theta \tau_j$. The two are proportionate but not the same if the tax schedule is progressive, meaning that $\theta > 1$.

The solution to the program solves the first-order condition.

[2] $\frac{\partial \mathcal{L}}{\partial \tau_j} = \xi \theta^2 \tau_j y_j^0 - \lambda (1 - 2\xi \theta \tau_j) y_j^0 = 0,\ j = 1, \cdots \infty$

The solution is the following.

[3] $\tau_j^* = \frac{\lambda^*}{\theta \xi (\theta + 2\lambda^*)} = \frac{D_0 + G}{Y}$, $\forall j$.

In this expression, $Y \equiv Y^1 = \Sigma_{j=1}^{\infty} \frac{y_j^1}{\prod_1^j (1+i_j)} = \Sigma_{j=1}^{\infty} \frac{(1 - \xi \theta \tau_j^*) y_j^0}{\prod_1^j (1+i_j)}$, depends on the tax-rate trajectory. Nevertheless, we will treat $Y \equiv Y^1$ as though it were unaffected by taxes, and thus impart a bias to our comparisons of various tax-rate trajectories later in the paper.[4)] We think the bias is slight.

The Eq. [3] reprises the tax-smoothing result of Barro (1979). The Lagrange multiplier represents the shadow price of debt, $\lambda^* = \frac{dB^*}{dD_0}$. It indicates the marginal increase in tax burden associated with an added increment of initial debt. In this framework, the excess tax burden arising from the distorting effect of taxes needed to service additional debt is the only thing reflected in the shadow price of debt. If there are other losses associated with debt, then that could be represented in this model by tightening the constraint to impose a higher shadow price of debt. We will return to this point below when comparing the tax-smoothing framework of Barro and the sustainable debt paradigm of Blanchard and others.

The derivation just related assumes that initial debt must at some point be paid in full. We might suppose to the contrary that the initial debt could be carried indefinitely — rolled over — if the annual interest obligations were met. This does not change the result. The present value of the annual stream of interest payments needed to roll over the debt equals the present value of the debt. If the interest rate is constant this is evident as shown here.

[4] $$\sum_{j=1}^{\infty}\frac{iD_0}{(1+i)^j}=D_0.$$

What if i changes over time? Arbitrage (not mathematics) requires that the present value of the annual stream of interest payments needed to roll over the debt equals the present (market) value of the initial debt, $D_0^*=\frac{(1+i_0)}{(1+i_1)}D_0$, (see fn. 3).[5]

[5] $$\sum_{j=1}^{\infty}\frac{i_jD_0^*}{\prod_1^j(1+i_j)}=D_0^*.$$

Fiscal equivalence between a tax stream sufficient to rollover a government debt indefinitely and a tax stream sufficient to eventually pay it off in full foreshadows a result that we will explore below: equivalence between the 'tax smoothing tax rate' of Barro and the 'sustainable tax rate' of Blanchard. This equivalence requires that the only cost associated with a rise in the government-debt-to-GDP ratio is an enlarged tax burden. That is the same framework we have so far elaborated.

3. Government debt

3.1. Alternative trajectories of government debt

We next characterize the trajectory of government debt, conditional on given trajectories of government spending and taxation. The government 'budget constraint' is the following.

[6] $$G_j-T_j+iD_{j-1}=D_j-D_{j-1}$$

Dividing both sides of the equation above by GDP$\equiv y_j$, and rearranging, we get

[7] $$\frac{D_j}{y_j}=\frac{G_j-T_j}{y_j}+\frac{(1+i)D_{j-1}}{(1+\eta)y_{j-1}}$$

where $\eta=\frac{y_j-y_{j-1}}{y_{j-1}}$ is the growth rate of GDP, which we here assume to be constant. Repeated substitution into this equation of the previous year's debt-to-GDP ratio, beginning with $j=1$, results in an expression for the debt-to-GDP ratio in future year, $j=N$, implied by the given future trajectory of primary deficits, $g_j-\tau_j$, for $j=1, \cdots, n$ (see Broda and Weinstein, 2005).

[8] $$\frac{D_N}{y_N}=\left(\frac{1+i}{1+\eta}\right)^N\frac{D_0}{y_0}+\sum_{j=1}^{N}\left(\frac{1+i}{1+\eta}\right)^{N-j}\frac{(G_j-T_j)}{y_j}.$$

If G_j and T_j are growing at the same constant rate η as GDP:

[9] $$\frac{D_N}{y_N}=\left(\frac{1+i}{1+\eta}\right)^N\frac{D_0}{y_0}+\frac{(G_0-T_0)}{y_0}\sum_{j=1}^{N}\left(\frac{1+i}{1+\eta}\right)^{N-j}.$$

The 'tax-smoothing' tax rate, from the above discussion, is $\tau^*=\frac{D_0+G}{Y}$. A thought experiment with conjectured values for the variables reveals its order of magnitude and how it would change if government debt were allowed to accumulate relative to GDP. Later in the paper we will attempt a finer calculation with assumptions better adapted to the Bangladesh data, but a preliminary foray is a good starting point.

In Bangladesh, $\frac{G}{Y}=0.15$, is a reasonable assumption (based on extrapolation of recent data for Bangladesh). As a thought experiment let's assume that $i=0.05$. Assume further that y_j is growing at a constant rate $\eta<i$ and we have the following.[6)]

[10] $$Y=\sum_{j=1}^{\infty}\frac{y_j}{(1+i)^j}=\frac{y_1}{i-\eta}$$

Noting that $y_1=(1+\eta)y_0$, then if $i=0.05$ and $\eta=0.03$

[11] $$\frac{D_0}{Y}=\frac{(i-\eta)}{(1+\eta)}\frac{D_0}{y_0}=\frac{0.01}{1.03}\approx 0.01\frac{D_0}{y_0}$$

As the debt-to-GDP ratio increases, the tax-smoothing tax rate also increases.

[12] $$\begin{aligned}\tau^*&=\frac{D_0+G}{Y}\\&=\frac{G}{Y}+\frac{(i-\eta)}{(1+\eta)}\frac{D_0}{y_0}\\&=0.15+0.01\frac{D_0}{y_0}\end{aligned}$$

Delay in implementing the tax-smoothing tax rate allows the debt-to-GDP ratio to rise. When the tax-smoothing rate is implemented it will have become a little higher than before, meaning the excess burden in each subsequent year will have also become a little higher than it would have been under immediate implementation, and stay higher in each year ever after. The excess burden in the years before implementing the tax-smoothing rate will of course be lower, but not by enough to offset the later increase in excess burden. Delay is costly. But by how much?

First, how much will government debt grow relative to GDP if the tax rate lies below the level that minimizes the present value of all future tax burdens — the 'tax-smoothing' tax rate? Suppose that $\frac{G}{Y}=0.15$, $\frac{D_0}{y_0}=0.4$, $i=0.05$ and $\eta=0.03$. The tax-smoothing tax rate is $\tau^*=\frac{D_0+G}{Y}=0.158$. If the tax rate is held at $\tau_j=0.10$ the debt-to-GDP ratio will rise to the following level after N years.

$$[13] \quad \frac{D_N}{y_N}=\left(\frac{1+i}{1+\eta}\right)^N\frac{D_0}{y_0}+\frac{(G_0-T_0)}{y_0}\sum_{j=1}^{N}\left(\frac{1+i}{1+\eta}\right)^{N-j}$$

$$[14] \quad \frac{D_N}{y_N}=\left(\frac{1+i}{1+\eta}\right)^N 0.40+0.05\sum_{j=1}^{N}\left(\frac{1+i}{1+\eta}\right)^{N-j}$$

Next, with a primary deficit $\frac{G_0-T_0}{y_0}=0.05$ and initial debt-to-GDP ratio $\frac{D_0}{y_0}=0.40$, find that after nine years and after 19 years

$$[15] \quad \frac{D_9}{y_9}=0.98$$

$$[16] \quad \frac{D_{19}}{y_{19}}=1.74$$

We next explore how much it might add to the tax burden to allow government debt to accumulate in this manner.

3.2. Rising government debt and the cost of delay in implementing the optimal tax rate

What are the effects of holding the tax rate below the level consonant with minimizing the present value of excess burden? How much does delay in implementing the 'tax-smoothing' tax rate increase the present value of the tax burden? To keep matters simple, posit an economy with a constant growth rate, η, which is less than the constant interest rate, i. Suppose also that government spending is growing at the same rate as GDP — meaning that government spending is a constant fraction of GDP.

The 'tax-smoothing' tax rate to be implemented starting the year after any initial time *0* is

[17] $\tau_1^* = \dfrac{G_0}{y_0} + \dfrac{(i-\eta)}{(1+\eta)}\dfrac{D_0}{y_0}$

If the tax rate is unchanging (meaning that the primary deficit is an unchanging fraction of GDP), the debt-to-GDP ratio will rise to the following level in year N,

[18] $\dfrac{D_N}{y_N} = \left(\dfrac{1+i}{1+\eta}\right)^N \dfrac{D_0}{y_0} + \dfrac{(G_0 - T_0)}{y_0}\sum_{j=1}^{N}\left(\dfrac{1+i}{1+\eta}\right)^{N-j}.$

The tax-smoothing tax rate to be implemented in any future year $N(>1)$ is thus the following.

[19] $\tau_N^* = \dfrac{G_0}{y_0} + \dfrac{(i-\eta)}{(1+\eta)}\dfrac{D_{N-1}}{y_{N-1}}$

[20] $\tau_N^* = \dfrac{G_0}{y_0} + \dfrac{(i-\eta)}{(1+\eta)}\left[\left(\dfrac{1+i}{1+\eta}\right)^{N-1}\dfrac{D_0}{y_0} + \dfrac{(G_0-T_0)}{y_0}\sum_{j=1}^{N-1}\left(\dfrac{1+i}{1+\eta}\right)^{N-j}\right].$

The present value of excess burden is minimized if the tax-smoothing tax rate is set in year 1 and maintained ever after. If the current tax rate, $\tau_0 = \dfrac{T_0}{y_0}$, is below the tax-smoothing tax rate, delay in implementing the tax-smoothing rate to a year $N>1$ will mean that the tax-smoothing rate has increased. And it will have increased more, the longer the delay.

[21] $\tau_N^* - \tau_1^* = \dfrac{(i-\eta)}{(1+\eta)}\left[\left(\dfrac{1+i}{1+\eta}\right)^{N-1} - 1\right]\dfrac{D_0}{y_0} + \dfrac{(i-\eta)}{(1+\eta)}\left[\sum_{j=1}^{N-1}\left(\dfrac{1+i}{1+\eta}\right)^{N-j}\right]\dfrac{(G_0-T_0)}{y_0}.$

The excess burden of taxes in year j varies with the square of the tax rate τ_j, and with the elasticity, ξ, of the income-compensated supply of labor: $1/2\xi\theta^2\tau_j^2 y_j$. Delay in implementing the tax-smoothing rate will mean that excess burden is lower in initial years than it would have been with immediate implementation, but then higher ever after, once the tax-smoothing tax rate is finally implemented. The change in present value (in year 0) of increased future excess burden attributable to delay in implementing the tax-smoothing tax rate is the following.[7]

[22] $\Delta B = \overbrace{\dfrac{1/2\xi\theta^2 y_0}{i-\eta}\left(1-\left(\dfrac{1+\eta}{1+i}\right)^{N-1}\right)(\tau_0^2 - \tau_0^{*2})}^{\textit{reduction in burden in years } 1 \textit{ to } N-1} + \overbrace{\dfrac{1/2\xi\theta^2 y_0}{i-\eta}\left(\dfrac{1+\eta}{1+i}\right)^N(\tau_N^{*2} - \tau_1^{*2})}^{\textit{increase in burden in years } N\cdots\infty}$

Let us continue the previous example in in which $\dfrac{G}{Y}=0.15$, $\dfrac{D_0}{y_0}=0.4$, $i=0.05$, and $\eta=0.03$, with a primary deficit $\dfrac{G_0-T_0}{y_0}=0.05$ and initial debt-to-GDP ratio $\dfrac{D_0}{y_0}=0.40$. We had determined that $\tau_1^*=0.158$, and that if the tax rate were left at its initial level, $\tau_0=0.10$, then after 9 years and after 19 years, the debt-to-GDP ratio would rise to

$\frac{D_9}{y_9}=0.98$ and $\frac{D_{19}}{y_{19}}=1.74$. As a result of this accumulation of debt, after 10 years and after 20 years the tax-smoothing tax rate, $\left(\tau_N^*=\frac{G}{Y}+\frac{(i-\eta)}{(1+\eta)}\frac{D_N}{y_N}\right)$, would rise from $\tau_1^*=0.158$ to $\tau_{10}^*=0.169$ or $\tau_{20}^*=0.184$.

Based on all the givens listed in the preceding paragraph, one calculates that the present value of excess burden (not just in year 1 but the stream of excess burden through the infinite future), scaled in relation to parameters, and contingent on enactment of the tax-smoothing tax rate starting in year 1 is as follows.

$$B=1/2\xi\theta^2\times 1.248\ y_0$$

Let us suggest ranges of plausible values of the parameters in this expression — parameter ξ denoting the elasticity of income-compensated supply of labor, and θ denoting ratio of marginal tax rate to average tax rate. As related at the previous section 2.1, estimates of labor supply elasticity for Japan and the US place it around 0.3, but we might suppose it to be higher for a developing country such as Bangladesh, say 0.5. Also the long-run elasticity of tax revenue with respect to GDP for Bangladesh has been estimated at 1.14, and as high as 1.2 for developed countries including Japan and the US with more progressive tax systems. This suggests that, for Bangladesh, a plausible assumption is that $1/2\xi\theta^2=\frac{1}{3}$. That means that the present value of excess burden is around 40 percent of current GDP: $\frac{1}{3}\times 1.248\ y_0=0.406\ y_0$.

With ten-year delay, the excess burden is increased by less than 3 percent compared to no delay (0.0348/1.248=2.7%). This can be broken down into the reduction in burden in the first years and increase in later years as shown here.

$$\frac{\Delta B}{(1/2\xi\theta^2)}=0.034\ y_0\ (=-0.118\ y_0+0.152\ y_0)$$

With twenty-year delay, the present value of excess burden is increased by about 6 percent compared to no delay (0.076/1.248=6.1%). Again, broken down into the two parts.

$$\frac{\Delta B}{(1/2\xi\theta^2)}=0.076\ y_0\ (=-0.228\ y_0+0.304\ y_0)$$

The increase in excess burden caused by delay in implementing the tax-smoothing tax rate is small — less than the effects of a single business recession or major natural disaster. Is it any wonder, then, that the governments of developing countries are disinclined to curb their rising debt-to-GDP ratios? On the other hand, maybe the focus on tax burden that is the heart of the Barro tax-smoothing approach misses important adverse effects of sovereign debt accumulation. This prompts us to consider the 'fiscal sustainability' paradigm of Blanchard and others. The Blanchard approach points toward a less sanguine view of sovereign debt accumulation than the Barro approach. Under narrow assumptions, the two frameworks propose identical tax-rate trajectories, but the Blanchard approach opens the possibility that accumulation of government debt has costs that the Barro approach elides.

4. Tax-smoothing and debt sustainability

4.1. Relation between 'tax-smoothing' tax rate and 'sustainable tax rate'

We shall next consider how the tax-smoothing tax rate is related to the 'sustainable tax rate' defined by Blanchard (1990) as the tax rate that would maintain the current debt-to-GDP. Let us adopt the Blanchard practical definition in which 'sustainable tax rate' is such that the debt in relation to GDP at some future year, J, is equal to its current level $\frac{D_0}{Y_0}$. The constrained-minimization problem that determines this sustainable tax rate is

$$[23]\quad \min_{\tau 1,\cdots,\tau J} \mathscr{L}=B-\lambda\left(T-\frac{D_J}{y_J}-G\right),\ \text{where}\ \frac{D_J}{y_J}=\lim_{j\to J}\frac{D_j}{y_j}$$

The 'sustainable' tax rate — deduced by Broda and Weinstein (2005) — is approximately the following.

$$[24]\quad \tau^*=\begin{cases} \frac{i-\eta}{1+\eta}\left[\frac{D_0}{y_0}+\left\{1-\left(\frac{1+\eta}{1+i}\right)^n\right\}^{-1}\sum_{j=1}^{n}\left(\frac{1+\eta}{1+i}\right)^j\frac{G_j}{y_j}\right], & if\ i>\eta \\ \frac{1}{n}\sum_{j=1}^{n}\frac{G_j}{y_j}, & if\ i=\eta \\ \frac{\eta-i}{1+i}\left[-\frac{D_0}{y_0}+\left\{1-\left(\frac{1+i}{1+\eta}\right)^n\right\}^{-1}\sum_{t=1}^{n}\left(\frac{1+i}{1+\eta}\right)^{n-j}\frac{G_j}{y_j}\right], & if\ i<\eta. \end{cases}$$

For constant GDP growth rate less than the interest rate $(\eta<i)$, and government spending relative to GDP that remains constant, $\left(\frac{G_j}{y_j}=\frac{G_0}{y_0},\ \forall j\right)$, the sustainable tax rate, τ^*, and tax-smoothing tax rate, τ_1^*, are identical.

$$[25]\quad \lim_{n\to\infty}\tau^*=\tau_1^*=\frac{G_0}{y_0}+\frac{(i-\eta)}{(1+\eta)}\frac{D_0}{y_0},\ if i>\eta$$

Invert this to show the steady-state debt-to-GDP ratio supported by a given unchanging tax rate, $\bar{\tau}$. That is, a steady-state debt-to-GDP ratio equal to $\frac{D_0}{y_0}$ requires an unchanging tax rate $\bar{\tau}=\tau^{*}$. But under the stated assumptions, any steady-state debt-to-GDP ratio — once attained — is supported by an unchanging tax rate that fulfills the following equation.

$$[26]\quad \overline{\left(\frac{D}{y}\right)}=\left(\bar{\tau}-\frac{G}{Y}\right)\frac{(1+\eta)}{(i-\eta)},\ ifi>\eta$$

If the interest rate is less than the growth rate, $i<\eta$, then the sustainable tax rate would fulfill these same expressions but would entail a persistent primary fiscal deficit rather than fiscal surplus. Under the same assumption, $i<\eta$, the tax-smoothing tax rate would be zero — minimizing the present value of excess burden would mean postponing *all* taxes indefinitely, and in this case the debt-to-GDP ratio would continue to rise without bound and not fulfill the Blanchard sustainability criterion. Here is an inkling that tax-smoothing and fiscal sustainability are indeed different concepts. To explore this idea further we next expand our framework so that a higher debt-to-GDP ratio itself has costs in addition to the increased excess burden caused by deviations from tax smoothing.

4.2. Fiscal sustainability when greater debt entails costs of its own

A recurring idea in the sustainability-of-government-debt literature is that a high government-debt-to-GDP ratio is perilous, a thing to be avoided. How can we represent this idea in the Barro framework? First, recall that the Lagrange multiplier in the Barro framework is the shadow price of relaxing the constraint. We reprint the optimization problem here.

$$[1]\quad \min_{\tau 1,\cdots,\tau\infty}\mathcal{L}=B-\lambda(T-D_0-G),$$

The solution, as noted before, is the following.

$$[3]\quad \tau_j^{*}=\frac{\lambda^{*}}{\theta\xi(\theta+\lambda^{*})}=\frac{D_0+G}{Y},\ \forall j$$

where λ^{*} is the shadow price of relaxing the constraint, which is equivalent to reducing the initial debt.

$$[27]\quad \lambda^{*}=\frac{dB^{*}}{dD_0}$$

In this sense, λ^{*} is the shadow price of debt itself. Solve Eq [3], $\frac{\lambda^{*}}{\theta\xi(\theta+\lambda^{*})}=\frac{D_0+G}{Y}$, for λ^{*} and we find this.

$$[28]\quad \lambda^{*}=\frac{\xi\theta^{2}\frac{(D_0+G)}{Y}}{1-\xi\theta\frac{(D_0+G)}{Y}}=\frac{\xi\theta^{2}(D_0+G)}{Y-\xi\theta(D_0+G)}$$

Now note the following.

$$\frac{d\lambda^*}{dY}<0 \text{ and } \lim_{Y\to\infty}\lambda^*=0.$$

The shadow price of debt is lower, the greater is Y, the present value of the stream of real GDP, holding constant the present value of the stream of government spending, G. And as the present value of real GDP increases without bound, the shadow price of debt approaches zero. Debt has a positive shadow price because allowing debt to rise in one period (by lowering the tax rate in that period), necessitates a higher tax rate in a later period to service the debt. And if the tax rate trajectory is optimal to begin with and the shadow price is positive, deviations from the implied optimal trajectory of debt must increase the total tax burden. At the margin, that increase in tax burden is λ^*. In this framework, the shadow price of debt is related to the amount of debt in relation to the future productive capacity of the economy. It is not the debt-to-GDP ratio that matters, but the debt-to-'future productive capacity' ratio. For example, if the interest rate is perpetually less than the GDP growth rate, then the present value of the stream of real GDP is infinite and the shadow price of debt is zero. The constraint that the stream of taxes must be enough to fund the stream of government spending and service the debt is not binding. Then debt can be allowed to rise without necessitating future tax increases.

Recall from Eq. [10] that if y_j is growing at a constant rate η and the interest rate is also constant and higher than the growth rate, then $Y=\frac{y_1}{i-\eta}$. If an economy is growing more slowly, or if the interest rate gap is greater, the future productive capacity of the economy has a smaller present value, Y, and the shadow price of debt will be higher.

Now suppose that debt itself has costs that arise other than by causing taxes to deviate from the burden-minimizing trajectory. For instance, sovereign debt that is 'too high' might trigger a default, a sudden stop in foreign exchange transactions, or a hyperinflation. How can we add this to the Barro framework? One way — the simplest way — is to introduce a parameter ω that amplifies the shadow price of debt. Change the constraint so that D_0 is replaced by $\widehat{D_0}=\omega D_0$, where $\omega\geq 1$ is the weight placed on debt in the objective function. The constraint becomes $T\geq\omega D_0-G$. Now the new shadow price at the optimum becomes the following.

[29] $$\hat{\lambda}^*=\frac{dB^*}{d(\omega D_0)}=\omega\frac{dB^*}{dD_0}=\omega\lambda^*$$

The higher shadow price of debt induces a higher optimal tax rate.

[30] $$\widehat{\tau_j^*}=\frac{\hat{\lambda}^*}{\theta\xi(\theta+2\hat{\lambda})}=\frac{\omega D_0+G}{Y},\ \forall j$$

[31] $\hat{\lambda}^{*}=\dfrac{\xi\theta^{2}}{Y/(\omega D_{0}+G)-\xi\theta}$

There is still tax-smoothing, but the optimal tax rate is higher, the greater is ω, reflecting the greater aversion to debt that a higher ω represents. Also, the cost of delay in implementing the optimal tax trajectory will become greater, the greater is ω.

Our analysis so far elides the possibility that the government debt-to-current-GDP ratio — in our notation, $\frac{D_j}{y_j}$ and not just $\frac{D_j}{Y}$ — may entail costs beyond those reflected in the shadow price of debt, $\omega\lambda^{*}$. This points to the idea that not just tax smoothing but also 'debt smoothing' is on net beneficial. For example, Blanchard (2019, pp. 1230–1231) discusses why a high debt-to-GDP ratio might be worrisome even in cases where the government budget constraint is not binding. In his telling, there might be two equilibria in the government debt market, one with low interest rates in which the government budget constraint is not binding and another with high interest rates in which the constraint is binding. Which equilibrium attains depends on the beliefs of investors. A high government debt-to-GDP ratio could shift beliefs from the low-interest-rate equilibrium toward the high-interest-rate equilibrium, an adverse fiscal shock that could cause a recession or worse. To add this element of the costs of government debt to our framework let us stipulate that the optimal trajectory of tax rates is constrained by an upper bound on the debt-to-GDP ratio.

[32] $\dfrac{D_j}{Y_j}\leq\kappa,\ \forall j$

When the constraint is binding, the debt-to-GDP ratio will be at its upper bound, implying a lower bound on the tax rate that is also binding. When the debt-to-GDP constraint is binding, the tax rate will be higher than it would have been without this constraint. But this also implies that the tax rate must become lower than it otherwise would have been in years when the debt-to-GDP constraint is not binding. Intuitively, to hasten a reduction in the debt-to-GDP ratio if it is initially too high, it may be best to raise the tax rate above its long-run optimal level and then lower it in steps, a kind of initially overshooting trajectory. This is an implication of the fiscal sustainability framework that is absent from the basic tax-smoothing model.

The 'fiscal sustainability framework' is our name for the class of models in which costs arise from an inflated government debt itself, unlike the tax-smoothing model in which the only cost is the excess burden of taxes needed to fund the given stream of government spending. As a practical matter, we wish to ascertain how large these costs might be, both the excess burden cost and the others. We will take Bangladesh as the concrete example for this exercise.

5. Estimating the cost of delay in implementing an optimal tax trajectory in Bangladesh

5.1. Forecasts of government spending and real GDP in Bangladesh, and implied sustainable tax-rate trajectories.

In a previous paper, (Begum and Flath, 2020), we estimated the sustainable tax rate for Bangladesh if implemented immediately — that is in 2020 — under three different but plausible future trajectories of government spending, a projected trajectory of real GDP, and alternative interest rates. Here we want to revisit those cases, to calculate the excess burden of taxes in each case and determine how delay in implementing an optimal tax rate trajectory might add to that burden. We also want to explore how imposing an upper bound on the debt-to-GDP would add to the tax burden. This is a first step in calculating how great the costs arising from government debt accumulation would have to be in order to justify the fiscal austerity needed to impose such a bound. The first step is to briefly describe the assumptions underlying the cases we consider. For more details, refer to the companion paper.

As in the companion paper to this one, we have constructed a forecast of the growth trajectory of real GDP from 2020 until 2100. We assume growth in per-capita real GDP of 2 percent per year and apply that to a United Nations Population Division forecast of Bangladesh population growth. The 2 percent per year per-capita growth combined with the UN population projection implies growth in real GDP averaged over the forecast interval, 2020 to 2100, of about 3 percent per year.

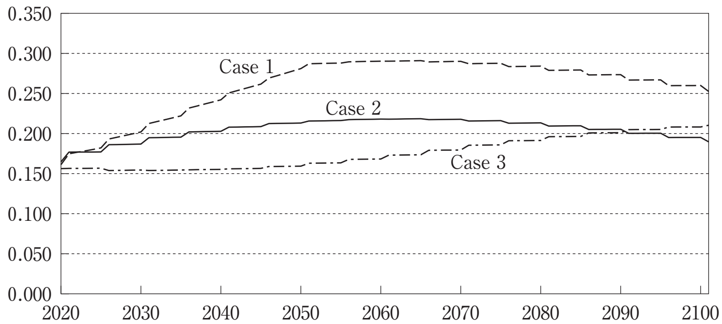
We make three separate forecasts of the future trajectory of government spending other than for debt servicing.

- Case 1: Government expenditures per person rise until 2050 at an annual rate that is one percent greater than the growth rate of GDP, and after 2050 grow at the same rate as GDP.
- Case 2: Government expenditures per person are always proportional to GDP.
- Case 3: Government expenditures per person are always proportional to GDP per worker.

These encompass a wide range of possibilities, from rapid growth in government spending per person (Case 1) to moderate growth in government spending per person (Case 2) to slow but persistent growth in government spending (Case 3). For Bangladesh, rapid growth in government spending is plausible because as income per person rises with development, the population may well demand greater provision of government services.

The initial values for government spending, real GDP and outstanding government debt are informed by recent observations for Bangladesh. As in the companion paper, we set

Figure 2. Bangladesh government expenditure as a fraction of GDP, for the three different cases, 2020-2100



Source: Authors' own calculation

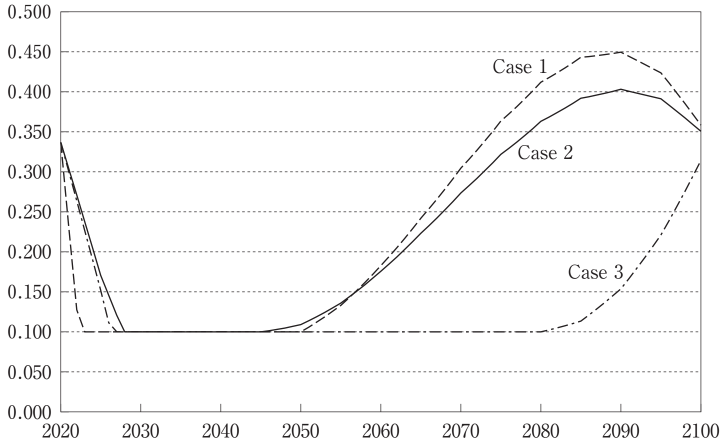
initial government-spending-to-GDP $\frac{G_0}{y_0}=0.171$, initial debt-to-GDP $\frac{D_0}{y_0}=0.337$, and initial average tax rate $\tau_0=0.10$. Here, we will set the interest rate $i=0.05$. In our constructions and simulations, we treat the forecast trajectory of real GDP as representing the actual real GDP, not the potential GDP consonant with zero tax distortion. This simplifies our calculations but imparts a small bias to some of the comparisons (see f.n. 4 and related discussion above for details).

The Figure 2 (which reprises Figure 3 from Begum and Flath, 2020) shows the forecast trajectories of government spending other than for debt servicing relative to GDP under each of the three cases.

We have constructed sustainable-tax-rate trajectories for each of the three cases, under varying assumptions. In the previous paper we presented the tax-smoothing tax rate for each case — the tax rate that if set immediately (in year 2020) and maintained through year 2100 would return the debt-to-GDP ratio to its initial level (=0.337). These are the tax rates that would, in principle, minimize the present value of excess burden of taxes, conditional on the assumed trajectory of government spending. But as we showed in the previous paper, implementing such a constant tax-rate trajectory would induce periods in which the debt-to-GDP is negative, in which the government is a (massive) net lender rather than a borrower. This is unrealistic, so we recalculated the sustainable-tax-rate trajectories subject to the constraint that the debt-to-GDP ratio is greater than or equal to 0.10. The Figures 3a and 3b (which reprise Figs 6a and 6b from Begum and Flath, 2020) show the constrained tax-rate trajectories and their corresponding debt-to-GDP ratio trajectories. Following the sustainable tax rate definition of Blanchard, and tax-smoothing logic of Barro — which here are one and the same — the tax rate is constant in any years when the constraint is not binding.

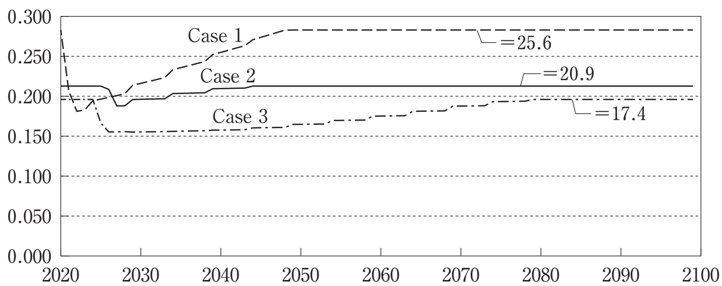
In years for which the assumed 0.10 lower bound of the debt-to-GDP ratio is binding, the tax rate is subject to an upper bound that is also binding. To return the debt-to-GDP

Figure 3a. Debt-to-GDP for each of the three cases, with sustainable tax rate constrained by lower bound of debt-to-GDP≥0.10.



Source: Authors' own calculation

Figure 3b. Sustainable tax rate trajectories for each of the three cases, constrained by lower bound of debt-to-GDP≥0.10.



Source: Authors' own calculation

ratio to its initial level (=0.337) in 2100, thus requires the tax rate to be higher than it otherwise would be in the years when the constraint is not binding. As we have argued, these sustainable-tax-rate trajectories — that entail the same tax rate in any year when the minimum debt-to-GDP constraint is slack — minimize the present value of excess burden. As shown in Table 1, we have computed this excess burden parametrically, for each case, with and without the minimum debt-to-GDP condition. The Table also describes the sustainable tax trajectories, if initiated without delay, calculated with and without debt ratio constraint.

It is evident from the table that the minimum-debt-ratio constraint increases the highest tax rates along the sustainable trajectory by about 0.02. This has a measurable but modest effect on the present value of excess burden. Note that the present value of excess burden is expressed parametrically. The parameters are income-compensated elasticity of

Table 1. Sustainable tax rates and excess burden with and without lower bound on debt-to-GDP ratio

	Sustainable tax rates		
	Case 1	**Case 2**	**Case 3**
Sustainable tax rates if initiated without delay and no constraint on debt-to-GDP ratio	0.256	0.209	0.174
Highest tax rates along sustainable tax-rate trajectories initiated without delay, with lower bound on debt-to-GDP ratio equal to 0.10	0.283	0.213	0.196
	Present value of excess burden *Units*$=\xi\theta^2\times$*multiples of 2019 real GDP* (y_0)		
	Case 1	**Case 2**	**Case 3**
No constraint on debt-to-GDP ratio	1.881	1.253	0.865
Lower bound on debt-to-GDP ratio equal to 0.10	1.988	1.262	0.917

Source: Authors' own calculation

labor supply, ξ, and ratio of marginal tax rate to average tax rate, θ. Bangladesh, like most developing countries, has a somewhat distorting tax system and moderately progressive tax structure. Because we presume that all taxes are shifted onto labor, the parameter, ξ, captures all elements of tax distortion. Developing countries like Bangladesh with large informal sectors are constrained in their capacity to collect broad-based and non-distorting taxes. To that extent, we might suppose that the parameter ξ is higher for Bangladesh than for high-income countries such as Japan. If ξ is around 0.3 for Japan, then perhaps it is closer to 0.5 for Bangladesh. The progressivity parameter for Bangladesh might be around 1.14 based on the estimates of Yousuf and Huq (2013). That places $\xi\theta^2$ around 0.65. Based on our estimates in Table 1, the present value of excess burden under the six different sets of assumptions ranges from 0.56 $(=0.865\times0.65)$ to 1.64 $(=2.521\times0.65)$ times 2019 real GDP $(=y_0)$. This is roughly the same order of magnitude we calculated for the constant-growth-rate example discussed earlier. In the calculations shown in Table 1 we included an estimate of the present value of excess burden from 2100 onward, based on assumed interest rate gap of 0.02 and unchanging tax rate. This matches the example from earlier. The GDP path and government spending trajectory in the three cases on which the Table 1 figures are constructed diverge a bit from the earlier example, maybe in a way that is closer to the actual situation of Bangladesh. As we did in the earlier example, we next want to consider the effects on excess burden of a ten-year or twenty-year delay in implementing tax sustainability.

5.2. Cost of delay in implementing a sustainable tax-rate trajectory

The Figures 4a and 4b, 5a and 5b, and 6a and 6b, show the trajectories of government-debt-to-GDP and their corresponding tax-rate trajectories for Cases 1, 2 and 3. The tax-

Figure 4a. Debt-to-GDP for Case 1, under various tax-rate trajectories.

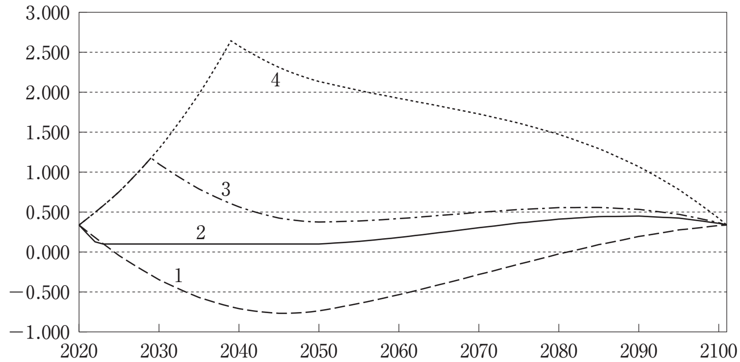


Figure 4b. Sustainable tax-rate trajectories for Case 1, under various conditions: 1. implementation in year 1 (2020) with no lower bound on debt-to-GDP, 2. implementation in year 1 with 0.10 lower bound on debt-to-GDP, 3. Implementation from year 10 (2040), and 4. Implementation from year 20 (2020).

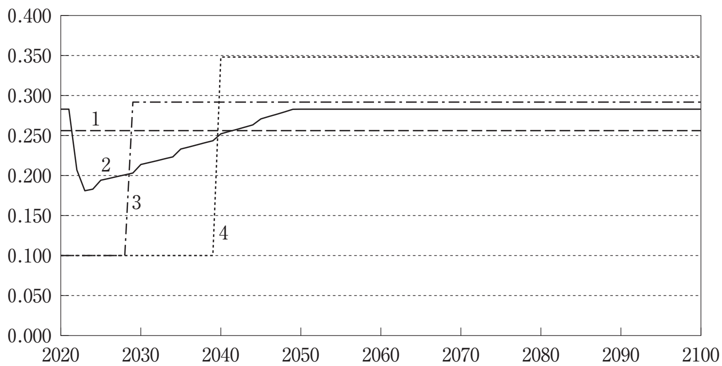
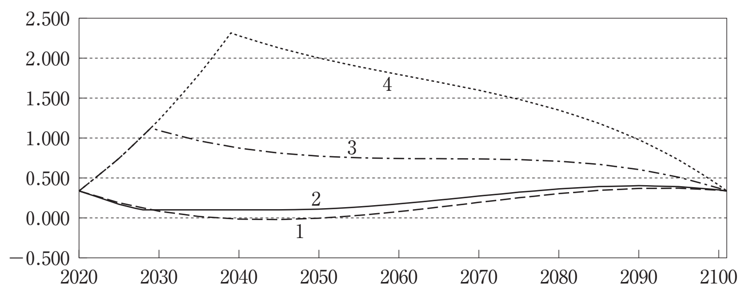


Figure 5a. Debt-to-GDP for Case 2, under various tax-rate trajectories.



rate trajectories are sustainable ones under various conditions: 1. implementation in year 1 (2020) with no lower bound on debt-to-GDP, 2. implementation in year 1 with 0.10 lower bound on debt-to-GDP, 3. Implementation from year 10 (2030), and 4. Implementation from

Figure 5b. Sustainable tax-rate trajectories for Case 2, under various conditions: 1. implementation in year 1 (2020) with no lower bound on debt-to-GDP, 2. implementation in year 1 with 0.10 lower bound on debt-to-GDP, 3. Implementation from year 10 (2040), and 4. Implementation from year 20 (2020).

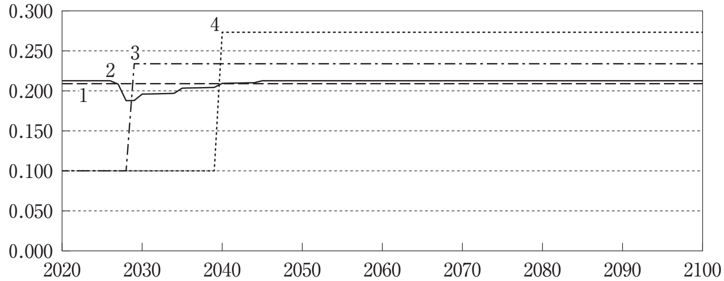


Figure 6a. Debt-to-GDP for Case 3, under various tax-rate trajectories.

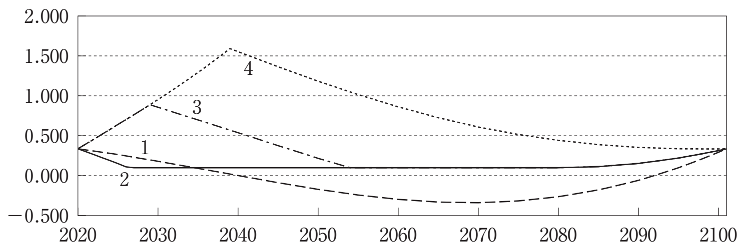
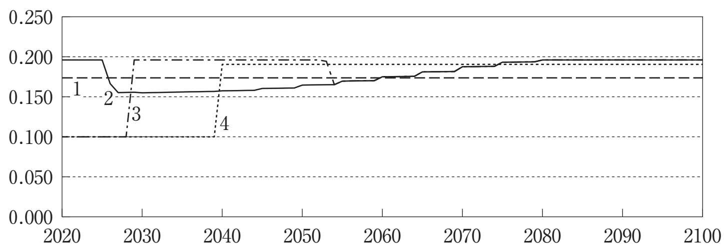


Figure 6b. Sustainable tax-rate trajectories for Case 3, under various conditions: 1. implementation in year 1 (2020) with no lower bound on debt-to-GDP, 2. implementation in year 1 with 0.10 lower bound on debt-to-GDP, 3. Implementation from year 10 (2030) with 0.10 lower bound on debt-to-GDP, and 4. Implementation from year 20 (2040).



year 20 (2040). The 0.10 lower bound on debt-to-GDP is binding for all three cases if a sustainable tax rate is implemented in year 1, and binding for Case 2 if implemented in year 10. It is not binding for the other instances.

In general, delay in implementing the sustainable tax-rate trajectory implies that the trajectory will be higher than if implemented earlier. This is because of the accumulation

Table 2. Excess burden under various sustainable tax-rate trajectories

	Present value of excess burden *Units=$\xi\theta^2\times$multiples of 2019 real GDP (y_0)*		
	Case 1	Case 2	Case 3
Implementation in year 1 (2020)			
1. with no lower bound on debt-to-GDP	1.881	1.253	0.865
2. with 0.10 lower bound on debt-to-GDP	1.988	1.262	**0.917**
Implementation from year 10 (2030)			
3. with no lower bound on debt-to-GDP	2.118	1.377	0.927
3. with 0.10 lower bound on debt-to-GDP			**0.941**
Implementation from year 20 (2040)			
4. with no lower bound on debt-to-GDP	2.521	1.586	**1.033**
4. with 0.10 lower bound on debt-to-GDP			

Table 3. Percent increase in present value of excess burden attributable to 10-year and 20-year delay in implementing sustainable tax-rate trajectory

	Present value of excess burden *Units=$\xi\theta^2\times$multiples of 2019 real GDP (y_0)*		
	Case 1	Case 2	Case 3
Implementation in year 1 (2020)			
1. with no lower bound on debt-to-GDP	1.881	1.253	0.865
2. with 0.10 lower bound on debt-to-GDP	**1.988**	**1.262**	**0.917**
	Percent increase in present value of excess burden compared to implementation in year 1 (2020)		
Implementation from year 10 (2030)			
3. with no lower bound on debt-to-GDP	12.6%	9.9%	7.2%
3. with 0.10 lower bound on debt-to-GDP	**6.5%**	**9.1%**	**2.6%**
Implementation from year 20 (2040)			
4. with no lower bound on debt-to-GDP	34.0%	26.6%	19.4%
4. with 0.10 lower bound on debt-to-GDP	**26.8%**	**25.7%**	**12.6%**

of government debt under a regime of low tax rates. This pattern is evident in the figures.

We want to know how delay in implementing a sustainable tax rate trajectory affects the present value of excess burden. Table 2 reports the present value of excess burden under the various sustainable tax-rate trajectories depicted in the previous figures. The first two rows reprise the estimates shown in Table 1. The other rows show how delay in implementing a sustainable tax-rate trajectory by ten years or by twenty years affects the present value of excess burden. Table 3 shows the percent increase in present value of excess burden caused by a ten-year and twenty-year delay in implementing a sustainable tax-rate trajectory. These percentages are calculated based on comparison of tax-rate trajectories either with 0.10 minimum lower bound on debt-to-GDP, or without such a bound.

Within these examples, delay in implementing a sustainable tax rate has a monotonic effect on the present value of excess burden. Delay increases excess burden. This effect is smaller for Case 3 under the 0.10 debt ratio lower bound constraint than for the other cases. There is an intuitive explanation for this. When the lower bound on debt-to-GDP and corresponding upper bound on the tax rate is a binding constraint, excess burden is made greater. Delay in implementing the sustainable tax-rate trajectory allows the debt-to-GDP ratio to rise which relaxes this constraint, and to just that extent dampens the increase in excess burden caused by the delay.

It seems from these estimates that, for Bangladesh at least, a ten-year delay in implementing a sustainable tax-rate trajectory would add to the present value of excess burden by less than ten percent of 2019 real GDP — and perhaps much less, as little as one or two percent of 2019 GDP. A twenty-year delay would be three or four times more costly than a ten-year delay. The upshot is that the potential gains from tax smoothing by Bangladesh are significant but modest and can withstand a delay of ten or even twenty years before implementation.

Our final question is, how large are the costs directly attributable to a high debt ratio.

5.3. Quantifying the costs of debt that are not included in the excess burden of taxes needed to fund government spending.

In the Blanchard sustainable tax-rate calculation, the sustainable tax rate is defined as the one that would eventually return the debt-to-GDP ratio to its current level. The aim is to determine whether taxes need to be adjusted to assure that the current debt-to-GDP ratio is approached in steady-state; that is the meaning of a 'sustainable' debt ratio. This makes sense if the current debt-to-GDP ratio is at its steady-state value under the optimal tax regime, even if the current tax regime is not optimal. But what is the optimal steady-state debt-to-GDP ratio? In the Barro tax-smoothing framework, the optimal steady-state debt-to-GDP ratio is the current one, whatever it might be. Taxes must be adjusted to collect just enough revenue to fund the given stream of government spending and pay down the current debt, whatever it is. But in the fiscal sustainability framework, the steady-state debt-to-GDP ratio should be considered a choice, not a given.

If there are costs of debt that are not included in the excess burden of taxes needed to fund government spending, then the shadow price of government debt will be greater than implied by the marginal increase in excess burden resulting from an increase in debt. Amplifying the shadow price of debt in the Barro framework by the parameter $\omega>1$ induces a higher optimal tax rate — given the initial debt — as shown in Eq. [30] which we reproduce here.

[30] $$\widehat{\tau_j^*}=\frac{\hat{\lambda}^*}{\theta\xi(\theta+2\hat{\lambda})}=\frac{\omega D_0+G}{Y},\ \forall j$$

The tax rate is made higher by the necessity of servicing not only the initial debt and stream of government spending D_0+G, but also of covering the special costs of debt $(1-\omega)D_0$ that are implied by $\omega>1$. One may think of the resulting greater flow of tax revenue this higher tax rate entails as the cost of debt that the Barro framework elides. Its present value is the following.

[31] $$(\widehat{\tau^*}-\tau^*)Y=(\omega-1)D_0,$$

It seems that the initial debt itself, D_0, places an upper bound on this special cost, because if the initial debt were zero, the special cost would be zero, and the initial debt could be fully retired with a payment equal to D_0. Then

[32] $$(\widehat{\tau^*}-\tau^*)Y=(\omega-1)D_0\leq D_0,$$

It follows that $\omega\leq 2$. With an interest rate of 5 percent as in our simulations, and a GDP growth rate of 3 percent, then at most, the special cost of debt would increase the sustainable tax rate by

[32] $$\widehat{\tau^*}-\tau^*=\frac{D_0}{Y}=\frac{D_0}{y_0}\frac{(i-\eta)}{(1+\eta)}\approx 0.01\frac{D_0}{y_0}$$

The upshot of this discussion is that the costs of Bangladesh government debt besides the excess burden of taxes needed to fund government are smaller than the costs that are included in the Barro framework, and would at most imply an increase in the sustainable tax rate less than one percent of GDP.

We hasten to add that there may well be costs of government debt in addition to those so far discussed. The possibility that a higher debt ratio increases the likelihood of a fiscal crisis of some kind, a sudden-stop in foreign exchange or foreign trade, a deep recession, or a disruption of the financial system needs to be explored further. It remains beyond the scope of the current paper.

We hasten to add that there may well be costs of government debt in addition to those so far discussed. The possibility that a higher debt ratio increases the likelihood of a fiscal crisis of some kind, a sudden-stop in foreign exchange or foreign trade, a deep recession, or a disruption of the financial system needs to be explored further. It remains beyond the scope of the current paper.

6. Conclusion.

The tax-smoothing framework answers a fundamental question: What is the purpose of government debt? It exists to allow a decoupling of government spending and government tax revenue. Period. This is useful because government spending is variable. It spikes in wartime. It rises over time as an economy develops. And tax revenue also varies. It is procyclic, plummeting during recessions and exploding during booms. It falls during natural disasters. The Barro tax-smoothing logic shows that requiring taxes to match the variation in government spending unnecessarily increases the tax burden. And requiring government spending to match variation in tax revenue would unnecessarily constrain the government provision of goods and services.

The Blanchard analysis focusing on the costs that attend government debt and that the Barro framework elides is also useful. The added tax burden caused by wide deviations from tax-smoothing seems to be small enough for governments to ignore without inciting political opposition. The added costs of debt issue that are absent from the Barro analysis — and that are potentially quite large — must be the main driver of variation in government debt and political attempts to stem its rise over time.

We conducted simulations aimed at quantifying the added excess burden of taxes that would result from a ten-year and twenty-year delay in implementing a tax-smoothing tax-rate trajectory in Bangladesh and found these costs to be relatively modest. Concern about primary fiscal deficits in Bangladesh — recently around 5% of GDP — must be based on the perceived perils of a high and rising government-debt-to-GDP ratio, not on the added excess burden caused by deviation from tax-smoothing.

注

1) The arc elasticity of income-compensated labor supply with respect to after-tax wage measured from the no tax point is $\xi=\frac{\Delta L}{(tw_0)}\frac{w_0}{L_0}$.

2) The arc elasticity of income-compensated labor supply with respect to after-tax wage

measured from the point with taxes is: $\xi=\frac{\Delta L}{(tw_0)}\frac{w_0(1-t)}{L_1}$. Tax receipts equal w_0tL_1.

3) As carefully noted by Barro (1979, pp. 952-954), if the period *1* interest rate is different from the period *0* interest rate that prevailed when the (one-year maturity) debt was issued, then the market value defined in this way will deviate from the initial par value D_0. The current (beginning of period *0*) market value at the beginning of period *1* of that debt is $D_0^*=\frac{(1+i_0)}{(1+i_1)}D_0$, where D_0 is the par value of the debt. To simplify the notation, except as noted, we will assume that $i_0=i_1$ and drop the asterisk.

4) If we define $Y^0=\sum_{j=1}^{\infty}\frac{y_j^0}{\prod_1^j(1+i_j)}$, then from Eq. [3], $\tau_j^*=\frac{1-\sqrt{1-4\xi\theta\frac{D_0+G}{Y^0}}}{2\xi\theta}$.

5) In a mathematical sense, the sequence of interest rates $(i_1, i_2, \cdots, i_j, \cdots,)$ is arbitrary. But if these interest rates, as assumed, reflect a general equilibrium with complete markets, they must afford no opportunities for profitable arbitrage, and in this economic sense are not arbitrary. Rolling over a debt indefinitely is equivalent to owning the principal outright and must therefore have the same general equilibrium market value as the principal.

6) See https://financeformulas.net/Present_Value_of_Growing_Perpetuity.html

7) Note that where the interest rate, i, is unchanging, the present value in year 0 of an $N-1$ year annuity that grows at the rate η from an initial amount a_1 received at the beginning of year 1, is

$$\left(\frac{a_1}{i-\eta}\right)\left(1-\left(\frac{1+\eta}{1+i}\right)^{N-1}\right),$$

and the present value in year 0 of a perpetuity that grows at the rate η from an initial amount a_1 received at the beginning of year 1, is

$$\left(\frac{a_1}{i-\eta}\right)(1+\eta)^N.$$

For the derivations of these formulas see the web site 'financeformulas. net,' https://financeformulas. net/Present_Value_of_Growing_Annuity. html, https://financeformulas. net/Present_Value_of_Growing_Perpetuity.html.

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