Abstract of Doctoral Dissertation
Title: Semi-classical approach to the Schrödinger operator with strong magnetic field

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This thesis presents the asymptotic distribution of eigenvalues of a magnetic Schrödinger operator in the limit of strong magnetic field. It is well-known that the two-dimensional Schrödinger operator with constant magnetic field is essentially self-adjoint and its spectrum consists of eigenvalues with infinite multiplicity (called Landau levels). On the other hands, eigenvalues with finite multiplicity may appear around the Landau levels caused by the perturbation of the potential. The distribution of such eigenvalues is a natural and interesting problem. In particular, its asymptotic behavior has been studied in various regimes.

For the high energy regime, we refer to E. Korotyaev and A. Pushnitski (2004), A. Pushnitski, G. Raikov and C. Villegas-Blas (2013), and T. Lungenstrass and G. Raikov (2014). The asymptotic behavior of the number of eigenvalues near the boundary of the essential spectrum for the two-dimensional constant magnetic Schrödinger operator operator with a power-like decaying electric potential was studied by G. Raikov (1990), and was recorded by V. Ivrii (1998). The cases of exponentially decaying and compactly supported electric potential of fixed sign were examined by G. Raikov and S. Warzel (2002), and M. Melgaard and G. Rozenblum (2003). In these works, the authors show that the eigenvalue asymptotics are non-classical. In particular, the leading terms of the eigenvalue asymptotics near a fixed Landau level are independent of the Landau level, and in the case of compactly support, they do not depend on electric potentials (see Theorem 2.2 in their paper). The paper of N. Filonov and A. Pushnitski (2006) contains an important improvement of the theorem. This improvement implies that the eigenvalue asymptotics recovers the logarithmic capacity of the support of the electric potential. The counting function in the strong magnetic limit has also been studied. G. Raikov (1990) proved a Weyl formula for the counting function in a non critical interval. M. Dimassi (2001) improved this result with a sharp remainder estimate and gave a complete asymptotic expansion of the trace formula. The shape resonances for the magnetic Stark Hamiltonian with strong magnetic field has been studied by X. P. Wang (1992). M. Dimassi and V. Petkov (2004) obtained a trace formula for resonances of the magnetic Stark Hamiltonian. In particular a connection between the resonance and the spectral shift function has been established.

Our interest in this thesis is the precise asymptotic distribution of eigenvalues for a fixed Landau level. The results consist of the asymptotics of the eigenvalues near the minimum and the maximum of the electric potential and the asymptotic behavior of the corresponding eigenfunctions, as well as the Bohr-Sommerfeld quantization rule for the eigenvalues near non-critical levels.