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Estimating Productivity Growth in the Korean Economy without Restrictive Assumptions

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1. Introduction

Motivation

- Solow residual fails to provide an accurate measure of TFP change if one of underlying assumptions is violated.
- The Solow residual under the restrictive assumptions varies with demand conditions, and many studies have eliminated this cyclical bias using flexible capital utilization.

Research Questions

- Is the invariance hypothesis that technology shocks should be orthogonal to demand shocks a necessary or even sufficient condition, as suggested by many studies?
- The main objective for removing cyclical movement from Solow residual is not to delete the correlation between TFP and business cycles entirely, but simply to eliminate any error that may exist in the Solow residual.
- What if an adjusted Solow residual continues to reflect demand shocks because one cyclical bias cancels out other biases when adjusting the Solow residual with markup, RTS, and capital utilization simultaneously?

Main Features of this Paper

- This paper uses aggregate data for the Korean economy to address these issues about the measurement and exogeneity of the Solow residual.
- By selecting a small open economy, we can avoid the problem of finding adequate instrument variables to represent exogenous shocks, which are difficult to find for the large American economy.
- The Korean economy is far from perfect competition, with many industries characterized by monopoly or oligopoly. As a result, markup in the economy is likely significantly greater than in the American economy.
- Scale economies have been regarded as an important factor in determining productivity change for the Korean economy

Contents of the Paper

- First, we estimate markup and RTS indices and derive their impacts on the Solow residual. Then, an alternative measure of productivity change is developed.
- Second, the link between both unadjusted and adjusted Solow residuals, and demand-side variables is investigated.

Main Results

- When we adjust the Solow residual by eliminating the effects of RTS, markup, and capital utilization, the adjusted Solow residual is much greater than the original one.
- The measured Solow residual for Korea reflects demand shocks, even when it is adjusted by eliminating the effects of RTS, markup, and capacity.
- In contrast to previous studies
  - Our Solow residual underestimates productivity changes due to demand shocks,
  - The Solow residual co-moves with demand shocks even after adjusting for cyclical bias arising from RTS, markup, and capital utilization effects.
  - However, our productivity measure is consistent with the neutrality of money.
  - Too much bias remains in the Solow residual measured with varying utilization to yield true productivity shocks
  - The residual does not necessarily overstate true productivity due to demand shocks.
  - The adjusted Solow residual suggests that true productivity changes can be procyclical even after all aspects are considered.

Contents

- Section 2: the theoretical background
- Section 3: data, empirical applications, and sources of the Solow residual
- Section 4: investigates the Granger causality between the Solow residual and the demand-side variables
- Section 5: conclusions.

2. Theoretical background

- **The Production Function**:

$$Y_t = A_t F(K_t, L_t) \tag{1}$$

- Take log and totally differentiate to get the following growth equation.

$$d\ln Y_t = d\ln A_t + \gamma_t (s_{kt} d\ln K_t + s_{lt} d\ln L_t) \tag{2}$$

  - where $d\ln Y_t$, as a log-derivative of output with respect to time, and γ, s_l, and s_k to denote RTS (=average cost/marginal cost), the shares of labor, and capital in total cost.

- Replacing $d\ln X_t$ with Δx_t for brevity, we can rewrite equation (2) as

  - $$\Delta y_t = \Delta a_t + \gamma_t \Delta k_t + \gamma_t s_{lt} (\Delta l_t - \Delta k_t) \tag{3}$$

- **Basic Relationship**

  - $$\gamma_t s_{lt} = (\frac{AC}{MC})_t (\frac{wL}{ACQ})_t = (\frac{P}{MC})_t (\frac{wL}{PQ})_t = \mu_t \varphi_{lt} \tag{4}$$

- The growth equation simultaneously allowing for RTS (γ) and imperfect competition (μ):
  - $\Delta y_t = \Delta a_t + \gamma_t \Delta k_t + \mu_t \varphi_{lt} (\Delta l_t - \Delta k_t)$ (5)
- **To consider capital utilization**, let $E_k = \delta K$ denote effective capital employed. Then we can rewrite the growth equation function as:
  - $\Delta y_t = \Delta a_t + \gamma_t \Delta k_t + \mu_t \varphi_{lt} (\Delta l_t - \Delta k_t) + (\gamma_t - \mu_t \varphi_{lt}) \Delta \delta_t$ (6)
- By deleting $\varphi_{lt} \Delta l_t + \varphi_{kt} \Delta k_t$ from both sides of the equation and rearranging the equation using $\varphi_{kt} = 1 - \varphi_{lt}$ to yield the following relationship:
  - $SR_t = \Delta a_t + (\gamma_t - 1) \Delta k_t + (\mu_t - 1) \varphi_{lt} (\Delta l_t - \Delta k_t) + (\gamma_t - \mu_t \varphi_{lt}) \Delta \delta_t$ (7)
  - The Solow residual can be decomposed into the effects of RTS, markup, and capital utilization, along with productivity change under realistic economic assumptions.

Data

- The dataset was constructed by compiling various sources derived from the Bank of Korea database for the period 1980 q1-2003 q3.

Empirical Application

- By adding an error term, ε_t, to Eq. (6), we have the following estimation equation:
  - $$\Delta y_t = \Delta a_t + \gamma(\Delta k_t + \Delta\delta_t) + \mu\varphi_{lt}[\Delta l_t - (\Delta k_t + \Delta\delta_t)] + \varepsilon_t, \quad (8)$$
- Estimating Eq. (8) failed due to multicollinearity. Thus, we estimate the following equation based on observed source of correlation as follows:
  - $$\Delta y_t = \Delta a_t + (\gamma_t - \mu\varphi_{lt})(\Delta k_t + \Delta\delta_t) + \mu\varphi_{lt}\Delta l_t + \varepsilon_t. \quad (9)$$
- Then, markup is estimated directly, and RTS (γ) can be retrieved by as:
  - $$\gamma = \hat{\beta}_2 + \hat{\beta}_3\varphi_t \quad (10)$$
- We adopt the generalized method of moments (GMM)
- Instruments: foreign demand (Δfd), real US GDP ($\Delta usgdp$), real Japanese GDP ($\Delta jgdp$)

Table 2: GMM Estimates of Markup and Returns to Scale in Korea

Estimating Equation

$$\Delta y_t = \beta_0 + \beta_1 SR_t + \beta_2(\Delta k_t + \Delta\delta_t) + \beta_3\varphi_{lt}\Delta l_t + \varepsilon_t$$

β_0	**0.0040** **(0.0016)**	**Markup and RTS**	
β_1	**1.3014** **(0.1011)**	**Markup:** $\hat{\beta}_3$	**1.3768** **(0.1792)**
β_2	**0.1501** **(0.0415)**	**RTS:** $\hat{\gamma}$	**0.9434** **(0.0995)**
β_3	**1.3768** **(0.1792)**	$H_0 : \hat{\beta}_3 = 1$ **(No markup)**	***p*-value** **=0.0383**
R^2	**0.7589**	$H_0 : \hat{\gamma} = 1$ **(CRS)**	**0.5689**
$\bar{R}^2$	**0.7515**		

Instruments for regression

Δfd_{t-1}, Δfd_{t-2}, Δfd_{t-3}, $\Delta usgdp_{t-1}$, $\Delta usgdp_{t-2}$, $\Delta usgdp_{t-3}$

$\Delta jgdp_{t-1}$, $\Delta jgdp_{t-2}$, $\Delta jgdp_{t-3}$

Overidentifying restrictions test

for the entire set of overidentification restrictions, p-value = 0.407

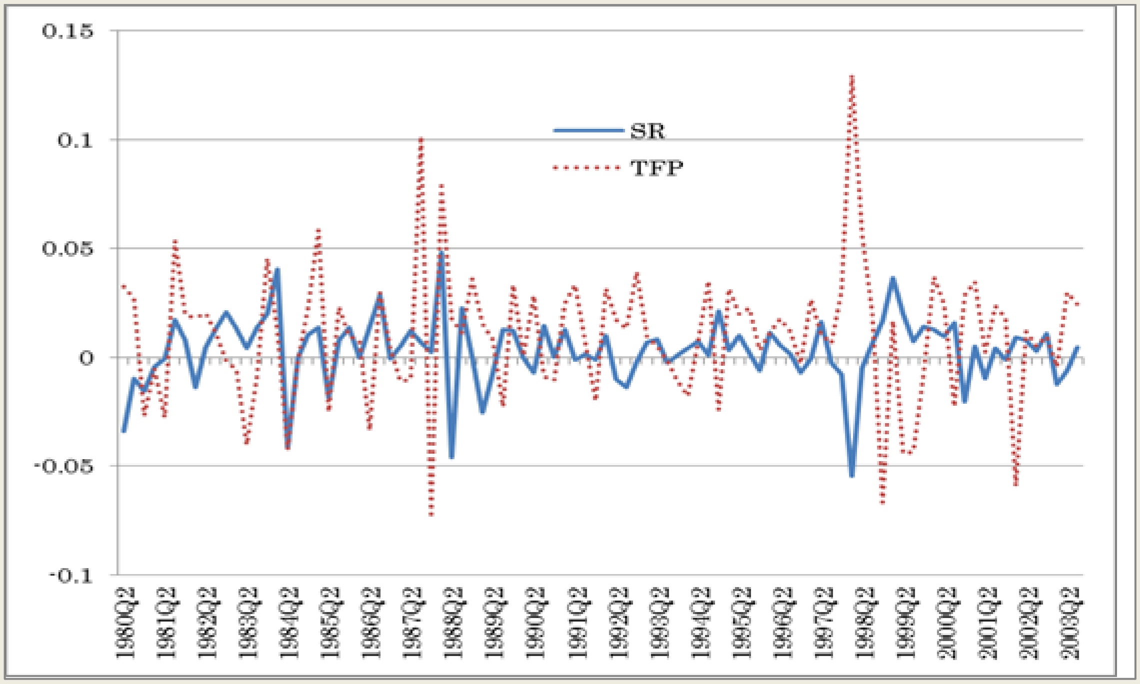
for the sub-sets of instruments: 1) Δfd, p-value = 0.304

2) $\Delta usgdp$, p-value = 0.476

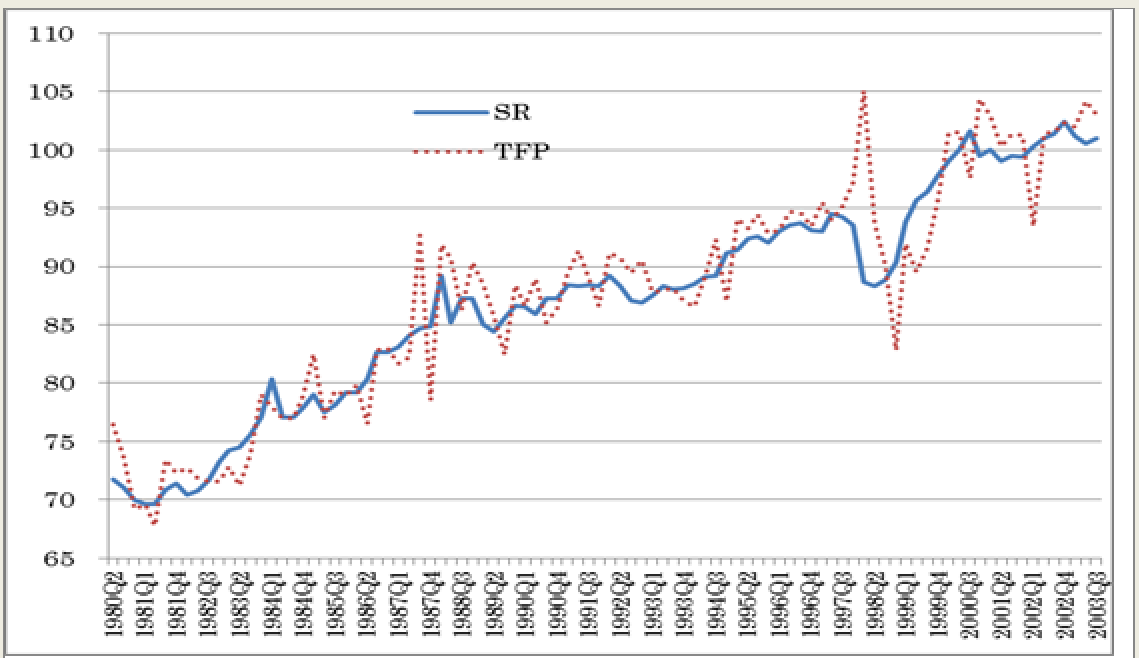
3) $\Delta jgdp$, p-value = 0.179

Table 3: Sources of Solow Residual Growth in Korea (1980-2003)

Period	Statistic	Unadjusted Solow residual	Adjusted residual (TFP): Δa_t	Effects of RTS: $(\hat{\gamma}-1)\Delta k_t$	Effects of Markup: $(\hat{\mu}-1)\varphi_{lt}(\Delta l_t-\Delta k_t)$	Effects of Capacity utilization: $(\hat{\gamma}-\hat{\mu}\varphi_{lt})\Delta\delta_t$
1980-85	Mean	**0.0027**	**0.0074**	**-0.0031**	**-0.0072**	**0.0056**
	SD	**0.0185**	**0.0278**	**0.0005**	**0.0073**	**0.0278**
	Min	**-0.0414**	**-0.0426**	**-0.0044**	**-0.0234**	**-0.0612**
	Max	**0.0404**	**0.0583**	**-0.0024**	**0.0095**	**0.0549**
1986-90	Mean	**0.0046**	**0.0102**	**-0.0024**	**-0.0072**	**0.0040**
	SD	**0.0192**	**0.0374**	**0.0006**	**0.0042**	**0.0383**
	Min	**-0.0458**	**-0.0719**	**-0.0030**	**-0.0132**	**-0.0833**
	Max	**0.0480**	**0.1015**	**-0.0012**	**0.0057**	**0.0870**
1991-96	Mean	**0.0026**	**0.0110**	**-0.0005**	**-0.0081**	**0.0003**
	SD	**0.0078**	**0.0177**	**0.0005**	**0.0030**	**0.0197**
	Min	**-0.0138**	**-0.0242**	**-0.0011**	**-0.0132**	**-0.0331**
	Max	**0.0211**	**0.0393**	**0.0007**	**-0.0029**	**0.0527**
1997-98	Mean	**-0.0039**	**0.0267**	**0.0002**	**-0.0093**	**-0.0215**
	SD	**0.0224**	**0.0547**	**0.0002**	**0.0058**	**0.0702**
	Min	**-0.0545**	**-0.0664**	**-0.0001**	**-0.0204**	**-0.1641**
	Max	**0.0166**	**0.1299**	**0.0006**	**-0.0021**	**0.0851**
1999-2003	Mean	**0.0058**	**0.0045**	**-0.0003**	**-0.0037**	**0.0053**
	SD	**0.0127**	**0.0283**	**0.0001**	**0.0035**	**0.0314**
	Min	**-0.0204**	**-0.0588**	**-0.0005**	**-0.0107**	**-0.0453**
	Max	**0.0367**	**0.0370**	**0.0000**	**0.0038**	**0.0685**
Total sample	Mean	**0.0032**	**0.0100**	**-0.0014**	**-0.0069**	**0.0015**
	SD	**0.0157**	**0.0309**	**0.0014**	**0.0051**	**0.0346**
	Min	**-0.0545**	**-0.0719**	**-0.0044**	**-0.0234**	**-0.1641**
	Max	**0.0480**	**0.1299**	**0.0007**	**0.0095**	**0.0870**



a) Original Estimates of SR and TFP



b) Normalized Estimates of SR and TFP (SR 2000q2=100)

Figure 1: Measures of Korean Total Factor Productivity Growth Derived from the Solow Residual (SR), and the Adjusted Solow Residual (TFP) after Eliminating RTS, Markup and Capital Utilization Biases from the Solow Residual.

Table 4: Predictability of Solow Residual for Korea

Granger causality test is based on:

$$SR_t = c + \rho(L)SR_{t-1} + \lambda(L)\Delta x_{t-1} + \varepsilon_t, \quad H_0: \lambda(L) = \mathbf{0}^+$$

Variable (x)	Lag			
	1	**2**	**3**	**4**
a) Unadjusted Solow residual				
$\Delta m1$	**0.001**	**0.004**	**0.016**	**0.029**
$\Delta m2$	**0.606**	**0.092**	**0.096**	**0.090**
$\Delta m3$	**0.706**	**0.868**	**0.688**	**0.393**
Δy	**0.435**	**0.462**	**0.415**	**0.004**
Δg	**0.922**	**0.835**	**0.186**	**0.001**
Δp	**0.013**	**0.033**	**0.107**	**0.029**
$\Delta oilp$	**0.544**	**0.654**	**0.494**	**0.187**
Δtot	**0.616**	**0.745**	**0.925**	**0.987**
$\Delta usgdp$	**0.127**	**0.120**	**0.184**	**0.222**
Δfd	**0.756**	**0.922**	**0.978**	**0.842**
b) Adjusted Solow residual				
$\Delta m1$	**0.103**	**0.104**	**0.109**	**0.147**
$\Delta m2$	**0.818**	**0.414**	**0.419**	**0.424**
$\Delta m3$	**0.755**	**0.680**	**0.616**	**0.772**
Δy	**0.119**	**0.169**	**0.149**	**0.027**
Δg	**0.051**	**0.077**	**0.111**	**0.149**
Δp	**0.049**	**0.180**	**0.006**	**0.001**
$\Delta oilp$	**0.839**	**0.579**	**0.137**	**0.251**
Δtot	**0.809**	**0.579**	**0.323**	**0.337**
$\Delta usgdp$	**0.321**	**0.456**	**0.633**	**0.701**
Δfd	**0.033**	**0.093**	**0.088**	**0.072**

Numbers are *p*-values for the null hypothesis that each variable do not Granger cause Solow residuals. All test variables are in log difference.

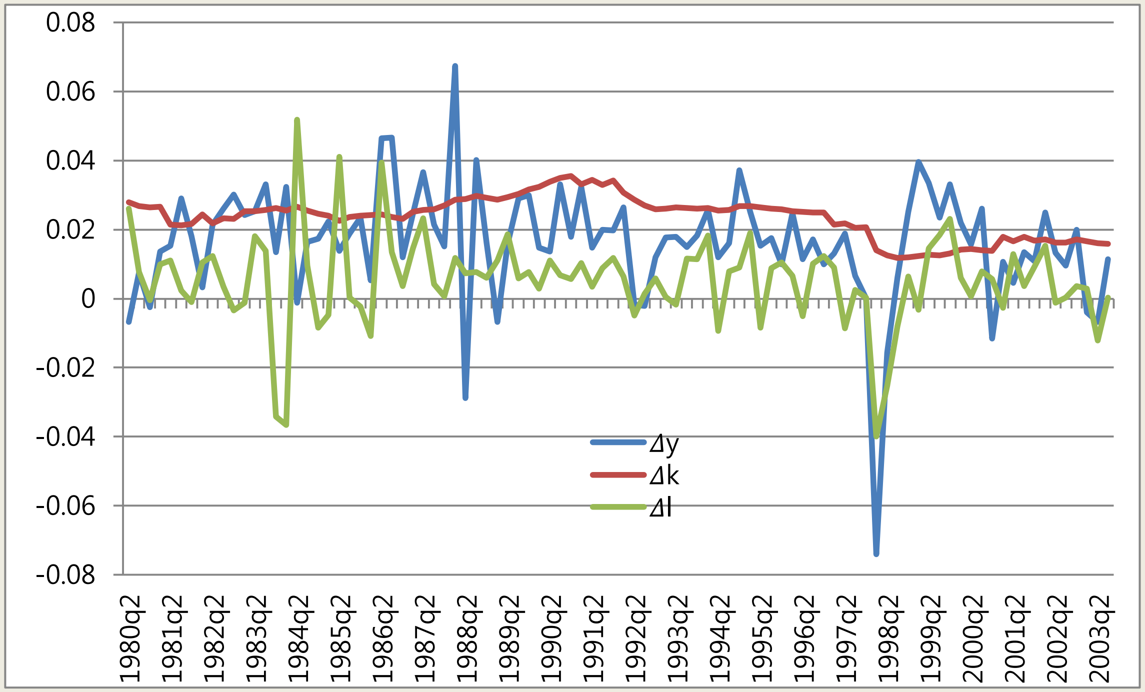


Figure 2: Growth of GDP (Δy), Capital (Δk), and Employment (Δl) in Korea.

Robust Check_1

- Estimate markup and RTS separately to avoid multicollinearity. First, we estimate markup by assuming CRS ($\gamma = 1$) as follows:
  - $$\Delta y_t - (\Delta k_t + \Delta \delta_t) = \Delta a_t + \mu \varphi_{lt} [\Delta l_t - (\Delta k_t + \Delta \delta_t)] + \varepsilon_t . \qquad (11)$$
- Likewise, RTS (γ) can be estimated by assuming perfect competition ($\mu = 1$) as:
  - $$\Delta y_t - \varphi_{lt} [\Delta l_t - (\Delta k_t + \Delta \delta_t)] = \Delta a_t + \gamma (\Delta k_t + \Delta \delta_t) + \varepsilon_t \qquad (12)$$

Table A1: Separate GMM Estimates of Markup and RTS in Korea

Markup equation (9): $\Delta y_t - (\Delta k_t + \Delta\delta_t) = \beta_0 + \beta_1\varphi_{lt}[\Delta l_t - (\Delta k_t + \Delta\delta_t)] + \varepsilon_t$

Returns-to-scale equation (10): $\Delta y_t - \varphi_{lt}[\Delta l_t - (\Delta k_t + \Delta\delta_t)] = \alpha_0 + \alpha_1(\Delta k_t + \Delta\delta_t) + \varepsilon_t$

Markup equation		Returns-to-scale equation	
β_0	**0.010** (0.003)	α_0	**0.001** (0.002)
β_1	**1.604** (0.285)	α_1	**1.021** (0.098)
$H_0 : \beta_1 = 1$ (No markup)	***p*-value =0.034**	$H_0 : \alpha_1 = 1$ (CRS)	***p*-value =0.827**
R^2	**0.413**		**0.413**
$\bar{R}^2$	**0.407**		**0.407**

Instruments for regression (9): $\Delta fd_t, \Delta fd_{t-1}, \Delta fd_{t-2}, \Delta usgdp_t, \Delta usgdp_{t-1}, \Delta usgdp_{t-2}$

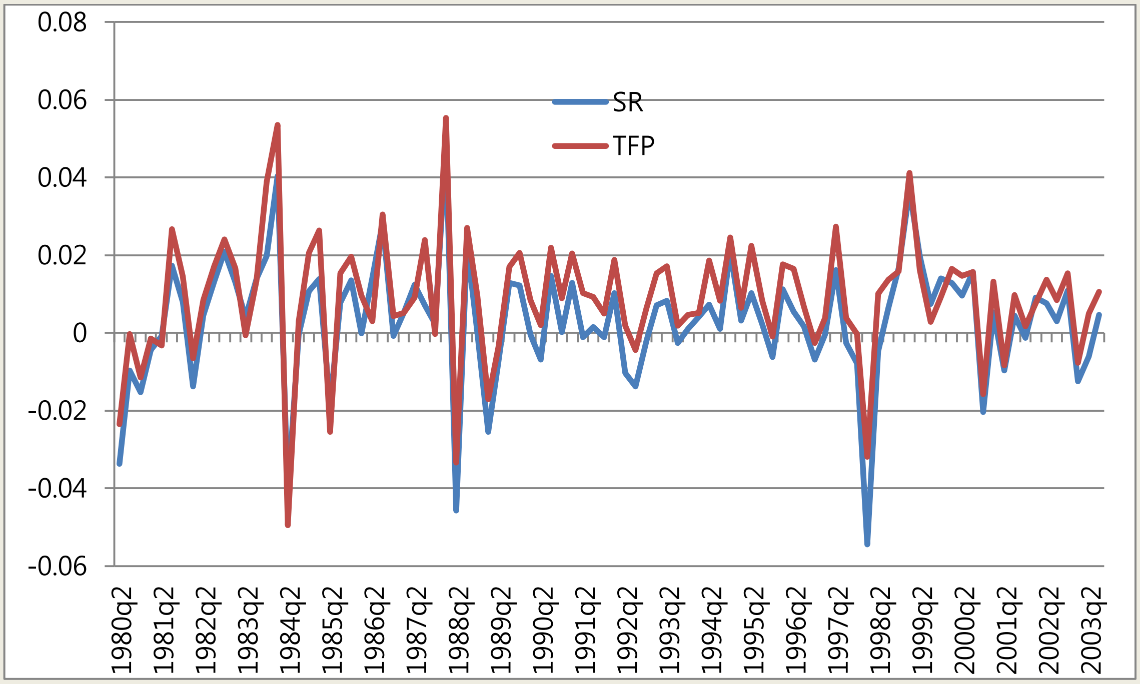
Instruments for regression (10): $\Delta fd_t, \Delta fd_{t-1}, \Delta fd_{t-2}, \Delta fd_{t-3}, \Delta usgdp_t, \Delta usgdp_{t-1}, \Delta usgdp_{t-2}, \Delta usgdp_{t-3},$

R^2 from regression of explanatory variable on instruments

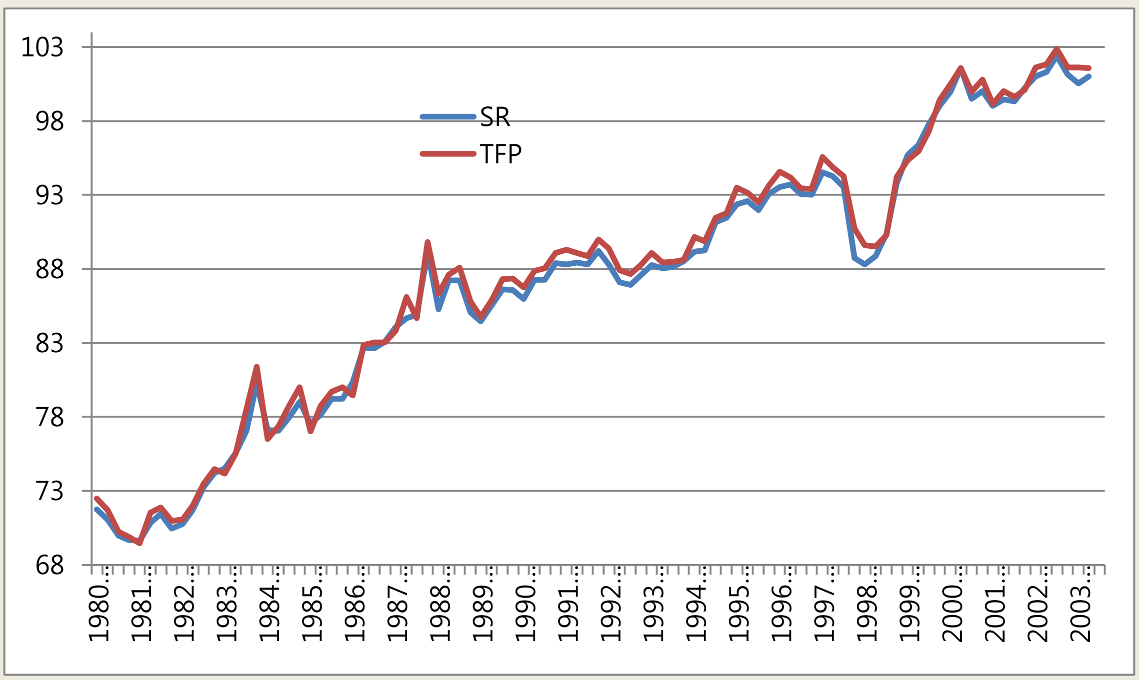
	Markup equation	Returns-to-scale equation
R^2	**0.526**	**0.577**
$\bar{R}^2$	**0.492**	**0.536**

Overidentifying restrictions test:

	Markup equation	Returns-to-scale equation
***p*-value**	**= 0.332**	**0.427**



a) Original Estimates of SR and TFP



b) Normalized Estimates of SR and TFP (SR 2000q2=100)

Figure A1: Measures of Korean Total Factor Productivity Growth Derived from the Solow Residual (SR), and the Adjusted Solow Residual (TFP) after Eliminating RTS, Markup and Capital Utilization Biases from the Solow Residual.

Robust Check_2

- We derive markup for each time period by using Eq. (11), after replacing Δa_t with the Solow residual and also assuming CRS technology $(\gamma = 1)$ as implied by previous hypothesis test.
  - $\Delta y_t - (\Delta k_t + \Delta\delta_t) = SR_t + \mu\varphi_{lt}[\Delta l_t - (\Delta k_t + \Delta\delta_t)] + \varepsilon_t$
- This markup has an average of 1.55, which is almost the same as a markup measure estimated under CRS assumption just explained as expected. and is slightly countercyclical against GDP growth with correlation coefficient of -0.144.
  - Adjusted Solow residual based on a period-by-period markup measure still contains very similar cyclical movement like the original residual. The simulation also shows that markup effects on the Solow residual and resulting adjusted TFP measure decreases (increases) if markup is countercyclical (procyclical)

4. Conclusions

Main Results

- The Solow residual co-moves with demand shocks even after adjusting for cyclical bias arising from RTS, markup, and capital utilization effects.
- Too much bias remains in the Solow residual measured with varying utilization to yield true productivity shocks
- The impact of monetary shocks on the Solow residual disappears and replaced by real shocks, when the Solow residual is adjusted.
- The residual does not necessarily overstate true productivity due to demand shocks.
- The adjusted Solow residual suggests that true productivity changes can be procyclical even after all aspects are considered.

Future Studies

- Investigate the same topic based on dataset from other countries.
- Check the cyclicality of productivity shocks itself.
  - Innovation activity increases at booms
  - Why and why not?

Thank you