

THEORY OF COMPLEX SYSTEMS and ENVIRONMENTAL CHANGE

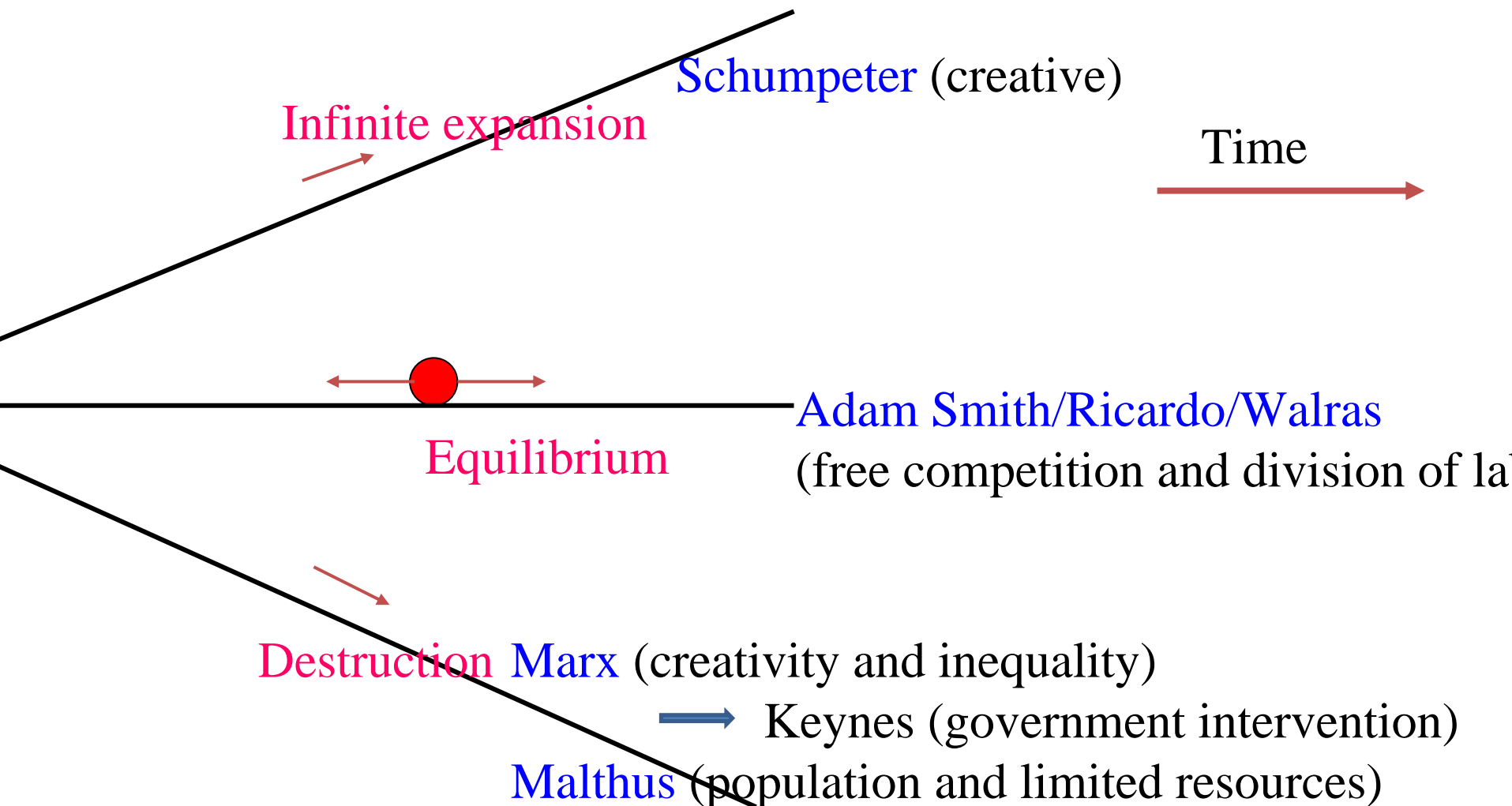
Wei-Bin Zhang

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Current economic globalization, capital redistribution, and fast knowledge diffusion

- Predictable economic order as assumed in the neoclassical/mainstream economics?
- Linear/linearized system not adequate representation of complicated system
- How to deal with economic orders?

Newtonian linear sciences – mathematically three possibilities ($dx/dt = A x + g(x)$)



traditional (linear/linearized) analytical economics

Samuelson's *Foundation*

Arrow-Debreu's general equilibrium theory

Solow's growth theory

Constant returns/unique equilibrium/stability/
homogeneous population/quantities, prices/
structural simplicity

No “grand theory” but a collection of structurally
unrelated (refined) models

Complex Systems/ Chaos/Nonlinear/Synergetics

- Prigogine (1977, Nobel Prize),
- Haken, Santa Fe Institute,
- Lock-in, path-dependent, bifurcation, catastrophe, chaos
- **Nonlinear economists** from the 1980s

Rationality and Chaos

- **Traditional rationality:**
 - structural changes is due to external forces
 - Chaos is not due to rational forces but random inputs
- **Modern rationality**
 - Structural changes and chaos is a part of evolution

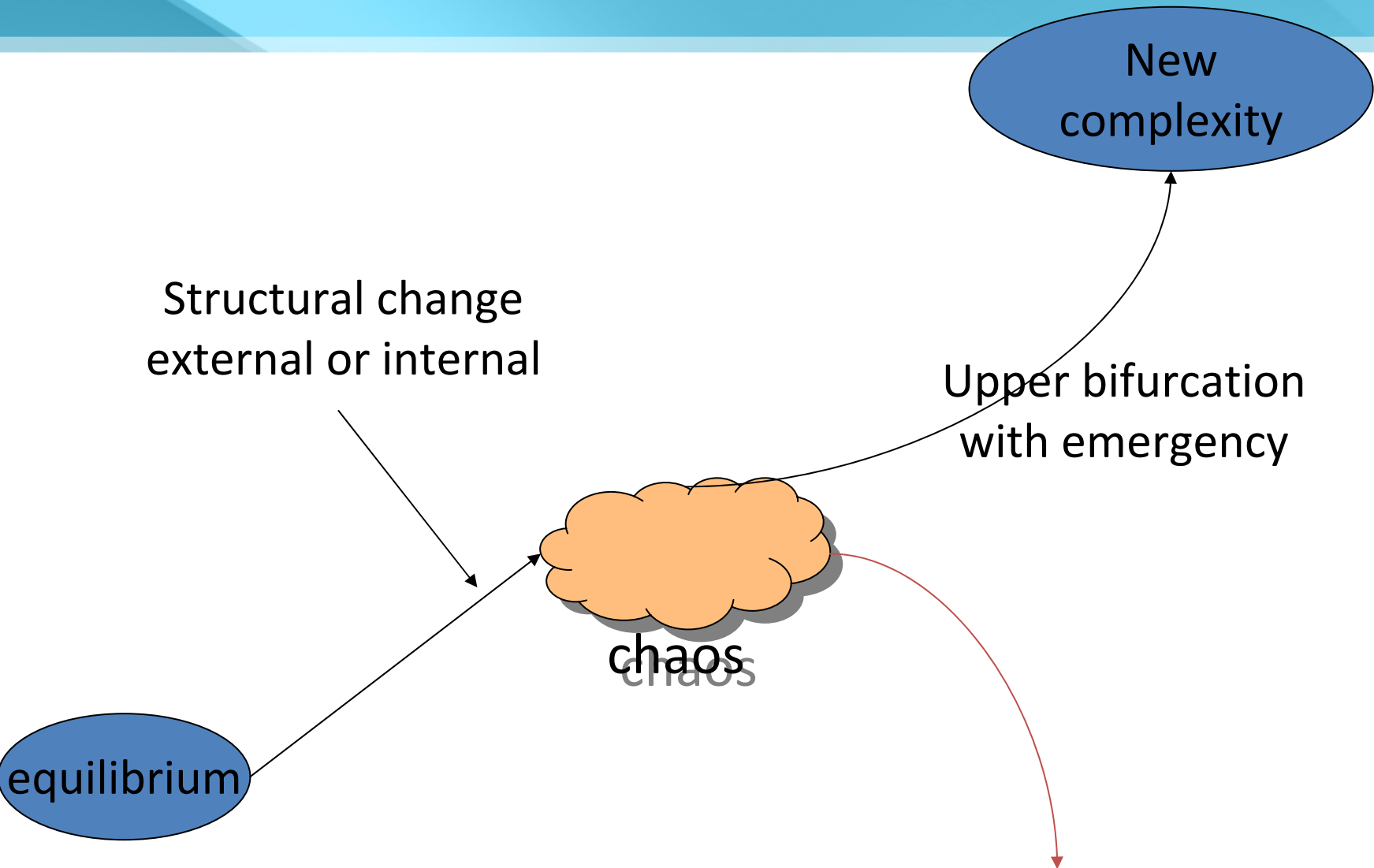
Living in unpredictable/chaotic world



Continued change

Changes with Bifurcations

Chaos and Organization



My research

- **By 1991:** *Synergetic Economics*
- applying different mathematical tools to various economic models/theories by generalization through introducing nonlinear terms
- **Since 1992**, build a general theory with complicated economic and policy structures (which includes all main economic models as special cases)
- **Historical conditions:** the traditional models are well examined and tested; nonlinear sciences, computer, collected data; collapses of socialism as well as capitalism

Wei-Bin Zhang

Synergetic Economics

Time and Change in Nonlinear Economics



Springer-Verlag

中央大学企業研究所翻訳叢書 7

時間と変化の経済学

——シナジェティクス入門——

W.B. チャン 著 有賀裕二 監訳
浅田統一郎／稲葉敏夫／輪湖博 訳

中央大学出版部

日本語版への序文

岱宗夫れ如何，齊魯まで青未だ了らず。
造化神秀を鍾め，陰陽に昏曉を割かつ。
胸を蕩かして曾雲を生ず，眈を決して歸鳥に入る。
曾ず当に絶頂を凌ぎて，一たび衆山の小なるを覽るべし。
杜甫「望嶽」〔訳注1〕

ポール・サミュエルソン教授は高名を博した『経済分析の基礎』の日本語版序文でつぎのように書き始めている。「経済学の古典より出来が良いものが1つだけあるがそれはその日本語訳である。」実際，私自身の本の日本語訳を見る機会を得て大変栄誉があり嬉しい思いです。私はまず最初に日本語で『経済分析の基礎』を興奮して読んだことを思い出します。私は、『経済分析の基礎』と『一般均衡分析』を読んで以来この方，理論経済学の冷徹な美しさに魅了されるようになった。科学者として，数個の方程式を操作することで経済生活に

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DIFFERENTIAL EQUATIONS, BIFURCATIONS, AND CHAOS IN ECONOMICS

Wei-Bin Zhang

World Scientific

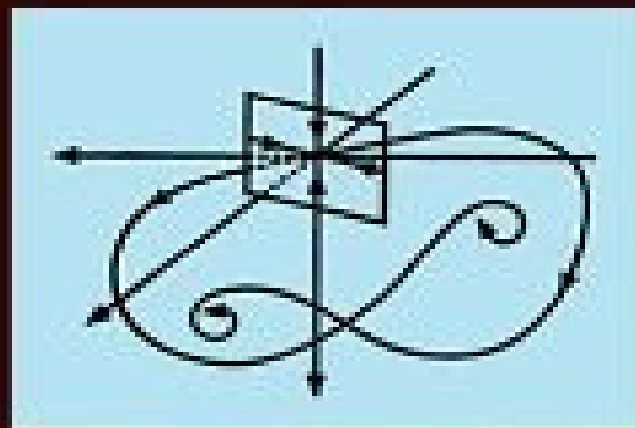
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MATHEMATICS IN SCIENCE
AND ENGINEERING Volume 204
SERIES EDITOR: C.K. CHUI

Discrete Dynamical Systems, Bifurcations *and* Chaos in Economics

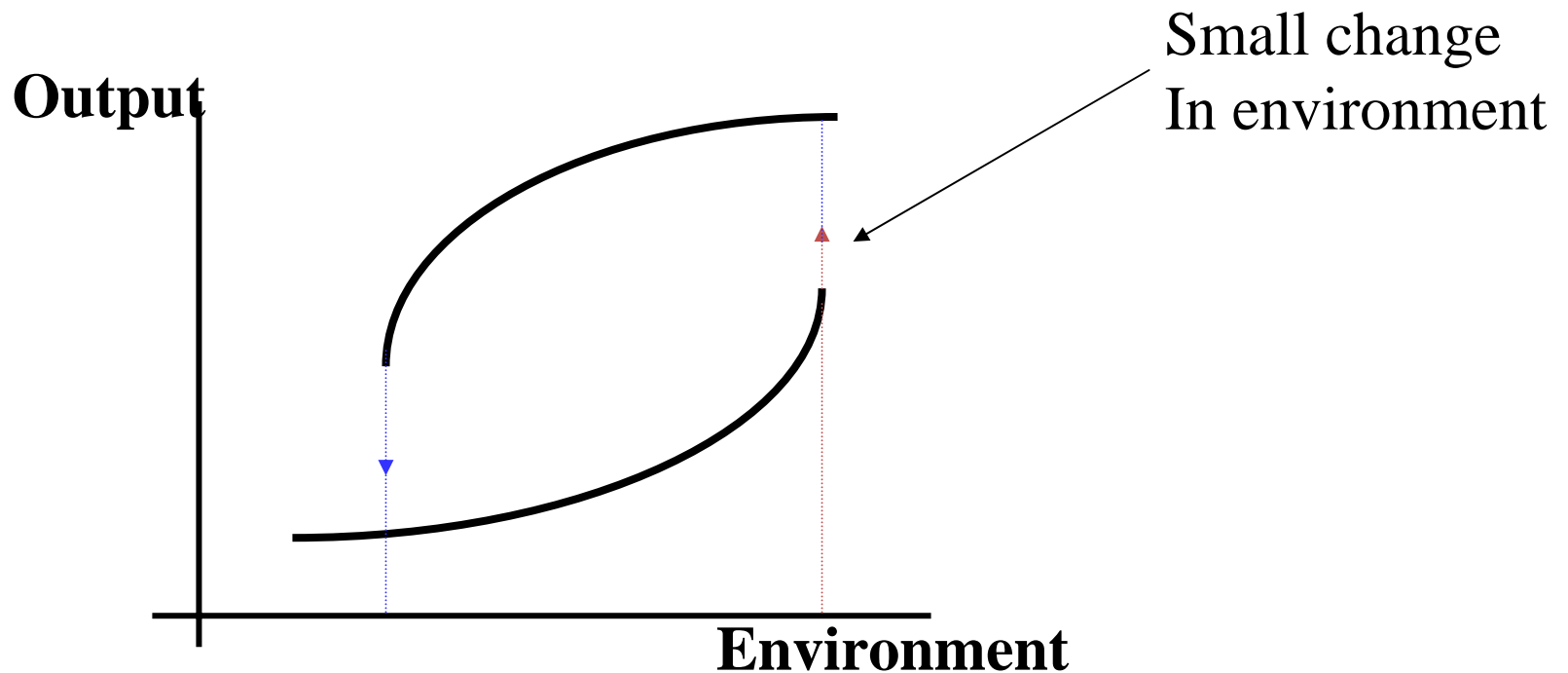


Wei-Bin Zhang

Traditional versus Nonlinear Economics

- | | |
|----------------------------|---|
| ✧ Neutrality or decreasing | ➤ Increasing returns |
| ✧ Stability | ➤ Instability |
| ✧ Unique equilibrium | ➤ Multiple equilibria/basins, chaos, Historical dependence, |
| | ➤ heterogeneous |
| ✧ homogeneous population | ➤ structurally unstable |
| ✧ structurally stable | ➤ Knowledge, environment |
| ✧ quantities, prices | ➤ structural complexity |
| ✧ structural simplicity | ➤ structurally evolutionary |
| ✧ structurally static | ➤ “structurally predictable” |
| ✧ predictable | |

- traditional: small change in environment leads to small change
- nonlinear science: small change leads to structural changes



Zhang (1992) A Development Model of Developing Economies with Capital and Knowledge Accumulation. Journal of Economics 55, 43-63

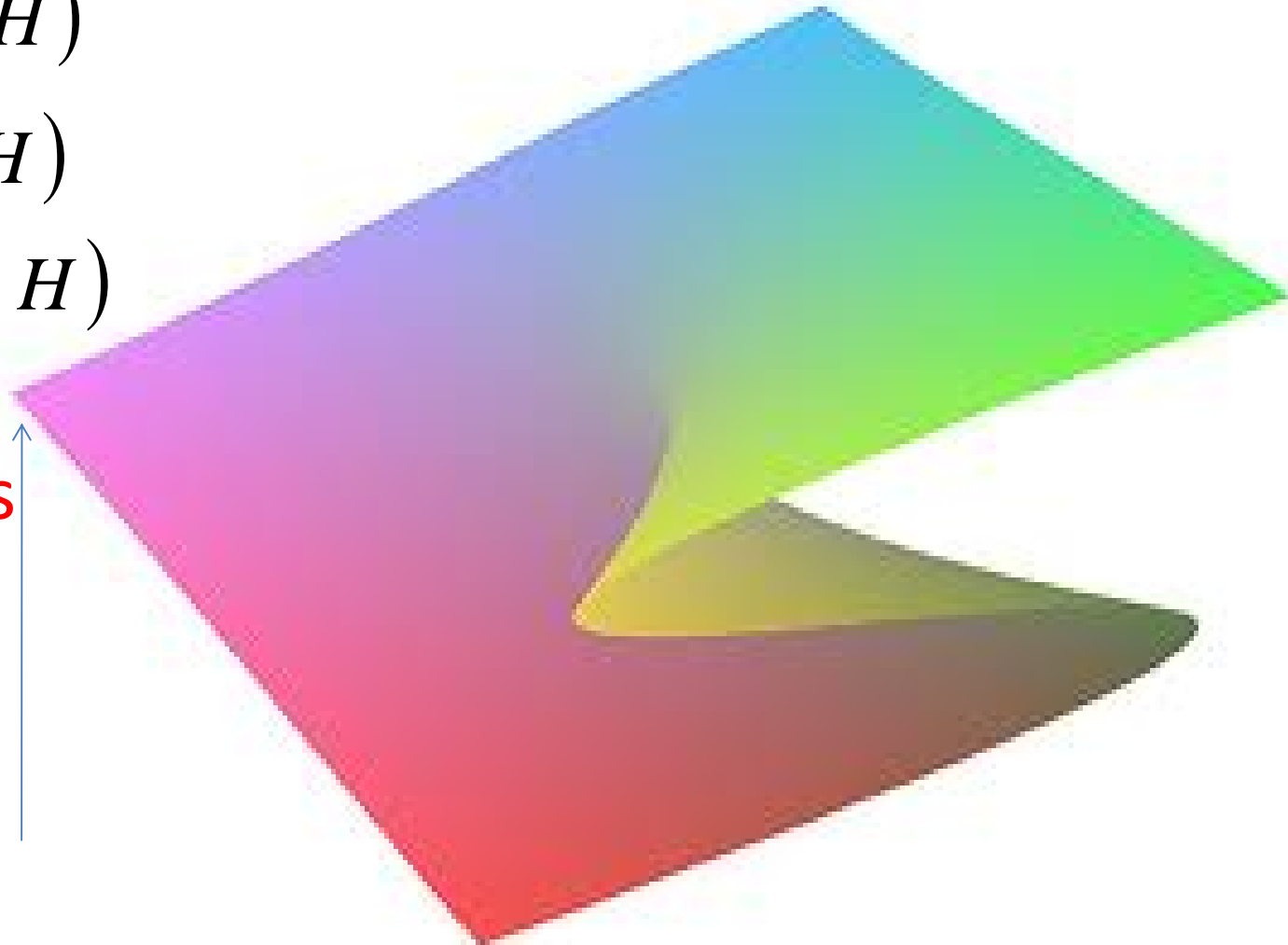
$$\dot{p} = \Psi_p(p, k, H)$$

$$\dot{k} = \Psi_k(p, k, H)$$

$$\dot{H} = \Psi_H(p, k, H)$$

•

p: openness



Chaos in a simple model

- **Haavelmo (1954)**

$$\dot{N} = N \left(a - \frac{\beta N}{Y} \right), \quad Y = AN^\alpha,$$

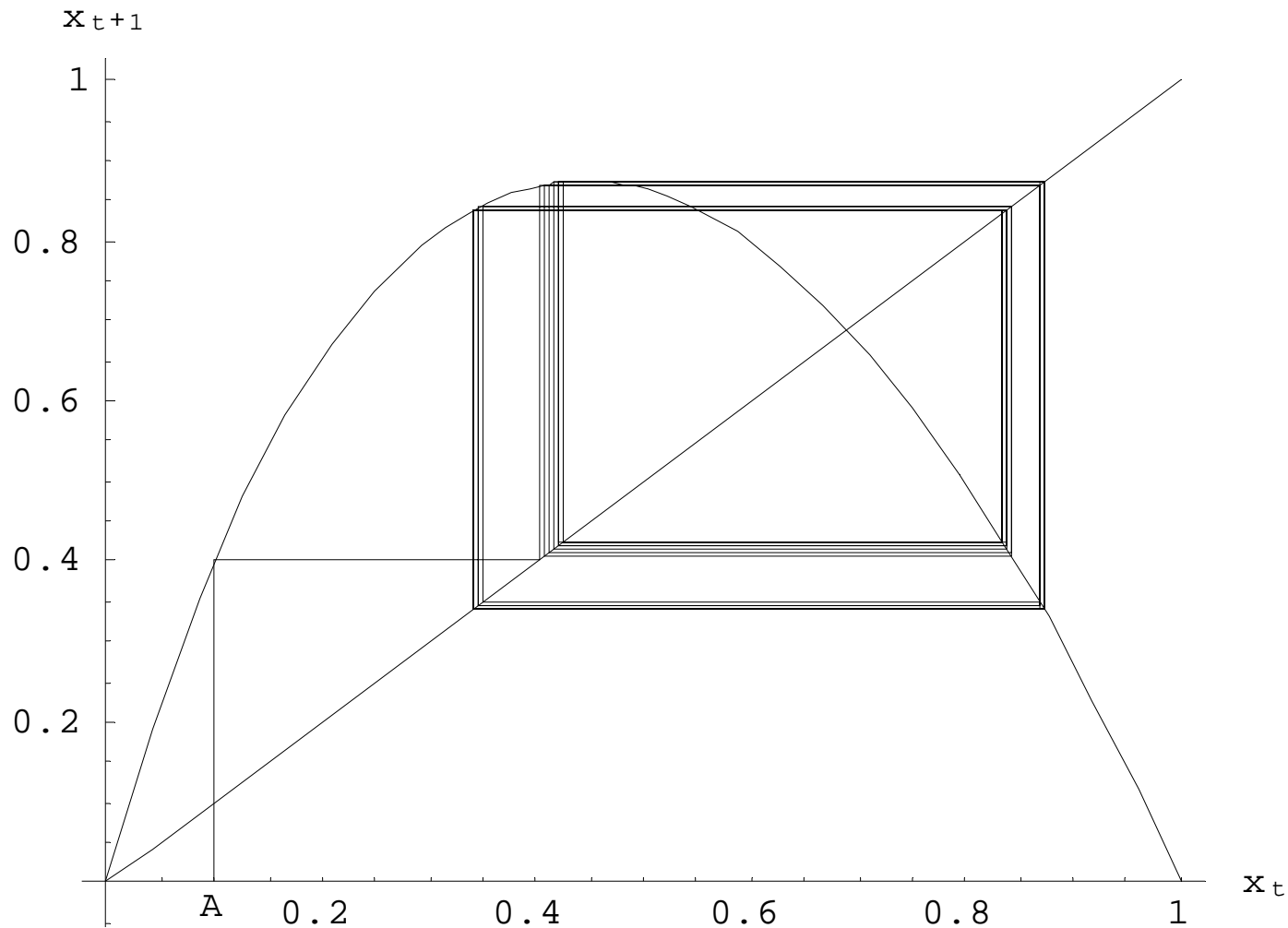
- **Stutzer (1980)**

$$N(t+1) = N(t) \left[(1+a) - \frac{\beta N(t)^{1-\alpha}}{A} \right],$$

$$x(t+1) = (1+a)x(t)(1 - x^{1/2}(t)) = F(x(t); a, 0.5),$$

$$N = [A(1+a)/\beta]^{1/(1-\alpha)} x, \quad \alpha = 1/2$$

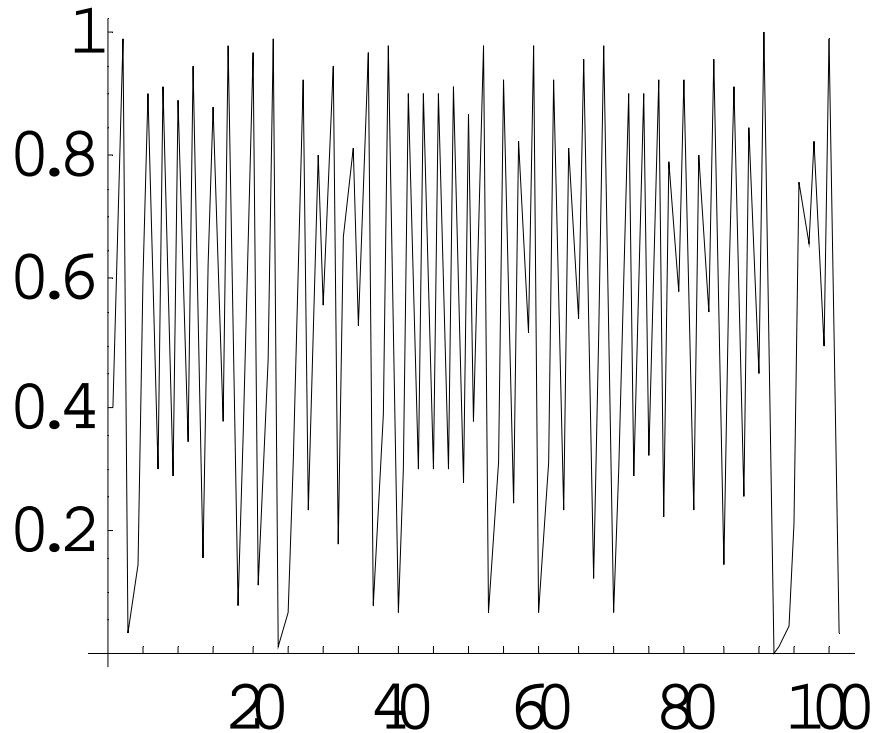
The 4-period Orbit for $a = 4.9$
 $2k(k = 2, \dots)$ harmonics prior to a reaching $a_c = 5.54$



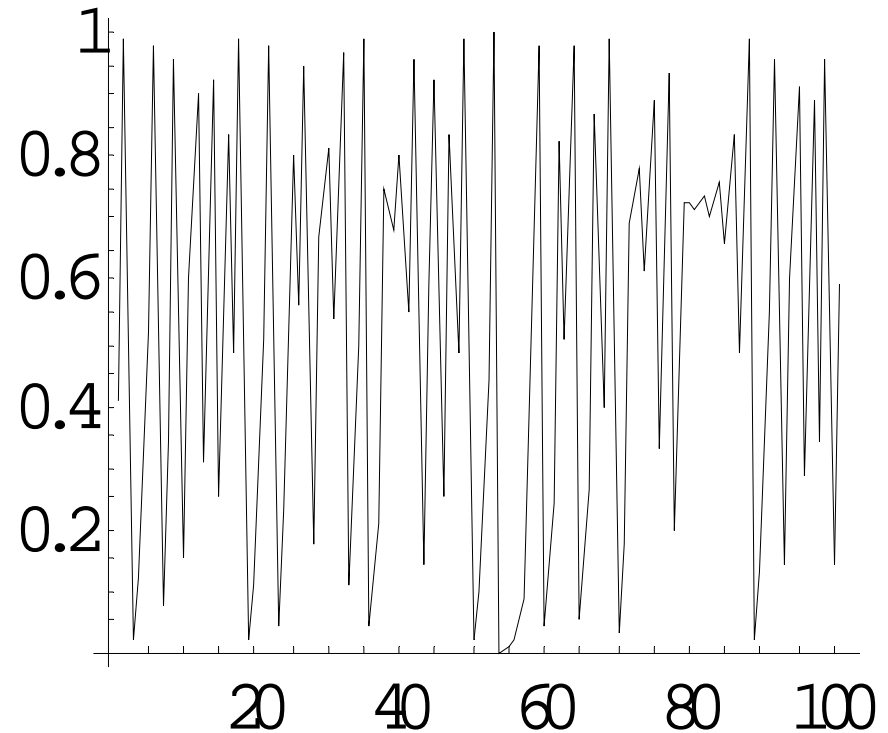
$a_c < a \leq 5.75$ is called chaotic region

- According to Li and Yorke (1975)
- for any $F(X(t), a)$ in which a non-degenerate 3-period orbit arises, there must also exist non-degenerate points of all periods, as well as an uncountable set of periodic (not asymptotically periodic) points whose trajectories wander “randomly” throughout the domain of F

$a = 5.75$; different initial states

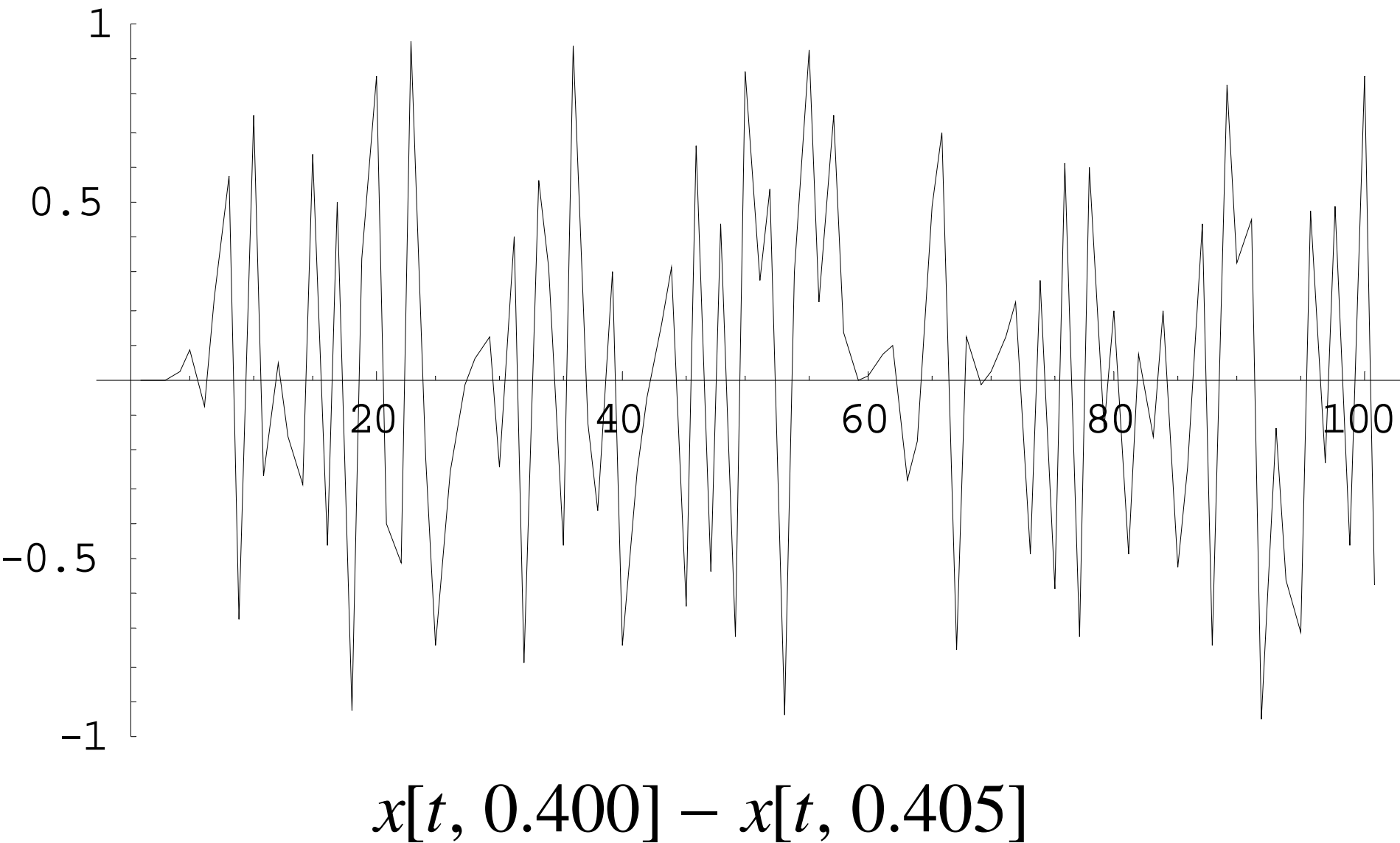


$x_0 = 0.400$

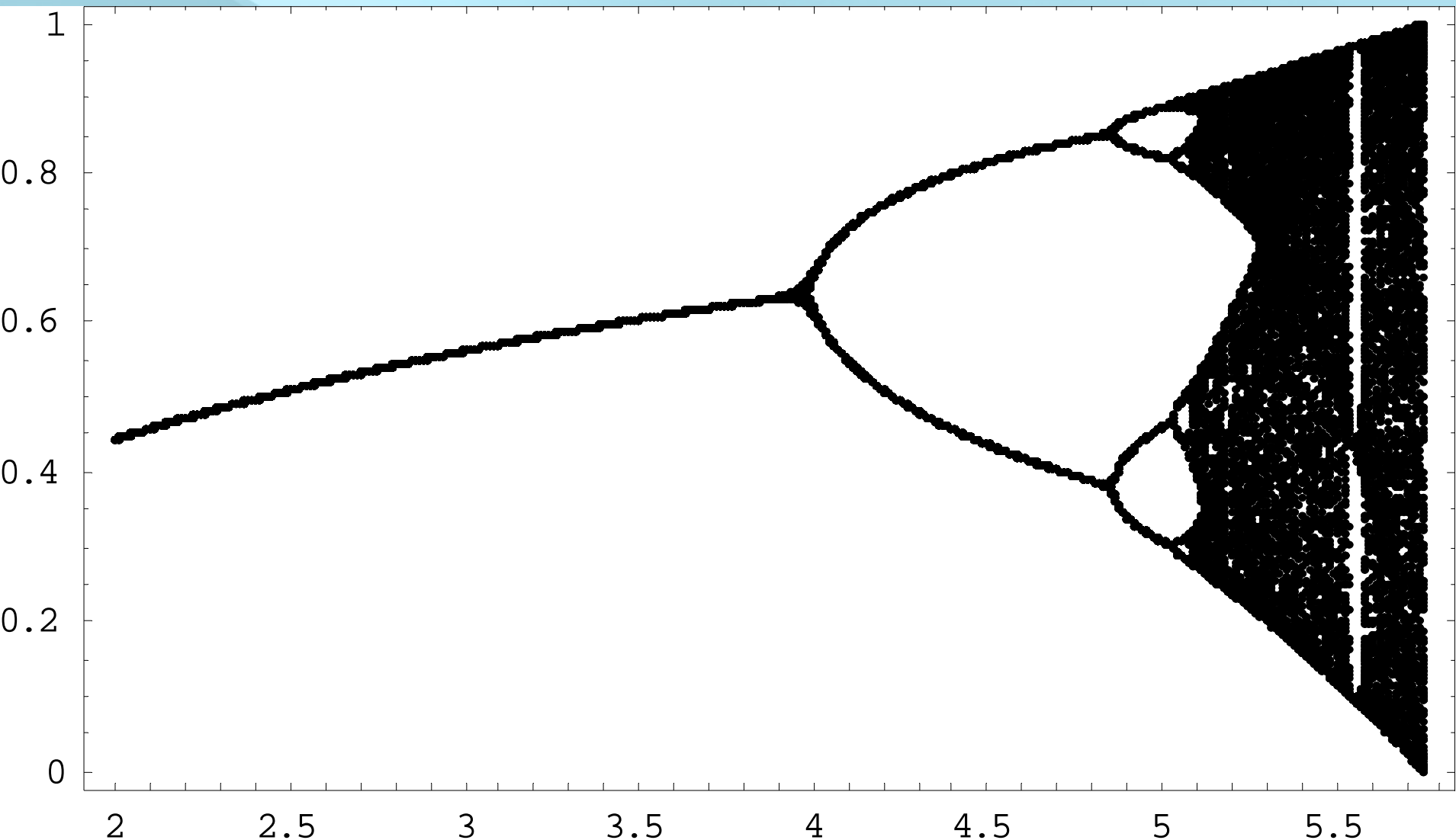


$x_0 = 0.405$

Small Differences at the Beginning Signify Much



The Map of Bifurcations for $a \in [2, 5.75]$



Chaos in a Small Urban Model

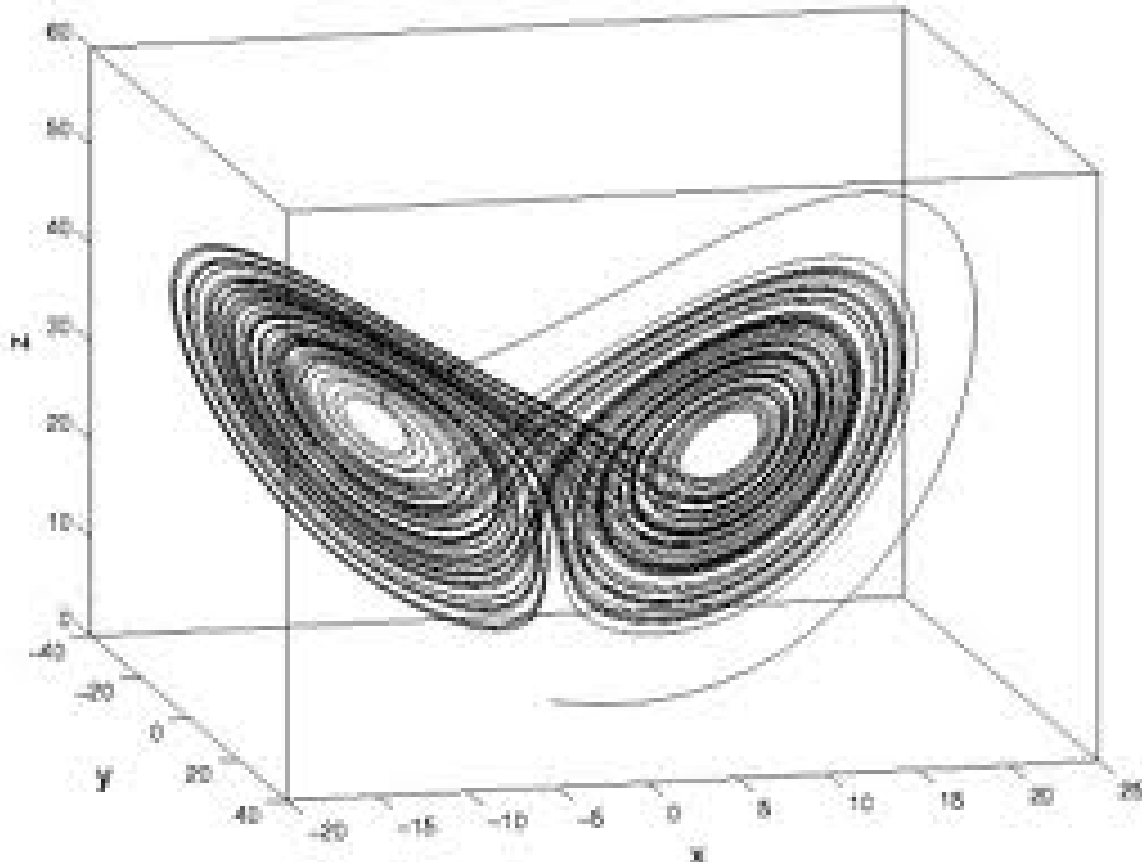
Zhang (Synergetic Economics, 1991, Chap. 6)

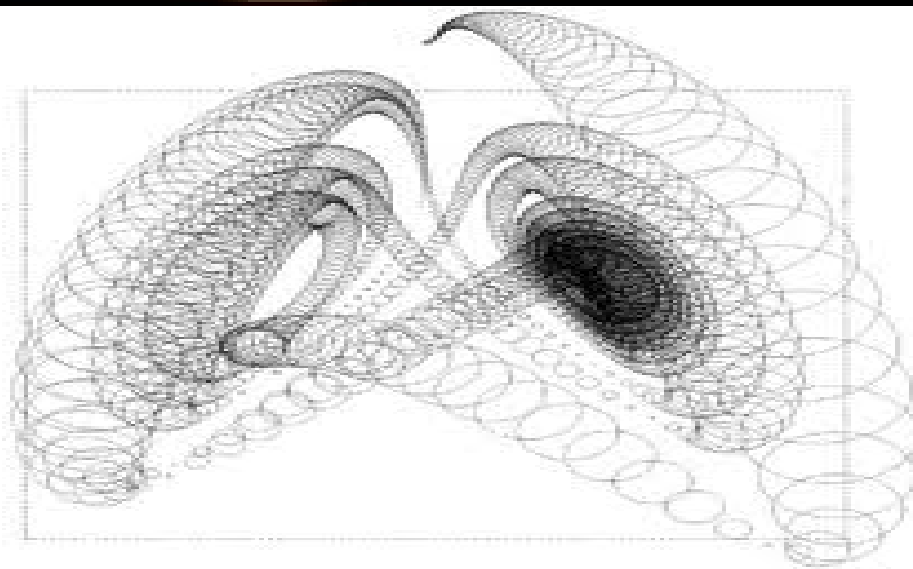
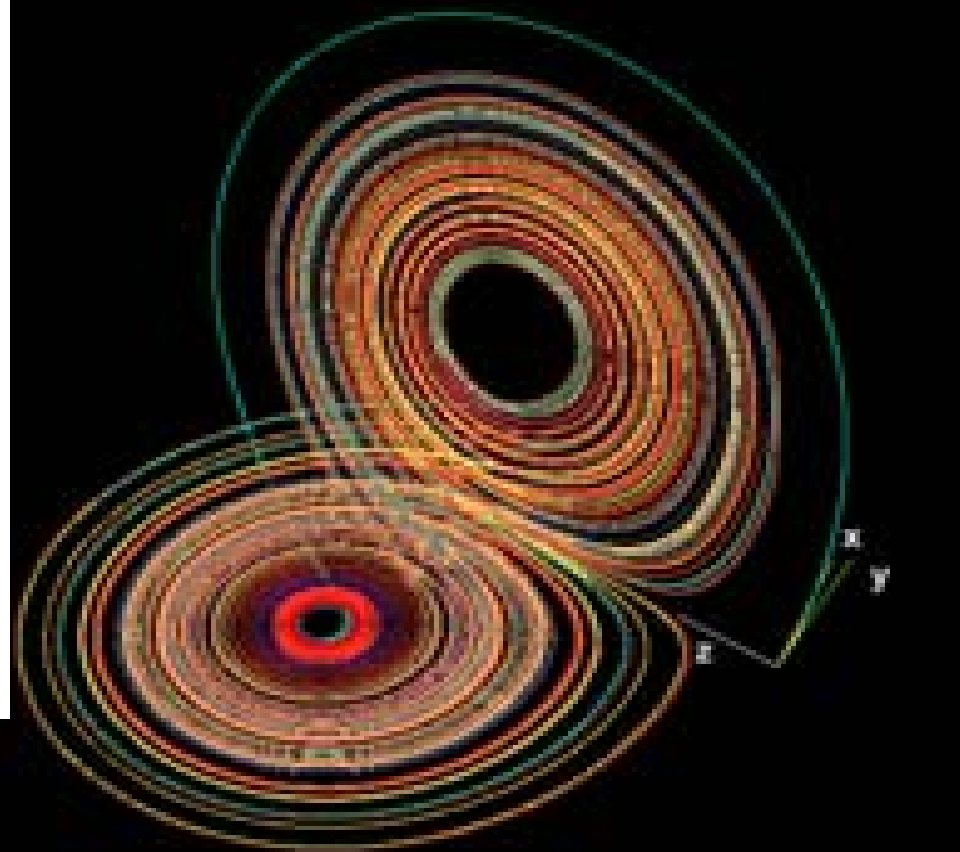
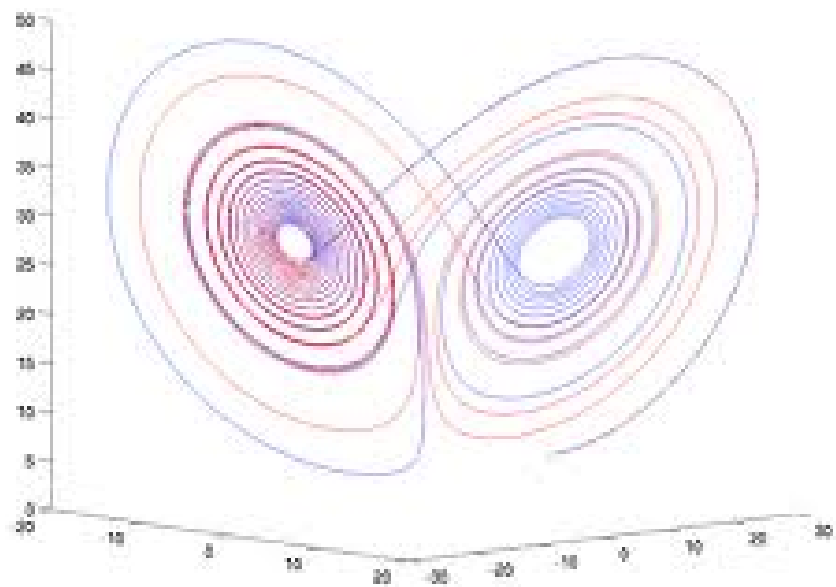
- x : output;
- y : urban population
- z : housing price

$$\dot{x} = \Psi_x(x, y, z)$$

$$\dot{y} = \Psi_y(x, y, z)$$

$$\dot{z} = \Psi_z(x, y, z)$$





- Western: religions and linear sciences
- East Asia: I-Ching (易经)

Chinese Nonlinear Vision of Change



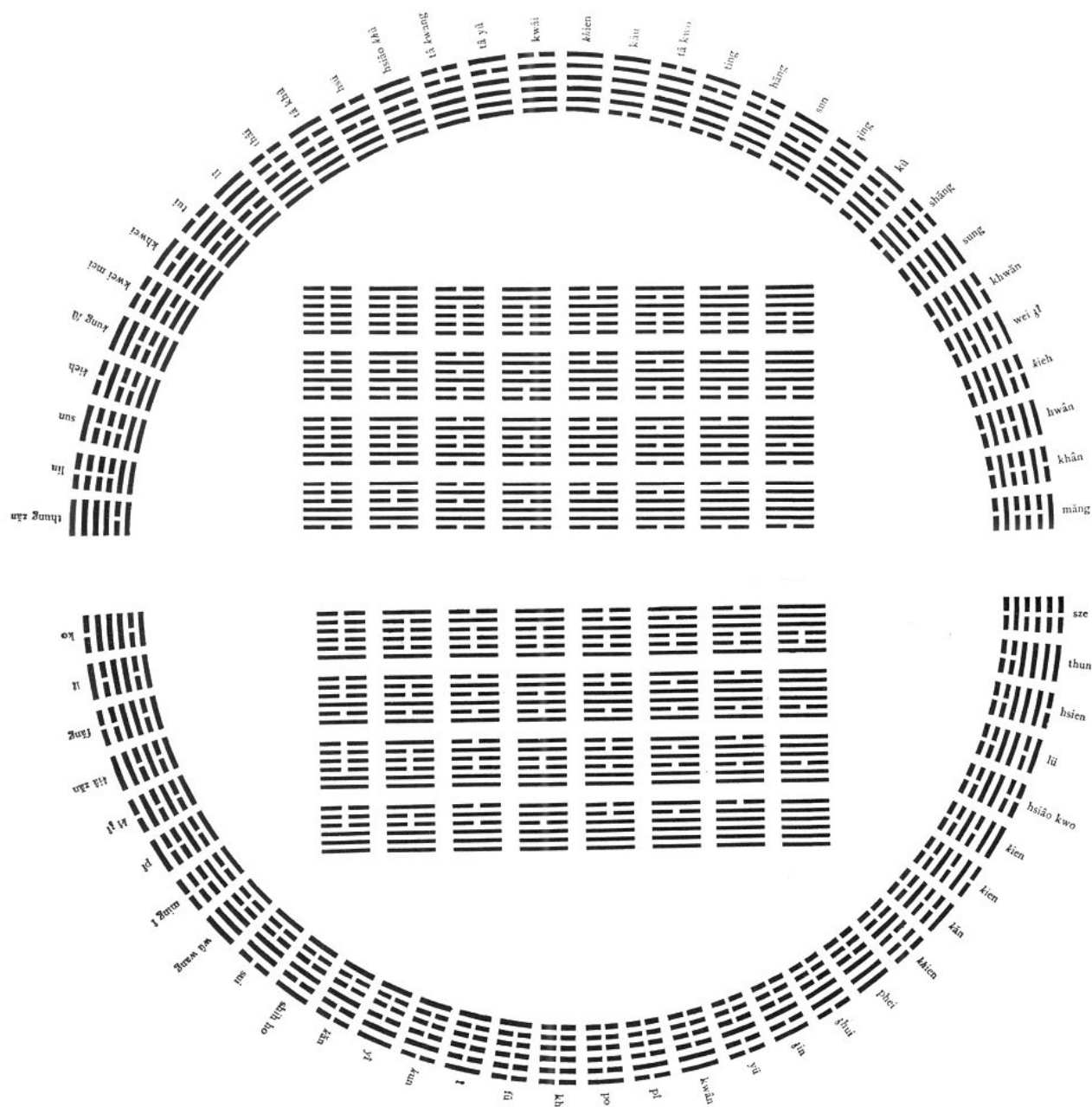






FIG. 1.

THE HEXAGRAMS, exhibited circularly and in a square, according to the natural process of development from the whole and divided lines, and the order of arrangement ascribed to Fû-hsi.



A general dynamic economic theory

Classical growth theory

Mathematical
Basic concepts

Harrod-Domar model

Neoclassical PF
Economic mechanisms

Solow-Swan growth
model

Main schools

$$\int_0^{\infty} U(c) e^{-\rho t} dt$$

Ramsey growth model

Household behavior

My analytical
framework:
capital, knowledge
population,
structures, prices
trade, urban
regional, sexual,
classes(races)

New growth theory
without capital

Growth and Environmental Change in the Uzawa's Two-Sector Model

Growth mechanism: the Solow model

Environment: environmental economics

Household behavior: my approach

Producer: neoclassical

Government: effective

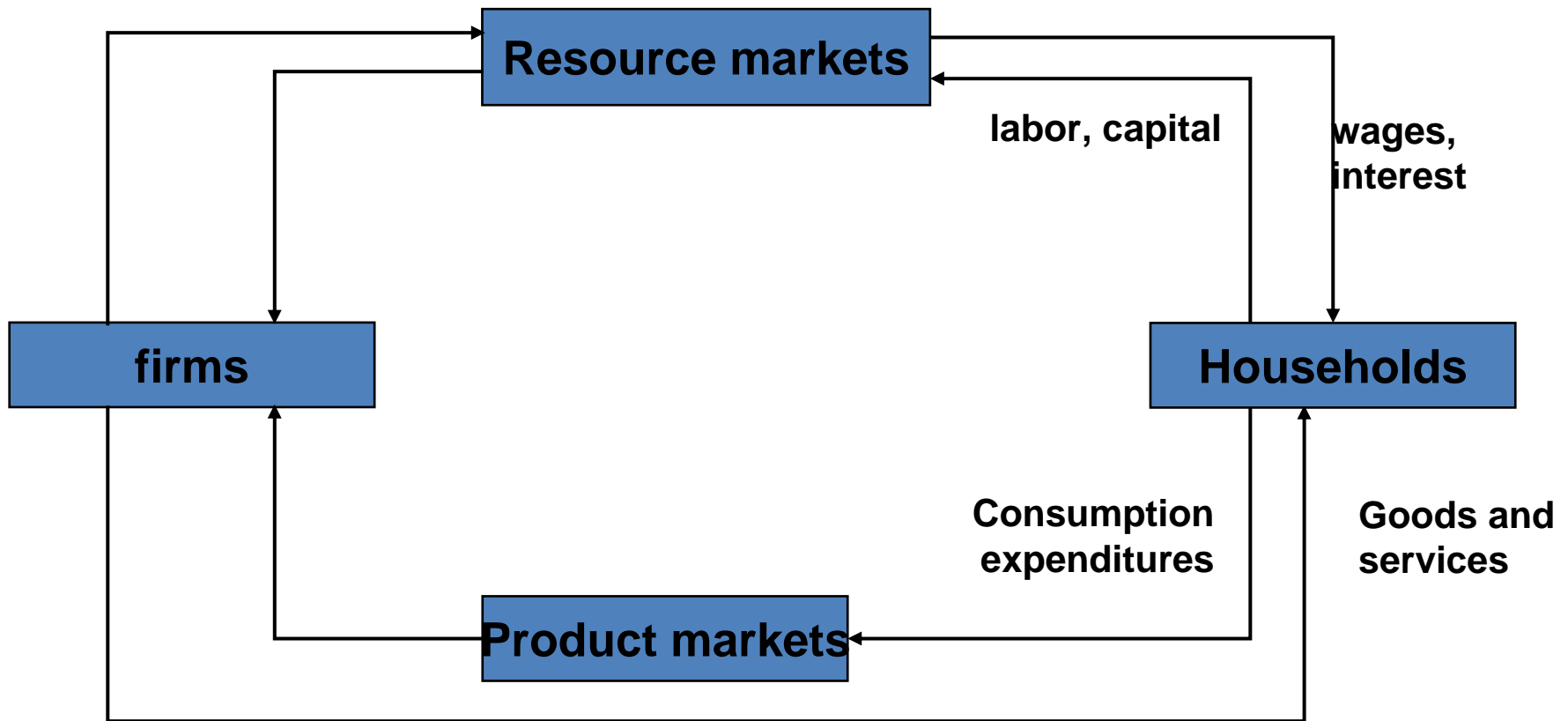
High dimensional dynamics: Computer simulation

Modeling environment

- three effects that are important in explaining the level of environmental pollution: (i) increases in output tends to require more inputs and produce more emissions; (ii) changes in income or preferences may lead to policy changes which will affect production and thus emission; and (iii) as income increases, the economic structure may be changed which will causes changes in the environment

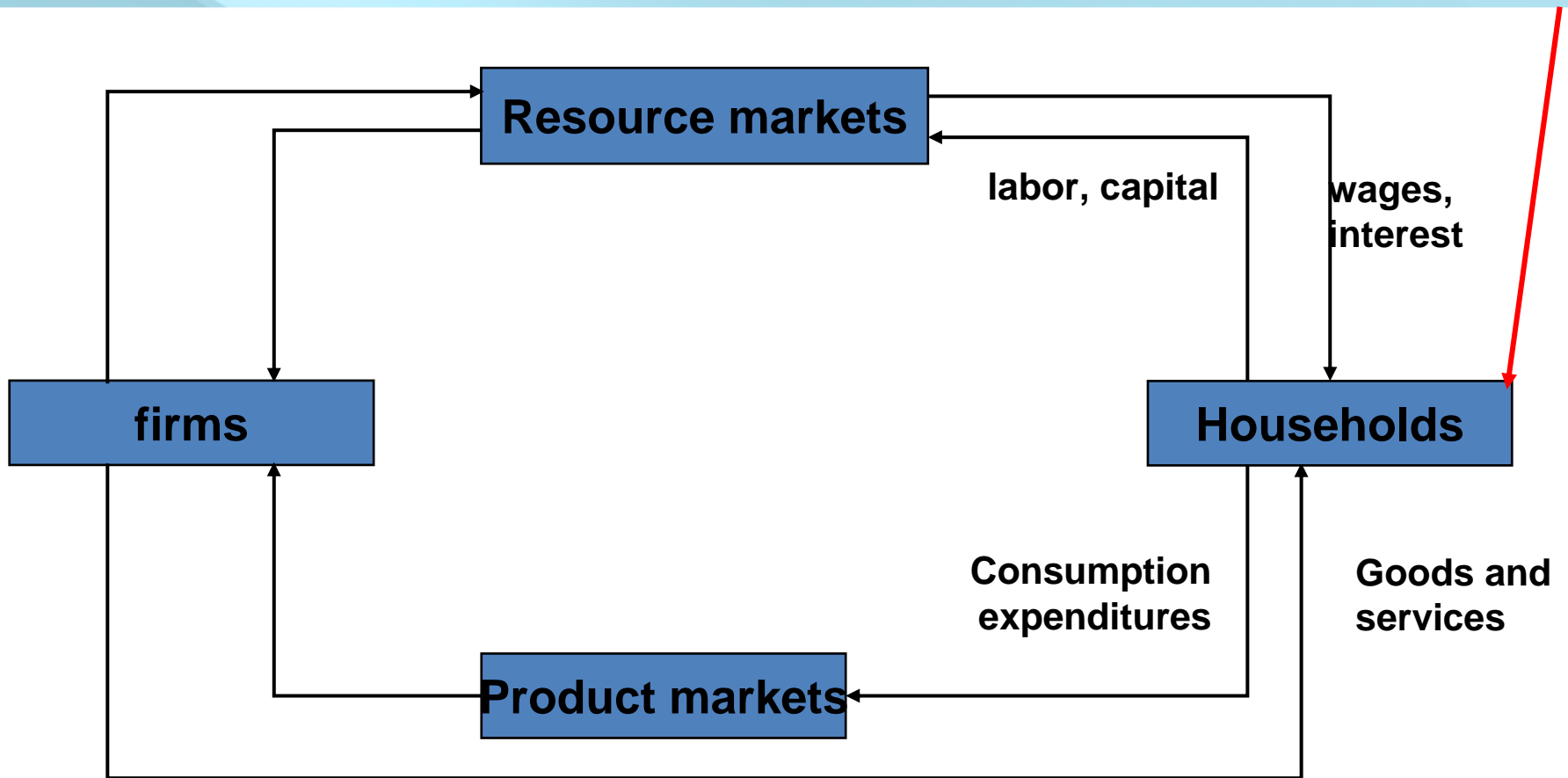
economic relations in a market economy *without* wealth and environmental change

Walras-Arrow-Debreu general equilibrium model

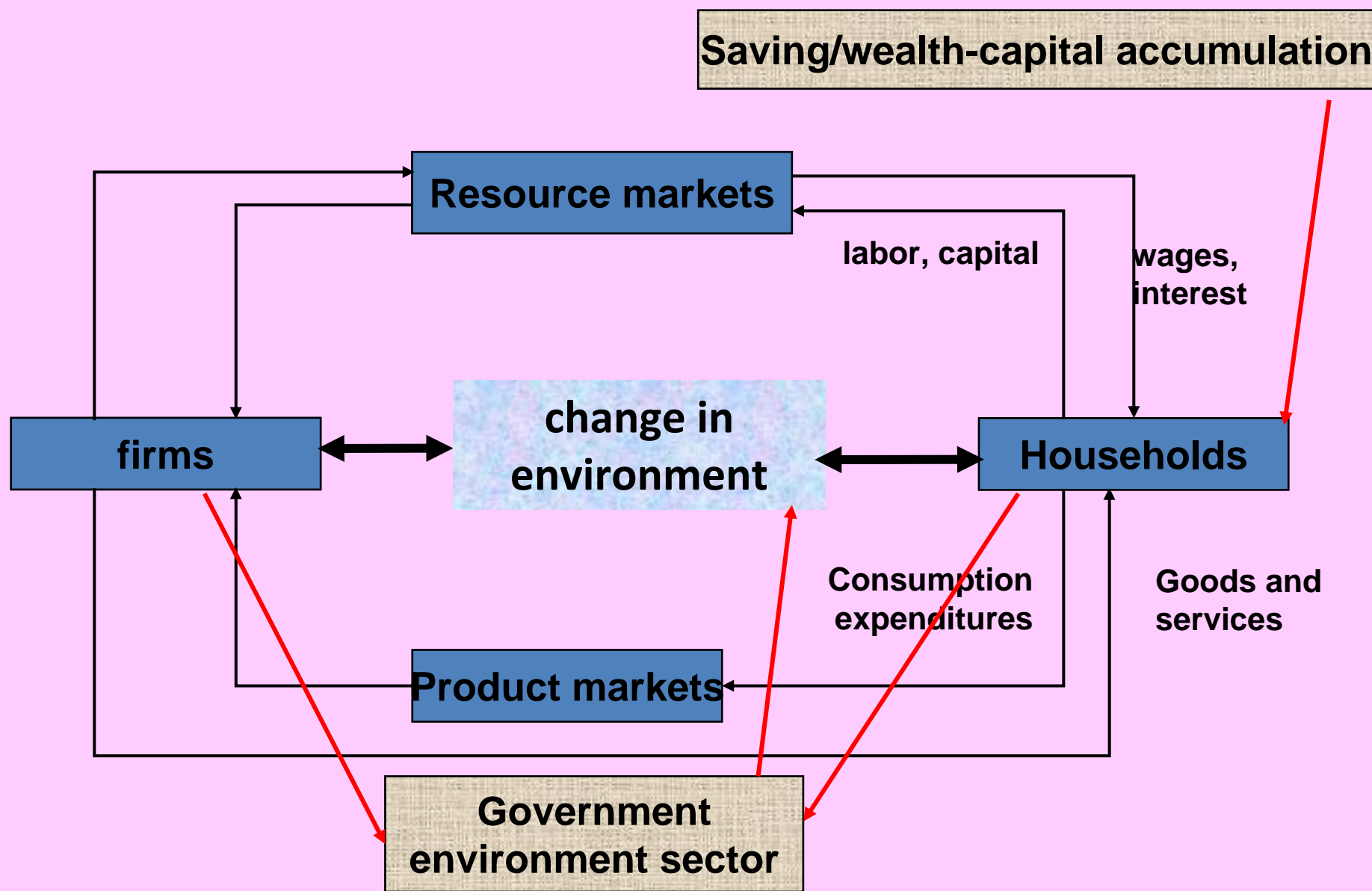


Dynamics of *Solow*-economy

Saving/wealth-capital accumulation



A special case of my modeling framework



Behavior of firms

neoclassical production function:

$$F_j = A_j \Gamma_j(E) K_j^{\alpha_j} N_j^{\beta_j}$$

The marginal conditions

$$r(t) + \delta_k = \frac{\alpha_j (1 - \tau_i) F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i (1 - \tau_i) F_i(t)}{N_i(t)},$$

$$r(t) + \delta_k = \frac{\alpha_s (1 - \tau_s) p(t) F_s(t)}{K_s(t)}, \quad w(t) = \frac{\beta_s (1 - \tau_s) p(t) F_s(t)}{N_s(t)},$$

Households behavior

Current income and disposable income:

$$y(t) = r(t)k(t) + w(t).$$
$$\hat{y}(t) = y(t) + k(t).$$

Budget:

$$c(t) + s(t) = \hat{y}(t).$$

Utility functions

$$U(t) = c^{\xi_0}(t) s^{\lambda_0}(t) E^{-\chi_0}(t)$$

Maximizing utility s.t. the budget yields

$$c(t) = \xi \hat{y}(t), \quad s(t) = \lambda \hat{y}(t),$$

The dynamics of wealth

$$\dot{k}(t) = s(t) - k(t).$$

Full employment and demand = supply

$$cN = F_s(t).$$

$$K_i(t) + K_s(t) + K_e(t) = K(t),$$

$$N_i(t) + N_s(t) + N_e(t) = N.$$

dynamics of pollutants

$$\dot{E}(t) = \theta_f F_i(t) + \theta_c C(t) - Q_e(t) - \theta_0 E(t),$$

where

$$Q_e(t) = A_e \Gamma_e(E) K_e^{\alpha_e}(t) N_e^{\beta_e}(t),$$

environmental sector

The government income:

$$Y_e(t) = \tau_i F_i(t) + \tau_s p(t) F_s(t).$$

The budget:

$$(r(t) + \delta_k) K_e(t) + w(t) N_e(t) = Y_e(t).$$

Maximizing Q_e subject to the budget yields:

$$(r(t) + \delta_k) K_e(t) = \alpha Y_e(t), \quad w(t) N_e(t) = \beta Y_e(t),$$

a few traditional models as special cases

Solow model for growth mechanism

Uzawa model for economic structure

Environmental dynamics

Synthesize within a comprehensive framework

Issues related to environmental Kuznets curve

Lemma 1

The economy is governed by the following 2 -dimensional differential equation

$$\dot{k}_t = \Lambda_k(k_t, E), \quad \dot{E} = \Lambda_e(k_t, E).$$

Moreover, all the other variables can be determined as functions of $k_t(t)$ and $E(t)$ at any point of time by the following procedure: k by (A11) \rightarrow $K = k N \rightarrow k_s$ and k_e by (A1) $\rightarrow N_t, N_s$ and N_e by (A8) $\rightarrow K_t = k_t N_t, K_s = k_s N_s$ and $K_e = k_e N_e$ by (A8) $\rightarrow r, w$ and p by (A3) $\rightarrow F_t$ and F_s by (1) $\rightarrow Q_e$ by (10) $\rightarrow \hat{y}$ by (3) $\rightarrow c$ and s by (11).

$$\begin{aligned} \Gamma_i &= E^{-b_i}, \quad \Gamma_s = E^{-b_s}, \quad \Gamma_e = E^{b_e}, \quad b_i = 0.1, \quad b_s = 0.05, \\ b_e &= 0.3, \quad N_0 = 5, \quad \alpha_i = 0.3, \quad \alpha_s = 0.35, \quad A_i = 1, \quad A_s = 1.1, \\ \alpha_e &= 0.5, \quad \beta_e = 0.3, \quad A_e = 0.5, \quad \lambda_0 = 0.4, \quad \xi_0 = 0.2, \\ \delta_e &= 0.05, \quad \tau_i = \tau_s = 0.05, \quad \theta_f = 0.2, \quad \theta_c = 0.25, \quad \theta_0 = 0.05. \end{aligned}$$

The equilibrium values are given as in (16)

$$\begin{aligned} K &= 7.58, \quad E = 13.63, \quad F_i = 1.19, \quad F_s = 3.79, \quad Q_e = 0.50, \\ N_i &= 1.46, \quad N_s = 3.39, \quad N_e = 0.15, \quad K_i = 1.76, \quad K_s = 5.14, \\ K_e &= 0.68, \quad k_i = 1.21, \quad k_s = 1.51, \quad k_e = 4.69, \quad f_i = 0.81, \\ f_s &= 1.12, \quad r = 0.14, \quad w = 0.54, \quad p = 0.79, \quad k = 1.52, \quad c = 0.7 \end{aligned}$$

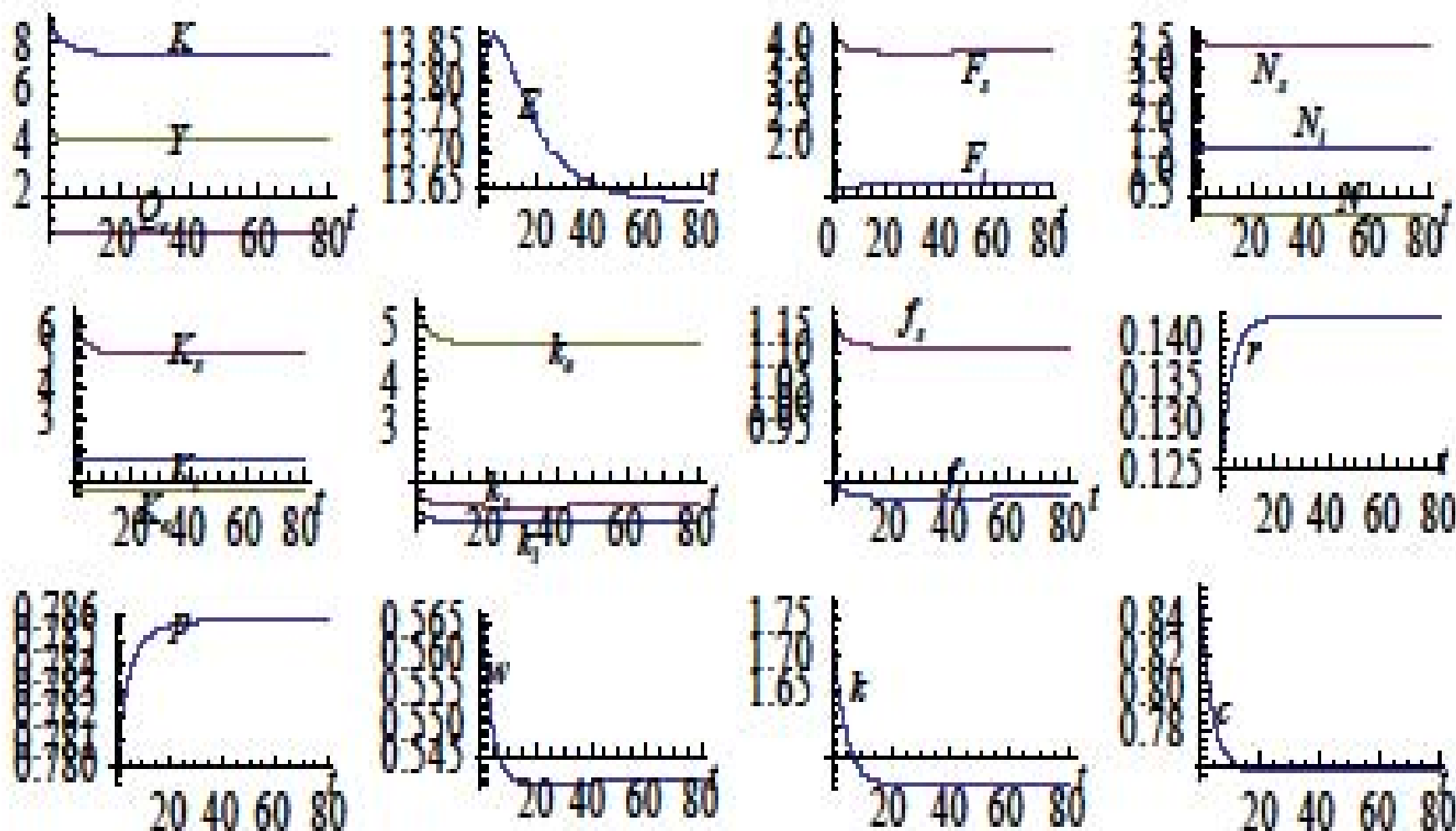


Figure 1 Motion of the Economic System

$\theta_c: 0.25 \Rightarrow 0.30$.

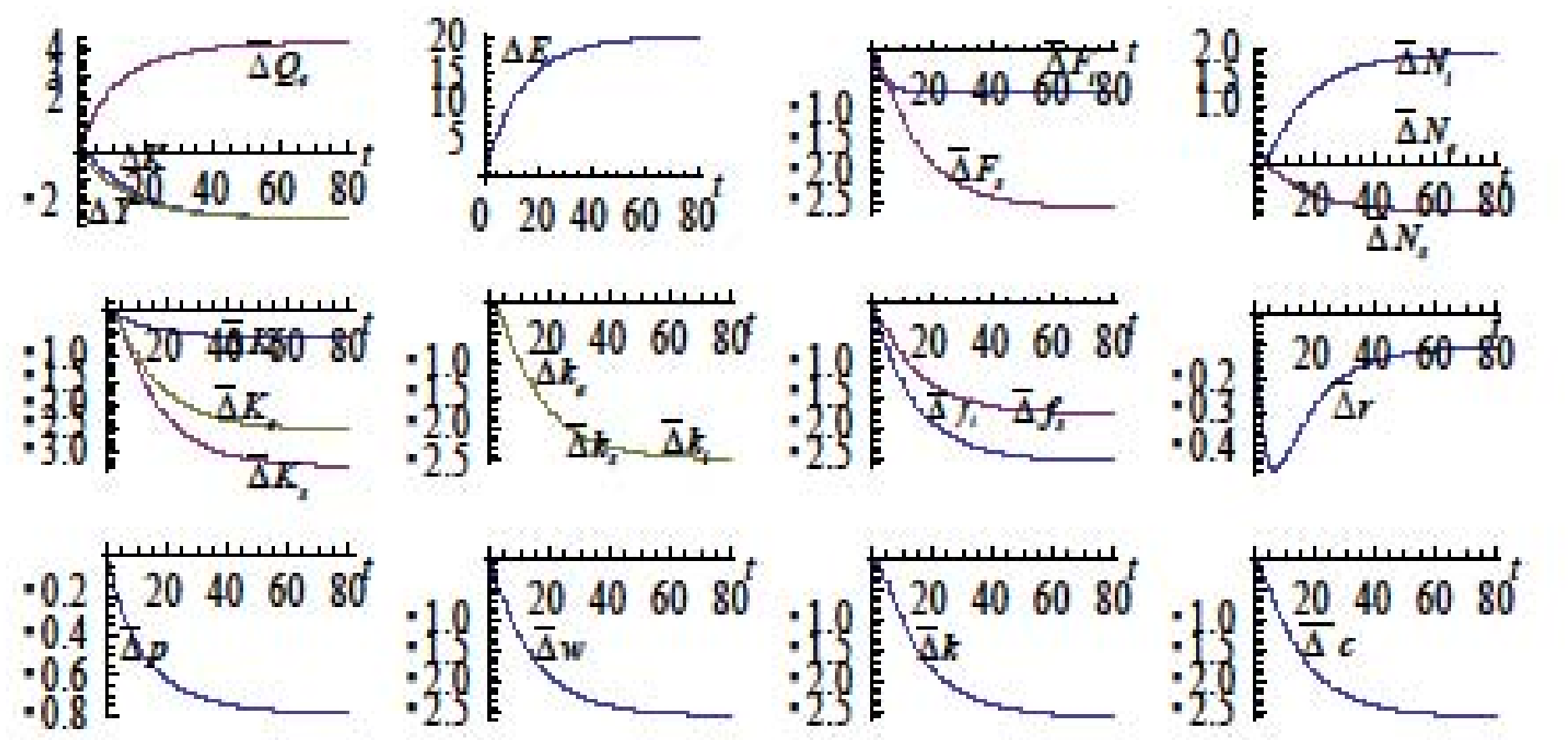


Figure 2 The Pollution by Consumption Becomes Stronger

to be generalized

- Perfect competition is not realistic
- relax full employment
- Endogenous environmental policy
- Endogenous knowledge

Complicating Reality and Fast Growing Theories

- Knowledge, education, globalization
- Available Computer: time and space
- separate theories such as Marxian, Keynesian, equilibrium theory, neo-classical ...
- The time for building a united economic theory