Loss Reserving Methods and Fibonacci retracement In the African Market

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Contents

LIST OF FIGURES

List of Tables

General Introduction

Knowledge of the future is almost impossible but very important for the success of businesses and operations that managers and actors undertake. For the purpose of minimizing the uncertainty inherent in the future, they use some forecasting tools to reduce the risks associated with business conditions. Forecasting is the art and science of predicting future events. It consists of taking historical data and projecting them into the future with some sort of mathematical models. It plays a central role in the management of firms and in a decision-making process. It can tell whether a project is feasible or not, whether it is profitable or not, and how to reduce non systemic errors. Forecasting is used to develop business model by changing the key assumptions and checking the results variations. Besides that, it is very useful in identifying resources and capital needed to carry out the tasks that matter for getting the expected outcomes. In addition the part we are partly interested in this paper, forecasting is used to demonstrate the potential and strength of a business in many ways. In the first part of the study, we are studying the applicability of the Fibonacci trading in the African market. So, the purpose of this paper is to see if round fractions and Fibonacci ratios can be found in the African stock market. We mean by African market the BRVM (Bourse Regionale des Valeurs Mobilieres), it is a Regional Stock Exchange for eight West African Countries and is based in Abidjan in Cote d' Ivoire. In the second part, we are trying to build a forecasting model for insurance companies to assess the reserves needed for future losses. Loss reserving is the function which is used to determine the present liability associated with future claim payments. It is a very important topic for insurance companies chiefly for property and casualty insurance companies which handle more frequent claims. The purpose of this part of the study is to see the extent to which some of the existing methods and models can be combined and enhanced to overcome their flows.

Chapter I: Introduction

Loss reserves are related to insurance companies. By purchasing an insurance policy, policyholders (those who buy the insurance policy) transfer their risk to the insurance company (the insurer) in exchange to a set of payments. When an accident occurs, the company is informed about it, and it has to pay benefits to those clients. The problem is that the company needs to know about the amount to be paid for future claims before they occur. Loss reserving is the function which is used to determine the present liability associated with future claim payments. It is a very important topic for insurance companies chiefly for property and casualty insurance companies which handle more frequent claims. Reserves are needed for accounting, calculation of sufficient premiums (whether to change policy), reinsurance (whether to transfer some risks) and for a better asset-liability management. The published profits of insurance companies depend not only on the actual claims paid, but on the forecasts of the claims which will have to be paid. It is essential, therefore, that a reliable estimate is available of the reserve to be set aside to cover claims, in order to ensure the financial stability of the company and its profit and loss account. Reserves have to be built to figure out how profitable the business is. The balance sheet and the income statement of an insurance company are prepared on an accrual basis. Under the accrual basis, revenues are recognized when earned. Costs are reported as expenses in the same period as the revenues giving rise to those costs are recognized. Insurance companies can pay today benefits for claims that have occurred may be, ten years ago. Policyholder benefits are expenses incurred by the firm, which must be matched to the revenues earned on the policies. It would clearly be inappropriate to count only paid losses and paid loss adjustment expense as expenses. The expenses incurred for policy benefits can be computed through use of the loss reserve liabilities. Loss reserves estimation is an important and unavoidable process insurance companies have to go through. The purpose of this study is to see the extent to which some of the existing methods and models can be combined and enhanced to overcome their flows.

1.1 Definition of terms

There are two kind of claims for which, we need to build loss reserves.

IBNR reserves (Incurred But Not yet Reported) represent those that have occurred but not yet reported to the insurance company. IBNER reserves (Incurred But Not Enough Reserved) for claims that have been reported but not fully settled. This means that the total amount of claim size is not known at the end of insurance period. The year in which an accident happened and the insurer was on risk is called the accident year. As for the number of years until a payment is made, it is called development year. Payments and reserve changes that are recorded in the financial statements during the period in question, without regard to the period in which the accident occurred or was reported. Case reserves can be defined as estimates of amounts required to settle claims that have already been reported but not yet fully paid.

A claim is a demand for payment by a policyholder or an alleged third party under the terms and conditions of an insurance contract. The policyholder or third party that is asking for a payment is called a claimant. A claim closed without payment is one that has been reported and did not require a loss disbursement and is finally closed. The portion of each premium dollar spent on claims and expenses represents the combined loss ratio. Change, in the estimated or actual losses or reserves over ulterior evaluations is called development. A development factor is the quotient of the paid or incurred value for incident record evaluated at time "t+1" divided by the amount of that same accident record evaluated at time "t". Earned premium is the part of the premium proportional to the segment of time a policy has been in force. The sum of all underwriting and operational expenses divided by the premium represents the expense ratio. The smallest divisible part of a claim is called a feature. It corresponds to a loss on coverage for one person. Incurred losses are the sum of payments and reserve changes for claims. An indication is an estimate based on analysis of the data. The loss ratio is the quotient of incurred losses over earned premium.

Finally, the reserves, that are the focus of this paper, are estimates of what the insurance company expects to pay out ultimately for losses that have occurred whether reported or not. Therefore a variety of mathematical methods for estimation of total loss amounts have been developed, and we are going to list some of them and give their advantages and limitations.

1.2 Significance of study

There are many methods and models developed for the purpose of estimating loss reserves. They all have strong points and some limitations. So there is room for improvement. This study is important in the sense that it tries to combine existing models and see the extent to which some flows can be overcome. The focus will be on the Bornhuetter-Ferguson method because it has some advantages over the chain ladder and as a deterministic method is more used in practice than the stochastic ones.

Chapter II Literature Review

During the last decades, practitioners and scholars have developed and used many methods of loss-reserving based on run-off triangles. All of these methods have a common assumption, that is: all claims are settled within a fixed number of development years and that the incremental and cumulative losses from the same number of accident years are well known until the present calendar year. In that way, the losses can be represented in a triangle shape that we call run-off triangle. This means that the development of the losses of every accident year is following a development pattern which is common to all accident years. The most famous and most used of these methods are the chain ladder and the Bornhuetter-Ferguson methods. In a paper by Halliwell (2008), it is said that that the origin of the chain ladder method is obscured in the Antiquity of the Casualty Society. He nevertheless acknowledged that the seeds of the method come from Tarbell (1934). Tarbell (1934) didn't come up with the chain ladder as it is known today, but it is deduced from what he had in mind the actual one.

The chain ladder is a common and easy to implement technique whereby actuaries project losses from less mature stage to a more mature future. At each phase of the development, the actuary calculates the link ratio, or age-to-age factor or development factor that is the ratio of cumulative losses at a development year to that of the previous.

Immature losses move toward maturity when multiplied at each stage by the corresponding development factor. This is a brief description of the chain ladder. It is practically very useful.

Kremer (1982) showed that the chain ladder technique is based upon a linear model. Verrall (2000) went further and explained how we can see in the chain ladder, the linear model Kremer (1982) was talking about. He pointed out that the linear model implied by the chain ladder technique is in fact that of a two way analysis of variance. In another paper by Verrall (1989a), he tried to extend and consolidate the statistical framework which enables the analysis of insurance data. In fact, that was how to enhance and improve the classical chain ladder technique. The improvements were designed to overcome two big problems that the chain ladder method faces. The first flaw was that there was no connection between the accident years resulting in an over-parameterized model and not stable forecasts. The second point is related to the assumption that is the base of the model itself: the development pattern is assumed to be the same for all accident years. The chain ladder cannot adapt to any change with which claims are paid, or any other elements that can affect the run-off triangle.

To establish a connection between the accident years, he proposed the use of the Bayesian framework in which people can assume that the row parameters have the same prior distribution. The over-parameterization that we talked about and that was one the main flaws of the chain ladder technique was due to the fact that in that model, the accident years were not seen as linked but rather considered as separate. He focused on a statistical analysis that allows the use of actuarial judgment. He came up with a methodology that permits other information available to be taken into account to extend the range of the analysis. This Bayes assumption can be a useful way of overcoming the very problem of over-parameterization. Anyway, there is another way to get around the problem, and Verrall thinks that it is more valid than the previous one; it is called the state space approach. It is thought to be superior to the Bayesian approach with the assumption we know in the sense that that assumption is kind of static. We assume there that all the rows are similar. As for the state space model, it assumes that each accident year is similar to the one it follows. This loosens the assumption imposed on the classical chain ladder method. Later, in that same paper, Verrall compares the chain ladder with the two- way analysis of variance. For the analysis of variance, he gave the unbiased as well as the maximum likelihood estimates of future liabilities. He also made a parallel between the different methods used to analyze claims runoff triangles and emphasized their closeness to the linear model.

In the Bayesian model, the management might use the information at its disposal as a specified distribution and incorporate it in the model. Verrall (1989a) went further in the same paper and pointed out the fact that the estimators from the Bayesian model can have credibility theory significance. The Bayesian model frees managers and they can chose the distribution they think fit the most to the data they have to deal with.

Recently, in 2008, Halliwell (2008) looked at a possible bias in the chain-ladder estimates. It was common knowledge that the chain ladder method, the most used reserving method is biased. Nevertheless, no one has previously attempted to prove or back it scientifically. Those that had that intuition believed the bias to be upward. Halliwell (2008) tried to see whether there is anything inherent in the chain ladder that predisposes it to bias. He discovered that, effectively, the chain ladder is biased. The bias is due mainly to two elements that make it over predict future losses. Those two elements cited earlier are the regression toward the mean and business expansion. His conclusion was what the chain ladder bias means for us, is that loss development models, in addition to being reasonable and empirically tested, should be free of proxy variables.

Wouve and Dhaene () emphasized the chain ladder's dependence on outliers. They considered scenarios with outlying data and found that forecasts depend strongly on outliers. They then tried to robustify the method in a way that the presence of outliers in the data won't affect the results any more. In that way, actuaries would find liabilities a bit similar to those they would have obtained if the data contained no outliers. The first method they proposed detects and adjusts the outlying values. They use the usual run-off triangle and some robust statistics to develop a robust chain-ladder. The latter will recognize the outliers in the run-off triangle and smooth the outlying data in the run-off triangle in a way such that the estimated liabilities will come close to what would be obtained without outliers. Their method is comprised of two items. They first developed a statistical tool to detect the outliers present in the data with a high probability. The second part consists of eliminating the impact of the outliers on the final results by some adjustment. This is how one can see the difference in the expected claims reserves with and without outliers. Their idea to detect the outliers in the run-off triangle is to change the way the development factors were calculated and use the incremental claims instead. The use of the cumulative claims for the development factors calculus makes the chain-ladder too sensitive to the presence of outliers. For instance, an outlier in the first column will affect all development factors. On the other hand, when using incremental claims, an outlier will affect at most two development factors. They proposed for that matter to use the mean instead of the mean to get the development factors. Once the outliers detected, they proposed the robust generalized linear model to cancel out the outliers' effects on outstanding claims.

England and Verrall (2002) made an almost complete review of the existing reserving methods and models actually in use in the insurance field.

The chain ladder method is based exclusively on the development factors; it often happens that the predicted result cannot be relied on with the confidence level we would like. This is particularly likely for more recent underwriting years where the development factor to predict from the actual to ultimate loss amount is relatively variable, due to the present lack of claims development.

That's way, actuaries thought of making use of an alternative ultimate amount, usually obtained from a supposed loss ratio. This leads us to a very used and useful loss reserving method called the Bornhuetter-Ferguson method. It is named after the two that developed it in 1972. Bornhuetter and Ferguson (1972) proposed predictors of outstanding ultimate losses and every predictor is obtained by multiplying an estimate of the expected ultimate loss by an estimator of the percentage of the outstanding loss with respect to the ultimate one. It is based on the run-off triangle like the chain-ladder but it restricts its use to the estimation of the percentage of the outstanding loss and uses the product the earned premium and an expected ultimate loss. This method tries to stabilize the chain-ladder method and makes it less sensitive to outliers.

Klaus and Zocher (2008) showed that despite their different appearances, the chain ladder and the Bornhuetter-Fergusson methods have very much in common. In that paper, they first pointed out the fact that they both have a multiplicative structure when they come to the ultimate outstanding losses. They introduced a new model named the extended Bornhuetter-Ferguson. They used the notion of development pattern to prove that the latest is not just one method among various others but a general one that comprises many other methods as special cases and leads to the Bornhuetter-Ferguson principle. After a thorough study of the Bornhuetter-Ferguson method they stated what its principle is. So, it consists of three elements: the simultaneous use of different versions of the Bornhuetter-Ferguson, the comparison of the different ultimate losses and finally, the selection of the best ones. With a numerical example, they showed that the Bornhuetter-Ferguson principle can be used to select an appropriate version of the extended Bornhuetter-Ferguson for any run-off triangle.

Up to this point, all the methods used are deterministic and give a single estimate without any information about its variability. In recent years, considerable attention has been given to discuss possible relationships between the chain-ladder and some stochastic models. So, in Mack (1993), a formula for the standard error has been derived and a programmable recursive way of calculating it was also given. Moreover, he shows how a tail factor can be incorporated in the calculation of the standard error. Schnieper (1991) used a mixture of Bornuetter-ferguson and the chain ladder for the same purpose; but Mack's formula is specialized for the pure chain-ladder method. Later, many stochastic models were developed to give an idea of the variability of the estimates or the prediction errors.

In this way, Mack published a paper where he described a stochastic model based on the chain-ladder without any assumed specific distribution. It is called the distribution free model and it reproduces the chain ladder estimates and provides a mean of getting the standard errors. In Mack and Gary (2000), they conducted a comparative study of the distribution free and the over-dispersed Poisson models. They concluded that, both of the two models reproduce the chain ladder estimates for the claims reserve. Nevertheless, they argued that the two models are different in the sense that the true expected claims reserves, let alone estimation issues, are different. Moreover, the over-dispersed Poisson model deviates from the classical chain ladder in many aspects that the Poisson model does not. In conclusion, they stated that only the distribution free model can qualify as a model underlying the chain ladder. Anyway, the two models are the only ones known that lead to the same estimators for ultimate claims as the chain ladder algorithm. Likewise, England and Verrall (2002) did an almost complete review of the stochastic claims reserving in general insurance. They discussed the limitations of various models actually in use in the reserving process. They emphasized a very important point about useful can be the stochastic models. Those stochastic models should not be seen as stopgaps when deterministic models fail. The usefulness of the stochastic models is that they can, in many circumstances, provide more information which might be useful in the reserving process and in the overall management of the company.

Kremer (1989) introduced the log-normal model and it was later used by Renshaw (1989) and others. According to England and Verrall (2002), use of that model usually gives estimates close to those from the simple chain ladder but it is not guaranteed, and there can be material differences.

Buhlmann (1967) first introduced Bayesian ideas and techniques in actuarial science. To date, Bayesian methodology is used in various areas within actuarial science, in particular in loss reserving. But up to recently, little has been done from in this case from a Bayesian perspective.

This might be due to the lack of appropriate software. Bayesian models sometimes can provide analytical closed forms for the distribution of the outstanding claims, and inference can be carried out from the distribution to have any of its characteristics and properties.

Bayesian method provides a natural way of incorporating the prior information. Alba (2002)

3 Chapter III Methods

3.1 The Expected Loss Ratio

3.1.1 Description

The Expected Loss Ratio (EPR) is the simplest method to estimate future payments associated with present liabilities. The business line manager or the actuary sets a rate that is going to be used to calculate the ultimate loss development. The ultimate loss development is the amount that is needed to settle all claims in the policy period. Generally, loss ratios are calculated by dividing some measure of losses (or claims costs) by the earned premiums.

EPL = Ultimate Loss / Premium Earned (1)

Premium is the amount paid by an employer to insurers for protection against risk of financial loss arising from a covered accident. This Expected Loss Ratio, once set, is multiplied by the earned premium for each year to have the corresponding ultimate loss.

The ultimate loss for the policy period is just the sum of the expected ultimate loss for the different years.

From the Expected Ultimate Loss, we deduct the loss paid up to-date to obtain the case reserve which is the amount of unpaid liabilities.

3.1.2 Advantages and Disadvantages

The main advantage of the expected loss ratio is its simplicity. It allows also the inclusion of actuarial judgment when setting the rate.

The last one may be seen as a disadvantage because there is no unique way of setting the loss ratio. The loss ratio is determined using data of the previous policy period, so it doesn't take into account the more recent development patterns. The estimate of overall losses depends *only* on the premium income and the stated loss ratio for the class of business. Important changes shown or incipient in, the claims development patterns will not be acknowledged or made use of in any way. This method estimates ultimate losses for a policy year by applying an estimated loss ratio to the earned premium for that policy year. Although the method is insensitive to actual reported or paid losses, it can often be useful at the early stages of development when very few losses have been reported or paid, and the principal sources of information available to the Company consist of information obtained during pricing and qualitative information supplied.

However, the lack of sensitivity to reported or paid losses means that the method is usually inappropriate at later stages of development.

3.2 The Chain-ladder method

3.2.1 Description

The chain-ladder is probably the most popular tool for estimating claim reserves. It will be assumed for this method that the data is in form of a triangle for notational convenience. There are no problems in extending the data to other shapes of data.

In this case, data are set up in what we call a development triangle where the rows represent accident years and the columns development years.

The elements X_{ij} in the development triangle represent what has been paid out during development year j for losses incurred during accident year i.

A paid loss triangle is only one of several types of triangles that can be constructed. We can have a triangle considering the number of claims opened; the number of claims closed, or incurred losses.

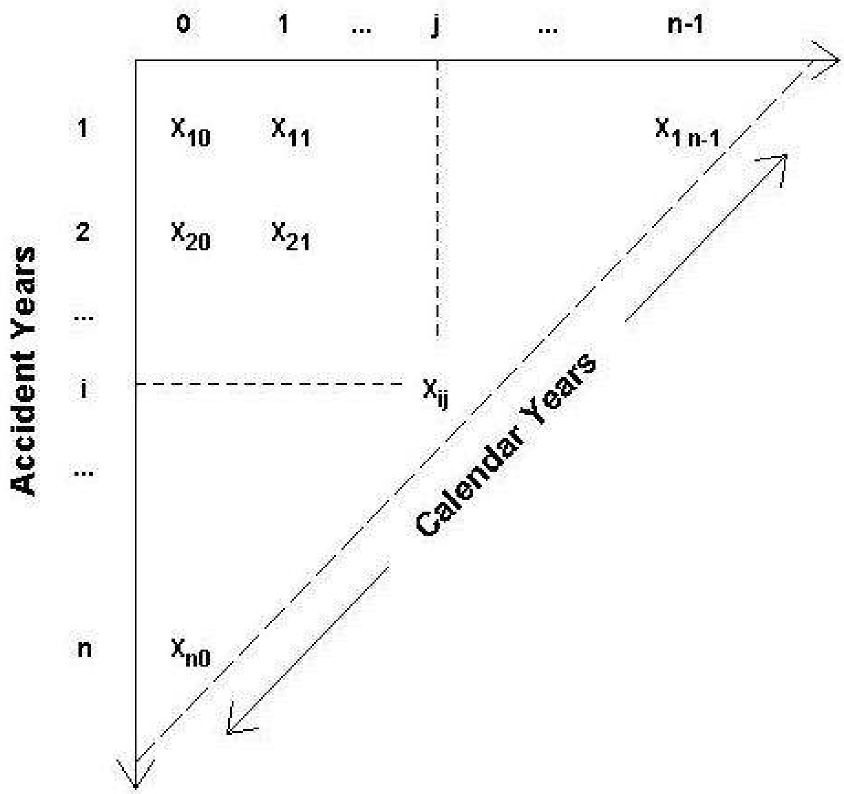


Figure 3.1 : Loss Triangle

Source: American Association Insurance

Now it is visible that the loss payments of the calendar year n appear on the lowest diagonal of the triangle. Similarly, the calendar year n-1 payments appear on the second lowest diagonal. This data organization greatly facilitates comparison of the development history experienced by an accident year.

We have that X_{ij} is known, it is observed when $I + j <= n$ and the task is to expand the triangle into a rectangle contained with predictions of future values.

For forecasting future values, we need the cumulative loss payments. The cumulative loss payment at development date k is the sum of the amount paid out up to the development date k. Therefore, cumulative claims can be expressed as follows.

$$S_{ik} = \sum_{l=1}^{k} X_{il}$$

From the cumulative loss paid, we calculate the link ratios or link factors or development factors to determine the ultimate expected loss.

Once I have determined the link ratios for each development date k, k=1…n-2, I can estimate the cumulative paid loss for each stage by multiplying the previous cumulative loss paid to the corresponding link factor. The process is repeated up to the last development date to have the ultimate expected cumulative loss for each accident year. As in the previous section, we deduct from the ultimate expected loss the amount paid up to date to obtain the case reserves for each accident year. The sum of these case reserves will constitute the case reserve for the policy year.

3.2.2 Advantages and Disadvantages

This method can be used to overcome the main problem of the Expected Loss Ratio which is it ignores the more recent payment patterns. The chain-ladder uses up to date data. It is an objective method that can be applied without any other considerations. In addition, unlike the Expected Loss

Ratio, as losses develop and time passes the estimate converges to the real loss value. It is also very simple and easy to use. However, the chain-ladder presents some problems.

We know that the relationship between losses at different development periods may not be multiplicative. The estimates given by the chain-ladder will be distorted by changes in the claim payment patterns. If the company decides to settle claims faster, normally the case reserves would be reduced, but with the chain –ladder it is going to be increased. If no losses have been paid yet for a given year, the method predicts ultimate losses of 0. The chain-ladder doesn't integrate any actuarial judgment, so it is difficult to take into consideration the company's policy when using it. This method is very sensitive. When an outlier exists in the data, it must be ignored or the chain ladder will overreact. For this reason, a more stable method is needed.

3.3 The Bornhuetter-Ferguson

3.3.1 Description

The Bornhuetter-Ferguson (BF) method was developed to combine advantages of the Expectation Loss Ratio and the chain ladder. So, it is somewhat in between these two methods. It is based on the idea of dividing the overall loss for each accident year into its past and future, or emerging, portions. The BF method uses what is actually paid plus what we expect to develop if the ultimate expected loss obtained with the loss ratio is correct. The claim payments are divided into 2 parts:

those already made, and those which will emerge in the future. It is to be estimated as a proportion of the final losses, which in turn are estimated by the simple application of the loss ratio to the earned premium. So, they used in the BF method a loss ratio and work with data in form of paid claims. The first stage is to work out the link ratios themselves. They are obtained the same way as in the chain ladder method. The final ratios used in the chain ladder are the ones we need for this part. We calculate then the inverse of those ratios and subtract the results from unity (1-1/ratios). If we apply these (1-1/ratios) factors to the ultimate loss, then we have the remaining claims which should emerge in the future. It gives us the case reserves for each year during the policy period.

3.3.2 Advantages and Disadvantages

The Bornhuetter-Ferguson method provides a happy judged combination of the expected loss ratio method and the earlier paid claims projections.

The expected Loss Ratio didn't pay attention to the actual claims development in the recent years. On the other hand, the chain ladder method relies on the continuation into the future of the patterns for claim payment. A sudden change in the pattern for the latest accident year in particular will distort the projections. The Bornhuetter-Ferguson lies between the two eventualities. It is a great advantage for the BF method. It is stable because it uses factors that are less than 1 for estimating emerging claims. It avoids overreacting unexpected losses. It allows the use of external sources and actuarial judgment to take into consideration the specific nature of the business in question. The main problem with the Bornhuetter-Ferguson is that it is also affected by changes in claims practices. There is a problem that is common to all the deterministic methods. Testing a single estimate is unlikely to be conclusive. Having just a punctual value doesn't tell how far we may be from the real one. So, we have no idea of the errors made in estimating losses. This weak point can be overcome by the use of stochastic models.

3.4 The Model of Mack

The primary advantage of stochastic reserving models is the availability of measures of precision of reserve estimates. To obtain the prediction error, we have to formulate an underlying statistical model making assumptions about the data. The model of Mack reproduces the chain-ladder estimates with limited assumptions as to the distribution of the underlying data. The data in the paid loss triangle are seen as outcomes of statistical distribution which is not specified. The model of Mack specifies only the first two moments.

$$E(S_{ij}) = \lambda_j \, S_{i,j-1}$$

$$V(S_{ij}) = \sigma_j \, S_{i,j-1}$$

We just need to estimate λ and σ to be able to fill in the lower part of the triangle.

It is simple and straightforward and easy to implement. However, the Mack model is distribution-free; no distribution is specified for this model. So, further assumptions are needed to predict errors. It cannot solve directly the problems encountered by the deterministic models. We need to look at some other stochastic models that permit to predict without any additional assumptions. The usefulness of stochastic models is that they can, in many circumstances, provide more information which may be useful in the reserving process and in the overall management of the company.

3.5 Over-dispersed

This predictor structure is still a chain-ladder type, in the sense that there is a parameter for each row i, and a parameter for each column j. There are some advantages and some disadvantages to this form of the model. As a generalized linear model, it is easy to estimate, and standard software packages can be used; the estimates should be well behaved. However, the parameter values themselves will be harder to interpret, making it necessary to convert them back into more familiar quantities. Note that constraints have to be applied to the sets of parameters, which could take a number of different forms. For example, the corner constraints would

put $a1= b1=0$. Bayesian models have much to recommend them, since estimation and prediction occur in the same model at the same time. Since model fitting and prediction is performed using simulation, the methods automatically provide a predictive distribution of reserve estimates, from which the prediction error, if required, can be estimated by calculating the standard deviation of the simulated results; there is no need to evaluate complicated

formulae. Bayesian models also have the advantage that actuarial judgment can be incorporated through the choice of informative prior distributions. This is also a major disadvantage, since it leaves the methods open to abuse. Practical difficulties associated with Bayesian models include choice of prior distribution and assurance that the software has converged on the optimum

solution. Although such reassurance can be gained by knowing what the results should be (using analytic methods), a pragmatic alternative is to repeat the analysis several times, starting the simulations from very different initial values, and checking convergence.

4 Chapter IV Results and Analysis

The Bornuetter Ferguson (BF) method presented earlier presents some advantages over the chain ladder. It remains somehow as simple as the chain ladder and is very easy to use. That is the reason why we chose to focus on the BF method to try to improve the objectivity of the results.

The flaws of the BF method mostly stems from the subjectivity involved in determining the loss ratio. The loss ratio as most of the quantities used in the claims reserving process is subject to uncertainty. In the BF method, if the calculated loss ratio does not seem very realistic, it can be adjusted with managerial insights to come close to the expected loss ratio.

This way of incorporating actuarial judgment can help to get a more realistic ratio but it lacks some scientific support. Therefore, the modified BF method will basically replicate the classical one but will use a more scientific of estimating the loss ratio instead of guessing it. For that purpose we propose a Bayesian estimating methodology to get the appropriate loss ratio in the given business line. Let us recall the BF formula

$$Reserves = Premium * ELR(1 - \frac{1}{f}) \qquad (2)$$

Where reserves is the claims reserves and premium the earned premium, ELR represents the expected loss ratio and f is link factor used in the chain ladder method. As stated before, many of the problems are related to the objectivity in determining that loss ratio. So, the empirical Bayes method will be used to estimate the expected loss ratio to overcome that weakness.

The method is described and implemented below.

I consider k business units or insurance companies in a given business line including the company or business line for which we intend to get the loss ratio.

I need to look upon the different business units for a quite long period. Let us assume that I have the loss ratios of the different business lines for N years.

μ_i is the expected loss ratio for the company i= 1…k.

For each company, we consider N yearly loss ratios and average them out.

Let y_{it} t=1…N the loss ratio for the company i at time t.

$$\dot{\hat{y}}_i = \sum_{j=1}^{N} y_{ij}/N$$

Having the times series mean, I assume that:

$y_i|\mu_i$ follows a normal distribution with mean $\mu_i$and variance V_i for 1=1…k independently. For the sake of simplicity and without loss of generality, I assume:

$$V_i = V$$

That means I assume constant variance across firms or business lines.

Furthermore, I assume that μ_i i=1…k are independently distributed an:

μ_i follows a normal distribution with mean μ and variance A for i=1…k

This is the prior distribution of the parameters $\mu_i \; i = 1 \ldots k$. Unlike in a full Bayesian model, these parameters are not assumed to be known and have to be estimated from the data collected.

Previous studies (Winkler 1972) have shown that it can be obtained from the two preceding results the distribution of $\hat{y}_i$.

$\hat{y}_i$ follows a normal distribution with mean μ and variance $A + V_i$ for each i=1…k independently.

I get from that the posterior distribution for each μ_i

$$\mu_i|\hat{y}_i \longrightarrow N\big((1 - B_i)\hat{y}_i + B_i\mu\big), (1 - B_i)V_i) \quad \text{for i=1…k independently}$$

Where $B_i = V_i/(A + V_i)$

From the distribution for $\mu_i|\hat{y}_i$ we get its mean, so

$$E(\mu_i|\hat{y}_i) = (1 - B_i)\hat{y}_i + B_i$$

The latter expression is known as the Bayes estimate for the parameter of interest μ_i

Since I don’t know μ norB_i, they should be estimated. This can be done by using their marginal distribution. By doing so, we will get the expected loss ratio for each company in the group.

$$\mu_{Ei} = \left(1 - \hat{B}\right)\hat{y}_i + \hat{B} * \hat{\mu}$$

Where

$$\hat{\mu} = \sum_{j=1}^{k} \hat{y}_j /k$$

And

$$\hat{B} = (k-3)/(k-1)[\frac{V}{V+A}])$$

And

$$A = \sum_{i=1}^{k} (\hat{y}_i - \mu)^2/(k-1) - V)$$

Now, it is possible to get the expected loss ratio for each company in the group in a more scientific way. Therefore, we will just need to plug the loss ratio in the classical BF formulae.

$$Reserves = Premium * ELR(1 - \frac{1}{f}) \qquad (3)$$

Instead of guessing or adjusting the expected loss ratio, it will be obtained through the method described previously and plugged in the above equation. The subjectivity that characterized the first way of doing is now removed.

I will apply the new method to a group of companies in the reinsurance field.

It has been applied to eighteen reinsurance companies whose loss ratios were collected over a period of twelve years from 1998 to 2009. The data is from the American Association of Insurance.

Following are the companies and the average loss ratio for each of them during that timeframe

The sample mean μ is used as an estimator for the population mean. It is equal to μ = 77.33% and the variance is equal to V = 0.00326.

$\hat{B}$ can be estimated as in the model and this is its value from the sample.

$$\hat{B} = 23.16\%$$

Now, these parameters have been estimated, we get the expected loss ratio for each company. That expected loss ratio will be used in the BF method for one company to calculate the reserves.

Companies	Expected Loss Ratios
American Re	95.12%
AXA Reinsurance	75.06%
EMC Rein	73.93%
Employers Re	81.57%
Everest Reins	75.67%
General Re Group	78.91%
Folkscamera re	74.11%
Odyssey Re	74.85%
Partner Re	78.76%
Platinium	60.93%
QBE Reins	72.33%
Swiss Reins	83.12%
The Toa Reins	78.50%
TRC/Putman	75.25%
XL Reinsurance	81.84%

Table 4.1: Ratios estimates

Partner's expected loss ratio is 78.76%. When we intend to get the reserves, we will only need to plug this ratio into the classical BF formulae in place of the ELR.

4.1 Merits of the modified Bornuetter-Ferguson method

I can recall that from the classical BF method, the main problem stemmed from the subjectivity in incorporating managerial judgment into the expected loss ratio calculation. So, the modified BF method addressed that problem and has proved to be more scientifically elaborated.

Another point is that to be able to use managerial insight in the determination of the loss ratio, one should look back at the company to see how resistant it is to exogenous shocks. This requires that the company to have been in business for a long time. Therefore the modified BF method is very appropriate for companies that have not been in business for so long. When managers don't know enough the company as to incorporate managerial insight in the computation, they can borrow information from the market.

As in the classical Bf method managerial judgment can be used in the new one not in terms of guessing or thinking but it will be scientifically backed. It can be used as a prior distribution and tested to effectively fit the assumed the distribution before being used.

Furthermore, even if it is a well and long established company, it has to be stable to enable actuaries to adjust the actual rate for future losses. Hence, this method is very helpful when the business line or company is stable. When it is too difficult to pinpoint a trend or a loss pattern, it is illusory to try to get a sound loss ratio from it. The modified BF method can address this kind of problem. In addition, the stability of the business line does not guarantee the soundness of the expected loss ratio. No company operates alone, and how competitors are doing affect the company. Whatever company limits itself to its own historical data is overlooking the market risk. Taking into account that fact can help get if not the right rate, one close to it.

There are many external factors that have an important impact on the company. Ignoring them and focus on only the company of interest can provide misleading results. We think that for all these reasons, the modified BF method has many advantages over the classical one. However, as in any deterministic method, it does not provide

Conclusion

So far we have seen a couple of models. I have listed the advantages and disadvantages of each of them. The first three are deterministic and provide a punctual estimate of the loss. So, no way to figure out the error made in the estimating the future losses. Any way the Bornhuetter-Ferguson method which allows the use of actuarial judgment may be updated from one evaluation to another.

If the company's operations change, or if other factors suggest an appreciable divergence from past development of input parameters, then to the extent that these changes can be quantified, "historical" inputs should be replaced by these "subjective" inputs that incorporate the changes.

The only stochastic model we have seen is distribution-free and doesn't incorporate any actuarial judgment in the evaluation.

For the sake of simplicity, we proposed to modify the BF method which of course deterministic to make it more objective. That modified BF method addressed many of the flaws faced by the classical one. Still it can be improved and that will be the focus of further research.

A strict formula approach to projecting loss and loss adjustment expense reserves will not work. A number of different projection methods must be used, and the experience and judgment of the analyst is critical. The analyst must be in constant communication with claims and underwriting management so that appropriate methods are used and proper adjustments are made. Of equal importance is communication of the results to management that might lead to improvement in claims processing procedures or development of underwriting initiatives. The reserving actuary should also communicate results to the pricing actuary so that pricing procedures can be modified if required. The use of several methods at a time can be a very good idea in determining the right method for a given company. There no panacea method for all companies all the time. The method in use should be carefully selected and can be changed over time if the conditions change.

5 Chapter VI Fibonacci Retracement

5.1 Introduction

Technical analysis is the use of trends and charts to understand and analyze investors' behavior and its effect on subsequent price action of financial instruments. It is believed that technical analysis holds the key to monitoring investor. For technical analysts, investor sentiment is the single most important factor in determining an instrument's price.

They are only interested in the price movements in the market. The field of technical analysis is based on three key assumptions:

The market discounts everything: the company's fundamentals, along with broader economic factors and market psychology, are all priced into the stock.

Price moves in trends: After a trend has been established, the future price movement is more likely to be the same direction than against it.

History tends to repeat itself: Market participants tend to react the same way to similar stimuli over time.

Technical analysis relies on the use of trends and chart patterns, or moving average when the trend is so clear or support and resistance levels obtained through Fibonacci ratios.

Over the past seven centuries, much has been written about *Leonardo Fibonacci*, the gifted Italian mathematician who discovered the Fibonacci sequence.

The sequence begins with 0 and 1, and then adds the previous two numbers to get the third one. Then, the sequence continues onwards to infinity.

Mathematically if the following is a Fibonacci sequence:

$$F_1, F_2\ , F_3\ , F_4\ , F_k\ \ldots$$

Then, from the third term one will have the following equality:

$$F_n = F_{n-1} + F_{n-2}$$

These numbers are believed to have a very important role in sciences and are seen as the key to nature. The Fibonacci ratio is found in the geometry of logarithmic spiral, which is widespread in nature. Fibonacci proportions are also found in the double helix of DNA molecule, in the reproduction cycles of rabbits, and branching pattern in plant life and are used in art and architecture. So, now Fibonacci ratios are found in many different fields in sciences and in nature, can we say that they are also present in the stock market?

Fibonacci retracement is a popular trading tool and it is used in trading strategies by many professionals. That is a trading methodology that has been given credence, and consequently, its impact has become self-fulfilling. Moreover, the Fibonacci sequence of numbers is also referred to by *Ralph Elliot* as the mathematical basis for the "Elliot Wave" principle. The "Elliot Wave" principle is a powerful tool for forecasting stock market behavior.

This idea has been challenged by Batchelor and Ramyar(2006), they stated that the idea that round fractions and Fibonacci ratios occur in the Dow Jones can be dismissed.

So, the purpose of this paper is to see if round fractions and fibonacci ratios can be found in the african stock market. We mean by african market the BRVM (Bourse regionale des valeurs mobilieres), it is a Regional Stock Exchange for eight West African Countries and is based in Abidjan in Cote d' Ivoire. This paper follows closely the analysis by Batchelor and Ramyar(2006). It will be divided in four sections. The second section below introduces our hypothesis and reviews relevant research findings. In section 3, we will talk about the data we will be using, it is from the West African Regional Stock Market and develops a method of finding the peaks and troughs in range data based on Pagan and Soussonov (2003) modified a bit because we are working on a four month data set. In the fourth and last section, we will compare the price ratios for successive trends with the Fibonacci ratios distribution using the Kolmogorov-Smirnov test.

5.2 Support, Resistance and Fibonacci Ratios

Anyone who is familiar with the financial press and the web-based financial services is aware of the popularity of technical analysis and the abundant literature about it. Allen and Taylor (1992) conducted a survey among chief foreign exchange dealers based on London in November 1988. Among other findings, it is revealed that a least ninety per cent of interviewees give some weights to technical analysis when performing views at one or more time horizons. They found that traders relied more on technical analysis than on fundamental analysis at shorter time horizons and that this would be reversed in the long run. A great proportion of respondents suggested that technical analysis may be self-fulfilling. Lui and Mole (1998) get almost the same conclusion when they conducted a survey in 1995 on the use by foreign exchange dealers in Hong Kong of technical analysis to form their forecasts of exchange rate movements. Technical analysis is slightly more useful in forecasting trends than fundamental analysis, but found to be significantly useful in predicting turning points.

Technical analysis is like a generic word that comprises a set of techniques and methods some based on visual recognition of chart patterns and trends, others on values indicators computed from past price and volume data.

Neely, Weller and Dittmar (1996) used a genetic programming technical trading rules, and find strong evidence of economically significant out-of-sample excess returns to those rules for each of six exchange rates, over the period 1981-1995.

Neely (1997) explained shortly the fundamentals of technical analysis and the efficient markets hypothesis as applied to the foreign exchange market, and evaluated the profitability of simple trading rules, and reviewed recent ideas that might justify extrapolative technical analysis. Many previous studies investigate filter rules that require a trader to buy if price rises more than k% above the most recent low price and vice versa Batchelor and Ramyar (2006).

Lebaron (1996) and Szakmary (1997) show that extrapolative technical trading rules trade against U.S foreign exchange intervention and produce excess returns during intervention periods.

Leahy (1995) shows that technical trades make excess returns when they take positions contrary to U.S. Batchelor and Ramyar (2006) show that recent studies investigate moving average that tell the trader to buy or sell if the market price or (short term moving average) exceeds or falls below a long term moving average. Gencay (1999) found that simple technical rules provide significant improvements for the current returns over the random walk model. A smaller amount was interested in the evaluation of pattern-based trades. Some traders look at trend line breaking rules that require to buy or sell if the price breaks above some resistance level or falls through some support level. This is based on the belief that as long as the share remains between these levels of support and resistance, the trend is likely to continue. Support level is the level at which brokers are willing to buy and resistance level is the one at which they are willing to sell. That is the reason that makes it difficult for shares to fall below the support level or exceed the resistance level. Once a support level is broken its role is reversed, that level will become resistance. If the price rises above resistance level, it will often become support. Traders look also at the pattern trades that require to go short if some sequence of prices characteristic of the an upward trend appeared. An upward trend is a succession of higher peaks (highs) and higher low (troughs).

Each new high is higher than the previous one and each new low is higher than the one before. The trend continues until its reversal. Likewise, a downtrend is a succession of lower highs and lower lows.

A widely used formation is the head and shoulders one. Its development is a generation of lower high in an uptrend rather than a higher high or an equal high. Sellers appear levels than they previously did, and the buyers no longer have the same appetite at these higher levels as before. In a comprehensive and influential study Brock, Lakonishok, and LeBaron (1992) analyzed 26 technical trading rules using ninety years of daily stock prices from Dow Jones up to 1987 and found that they all outperformed the market. Neftci (1991) showed that a few of the rules used in technical analysis generate well-defined techniques of forecasting, but even well-defined rules were shown to be useless in prediction if the economic times series is Gaussian.

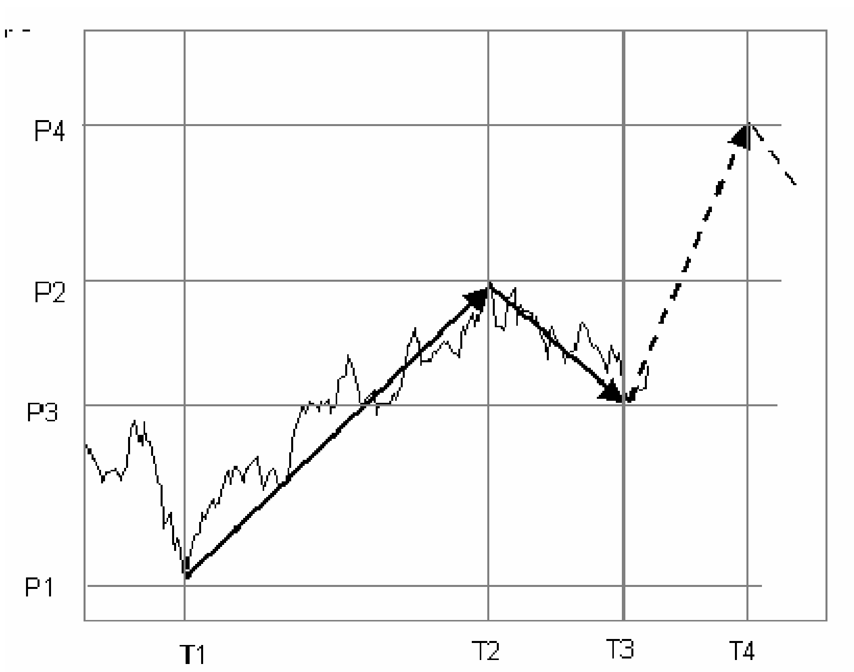
Brown and Jennings (1989) pointed out that technical analysis has value in a model in which prices are not fully revealing and traders have rational conjectures about the relation between prices signals. However, Blume, Easley and O'hara (1994) show that volume provides information quality that cannot be deduced from the price. They show also that traders who used information contained in market statistics do better than traders who do not. It was found by Lo, Mamaysky and Wang (2000) after an examination of the effectiveness of technical analysis on U.S stocks from 1962 to 1996 over the thirty one year-sample period that several technical indicators do provide incremental information and may have some practical value.

Ferdandez-Rodriguez, Gonzalez-Martel and Sosvilla-River (2000) applied a kind of neural network to the Madrid Stock Market and found that, in the absence of trading costs, the technical trading rule is always superior to a buy-and-hold strategy for both "bear" and "stable" market episodes but not in a "bull" market.

Kavajecz and Odders-White (2004) stated that support and resistance levels coincide with peaks in depth on the limit order book and moving average forecasts reveal information about the relative position of depth on the book. Among the different techniques of technical analysis, Batchelor and Kwan (2000) find that support and resistance trend lines, are used much more often than moving average rules and other indicators, in both stock markets and currency markets. In the technical analysis literature, there is more emphasis on technical indicators than on chart patterns and support and resistance levels identification. Nevertheless, Lucey (2005) examined the issue of whether or not there is some psychological barriers in gold prices. He used the standard M-values of the various time series and claimed to have found some evidence to back the existence of psychological barriers. Why the barriers and support and resistance levels are set at those given values instead of others remains an issue.

Now let us pass to the definition of some technical terms to contextualize this study.

Figure 1: Retracements and Projections



Even, when a trend is clearly identified in a market, either an uptrend or downtrend, price will never move in a straight line. There will be many short-term countertrend price movements known as pullbacks and corrections at different extent before the current trend resumes. It is well known in the trading environment that to gain more profits while taking less risk, one the best ways is to enter a position at the end of a correction period. A correction period takes place when a full swing has come to an end. A full swing means a straight move from one significant high to the next significant low for a downswing or a straight move from one significant low to the next significant high in a upswing. The different points chosen and considered as significant highs and significant lows might vary from one person to another but in some cases; there will be uniformity in the selection.

Let us have a look at the following figure. The price has hit a significant low at time T1 and the corresponding price is P1. It then went up in an uptrend move until it reaches a significant high at time T2 and P2 price. P2 represents a kind of ceiling for that move and can be seen as resistance level. The price then experienced a reversal and moved in a downtrend until another significant low is reached at time T3 and price P3. So P3 is like a floor for the price and can be regarded as a support level. Since the support level was not broken, the price started to turn up into another uptrend move or a bull phase. The downfall from (T2, P2) to (T3, P3) is called a retracement of the full swing (T1, P2) to (T2, P2). The following reversal into an uptrend move that is the rise from (T3, P3) to (T4, P4) is known as a projection of the previous bull phase (T1, P1) to (T2, P2). It is clear at a start

These different turning points are the basis for technical trading rule if the support and resistance levels are well defined. The idea behind is to sell when the price neared the resistance level from below but did not break it. If the price approached the support level from above, it would be required to buy if it failed to break it. If traders share the same beliefs about the support and resistance levels, the supply and demand mechanisms will make them hard to break. In 2000, Osler (2000) did a rigorous test of the levels specified by six trading firms during the 1996-1998 period reveals that these signals were quite successful in predicting intraday trend interruptions. He also noted that the technical trading signals provided to customers differ over time and across technical analysts, but the vast majority of the daily technical reports include support and resistance levels.

5.3 Hypotheses and Methods

My overaching hypothesis is that in the African market, prices reverse near the Fibonacci ratios. This hypothesis is based on beliefs and studies conducted earlier on different markets. Osler (2000) finds that exchange rates bounce off the levels quoted by the analysts much more often than from randomly chosen levels. This implies that reversal points take place when prices move near support and resistance levels and that there is a logic behind the choice of those levels. This is in line with the results obtained by Doucouliagos (2003). He found that certain price levels tend to act as psychological barriers and that they are associated with price retracements. Likewise, Osler (2001) shows that technical analysis is useful for predicting short-term exchange rate dynamics. He also finds good market-driven reasons expecting support and resistance levels at round numbers.

Now, let us go back the following figure.

Consider that I have just passed T3 and the price reversed and began to go up above P3. The question is how we determine the next resistance level that coincides with the P4 price level. An effective way of determining support or resistance levels is to have a look at a bar chart and its past price history and then find at what price levels the highs and lows seem to be clustered the most. The longer the window size, the wider the band between support and resistance, and analysts have set a number of possible support and resistance levels according to different window sizes. The support level is a price level which a stock has difficulty falling below. That is where many buyers tend to enter the stock. Similarly, a resistance level represents a price level above which a stock has difficulty climbing. This is where a lot of buyers take profits and sell it. The rationale behind this approach is that the past maxima and minima reflect price levels at which sellers and buyers have caused reversals in price recently. Unless there is a fundamental change in the actors' behavior, they are expected to enter the market again at those points. Major Price tops and bottoms in markets are also major resistance and support levels. Another way to discover support and resistance levels and in this case to get $P4$ is by using the Fibonacci ratios that are the focus of this report. For example, consider that a market is in a solid uptrend, that uptrend began at the 100 price level and prices moved up to 200. But then prices backed off to 150, only to then turn around and continue to rally higher. This would be considered a 50% retracement of the move from 100 to 200:

$$|150 - 200|/|200 - 100|$$

Namely, the 50% retracement level proved to be a solid support level because prices dropped by 50% and then moved back higher. The same holds true for downtrends and corrections to the upside.

To make the parallel with the chart on the figure, we look at the ratio of the size expected rise |P4 - P3|to the size of the previous fall |P3 - P2| is not stochastic, but it is likely to fall close the Fibonacci ratios. Batchelor and Ramyar (2006) included the ratios of durations of subsequent runs, like expecting the |T4 - T3|over |T3 - T2| to be a Fibonacci ratio. In this case, we will stick to prices ratios and leave that part for further research.

It makes sense to use round numbers or past turning points to predict reversal levels, but what it is not explained is why the ratios lie near those values. There is a lack of scientific explanation that the ration of |p4 -P3\right| over |P3 - P2| should be equal to 0.382 instead of 0.238 or 0.618 instead of 0.168. Batchlor and Ramyar (2006) thought a plausible argument can be aesthetic. They argued that the length of a Fibonacci-determined bull run looks right on a chart compared to the preceding bear phase. It is never too high or too low and around that point sellers and buyers agree that it has gone up far enough. This argument far from giving a scientific explanation reinforces the idea that there is a lack of scientific backing for this matter.

Another attempt to justify why the retracement and projections ratios are close to Fibonacci ratios is an empirical one. Looking at the history of the stocks market, the Dow theory claimed to have pinpoint successive cycles in the market. The market movement is divided into three cycle waves. There is a day to day movement known as daily trend, and movement that lasts between one and three months named secondary movement and those that are longer or equal to one year and called primary movement. Hamilton (1922) stated that in a secondary movement, prices usually retrace at 33% to 66% of the primary move and 50% is considered as the typical amount.

Elliot Wave (1938) introduced a wave theory that validates much of the Dow Theory but not vice-versa. Elliot wave theory maintains that prices rise in five waves and then fall in three phases. This means that whenever you look upon a long term wave, you can notice five rising waves and three fallings and the same holds for in each wave taken individually in the trend.

This can be seen in every kind of stock market data. However in a late publication, Elliot theory refocuses the debate, and asserts that waves are not only a function of time but involve both time and prices. According to Batchelor and Ramyar (2006) who was citing Elliot (1940), the ratios of price and time retracements and projections in successive waves were likely to conform to fibonnacci ratios. From all that has been said previously, nothing can prove why the retracement and projection ratios conform to Fibonacci ratios. Since it is not the aim of this paper to show why this happens, we will leave it and move on to the method we will be using in determining the retracement and projection ratios in the data on hand.

5.3 .1 Data

The data of the analysis are daily observations on the BRVM10 which is a West African Regional Stock Exchange index for 119 trading days between September 7 2010 and February 7 2011. I am using closing prices during this period for the BRVM10 index. Dividends are not taken into account in the index, because as in a technical analysis, I am interested in identifying peaks and troughs that might be observed by technical analysts rather than looking at the intrinsic value of the index.

In a time series that do have the same distribution over time, determining the peaks and troughs has been a real challenge for business cycle analysts.

The techniques they developed then were borrowed by traders and applied in the stock market to find bull and bear phases. I need to isolate the noise from the time series, so I will be able to determine peak and trough prices and recognize the dates they happened.

I could not see any generally accepted technical definitions of bull and bear market in the finance literature. However, some scholars have attempted to give some acceptable definitions. Delong (1992) defines those terms in the New Palgrave Dictionary of Money and finance as follows.

- Bull market is a period of time when the prevailing trend of stock prices was sharply and substantially enriched shareholders as a group.
- Bear market is an era when the prevailing trend of stock prices was sharply heading downward and

Likewise, in their paper, Chauvet and Potter (2000) asserted that bull or bear market coincide with a timeframe of generally increasing or decreasing stock prices.

The definition by Lunde and Timmermann is similar but a bit more informative. Theirs can be summarized as follows:

- A bull market is defined as a long-term upward price fluctuation characterized by a series of higher highs interrupted by a series of higher lows.
- As for a bear market, it is a long-term downward trend characterized by lower lows and lower highs.

These definitions are not totally informative and do not provide clear guidelines to operationalize them. They are a bit one way leading to abuses. So, to avoid that, more technical definitions should be used that can be implemented based on causual analysis instead of using this kind of theoretical and hardly quantifiable arguments.

Zumwalt(1979) and Chen (1982) consider definitions of bull markets based simply on return in a given time period exceeding a certain level. This ignores the long-term link between stock prices and returns and doesn't use that information in the computation.

One way to do so is by having a close look at the chart and pinpoints the changing trends. However, in this study, we will be using a more consistent of determining turning points and make sure the corresponding are accurately identified.

The easiest way to locate peaks and troughs and in trend is to have a predetermined threshold. If for example, we pick k% as our threshold, assume we are in bear market, if the lowest price achieved in the recent past took place at t; if the following rise in price from the low is greater than that predetermined then I conclude that a trough has occurred then and there has been a trend reversal. The same applies when we want to identify peaks in a trend.

This is the technique used by Lunde and Timmermann (2002) to get peaks and troughs in their paper. They consider that the stock market moved from a bull to a bear phase if stock prices have experienced a decline by a certain portion from their preceding peak within that state. It is time now to focus on stylized methods used to bull and bear markets. For that matter, I will be using the techniques presented by Harding and Pagan (2001) on turning points of the stock market, modified due the window of data we have on hand.

Before I can calculate the retracement and projection ratios, we should first get the different peaks and troughs in the series. The algorithm used for that purpose is detailed below and is from the paper by Sossounov and Pagan (2000). This one is a modification of the method proposed earlier by Bry and Boschan (1971), that dealt with business, to allow taking into consideration features peculiar to stock prices.

The determination of initial turning points in the series is done by picking peaks and troughs when they occur as the highest or the lowest prices in range of size 8 months on both sides of the date Pagan-Sossounov (2003). To make it appropriate for our data, we will be considering a 4 the highest or lowest values in a four days window on either side of the date to qualify as a turning point either a peak or a trough.

They consider that there is the price p is a peak at time t if the following inequality is verified:

$$p_{t-8,} \dots , p_{t-1} < p_t > p_{t+1}, \dots , p_{t+8}$$

As for a trough, it is considered to have occurred when:

$$p_{t-8}, \dots , p_{t-1} > p_t < p_{t+1}, \dots , p_{t+8}$$

This is the algorithm used by many scholars in different papers to determine turning points. To make it appropriate for this study, I will have to reduce the window to a four days window on both sides of the date. Therefore, our algorithm for detecting peaks and troughs will be:

This is the algorithm used by many scholars in different papers to determine turning points. To make it appropriate for this study, I will have to reduce the window to a four days window on both sides of the date. Therefore, our algorithm for detecting peaks and troughs will be:

- For a peak

$$p_{t-4}, \dots , p_{t-1} < p_t > p_{t+1}, \cdot\cdot p_{t+4}$$

- For a trough

$$p_{t-4}, \dots , p_{t-1} > p_t < p_{t+1}, \dots , p_{t+4}$$

A peak occurs when at time t, the corresponding price is the maximum of the following set:

$$p_t = \max\{ p_{t-4}, \dots , p_{t-1}, p_{t+1}, \dots , p_{t+4}\}$$

A trough will be considered to have occurred when at time t, the corresponding price is the minimum of the following set:

$$p_t = \min\{ p_{t-4}, \dots , p_{t-1}, p_{t+1}, \dots , p_{t+4}\}$$

The choice of the window appropriate size is not clearly given in the papers we rely on. Sossounov and Pegan (2003) chose an eight month window on either side to determine turning points. Bry and Boschan (1971) opted for a six months window for the same purpose.

Given this lack of clarity in the way to choose the suitable window size for the analysis, we think that given the size of our data set, a four days window is appropriate.

5.3.2 Software

I am using Scilab to implement this algorithm. Scilab is a free-software for numerical computations that contains many mathematical functions and has a high level programming language.

I couldn't find more sophisticated software like those used in the reference papers. So I tried to do the programming, and get the turning points. Then, I am going to calculate the retracement and projection ratios.

Once those ratios are obtained, I will test if those ratios and the Fibonacci ratios have statistically the same distribution. For that matter, I will be using R Software. The K.S test that is an R function is an excellent tool for the execution of that task.

After implementing the algorithm, the following tables contain the results. The peaks and troughs obtained are grouped in two different tables.

Date	Peaks of stock prices
11	170.27
18	170.48
25	172.14
40	183.64
58	181.58
92	167.19
99	168.65
105	167.86

Table 5.1: Peaks

Date	Troughs of Stock Prices
20	169.57
31	169.48
50	178.51
61	166.29
68	166.2
75	159
87	162.87
109	162.6

Table 5.2: Troughs

A censoring process should be applied to make sure that any trough is followed by a peak and any peak by a trough. We will have to go through the tables and compare the occurrence dates to pick those that fulfill these conditions.

In this part of the study, we want to perform the KS.test to the distributions of the ratios found in the market to the Fibonacci ratios. The ratios found in market are both retracement and projection ratios. There exist two types of retracements as emphasized previously; a bull retracement and a bear retracement.

- A bull retracement takes place when the price moves from a falling trend to a rising one.
- A bear retracement appears when the price moves from a rising to a falling trend

We can see a retracement is the ratio of one phase to the most recent opposite phase. As for a projection ratio, it represents the ratio of one phase to the most similar phase.

After getting both projection and retracement ratios, I performed the KS and here are the results.

```
ks.test (x, y, alternative = c("two.sided"), exact = 0.001)

        Two-sample Kolmogorov-Smirnov test

data:   x and y

        D = 0.3571, p-value = 0.7818

        alternative hypothesis: two-sided
```

From the KS test, I fail to reject the null hypothesis that Fibonacci ratios can be found in the African market. The test is not significant even at 50%.

5.4 Results Analysis

Now, I turn to test how close are the retracement and projection ratios to the Fibonacci ratios and what are the most frequent Fibonacci ratios that have occurred in the period of study. Technically I will see if the ratios that have occurred are within ε of Fibonacci ratios:

R in [f-ε ; f+ε]

where R is a ratio whether a retracement or a projection ratio and f is a Fibonacci number and ε a kind of error term. I don't expect the ratios found in the market to be exactly the same as the Fibonacci ratios, so I will need to consider those that are not very different.

To define the interval numerically, I choose ε to be equal to 0.025. Therefore, we can express our new interval and test if the ratios obtained are within ε of a Fibonacci ratio. It means if:

R in [f-0.025; f+0.025]

Before I focus on the computation of the retracement and projection ratios, it is useful to have a look at the graph that represents the prices trend during the period of interest.

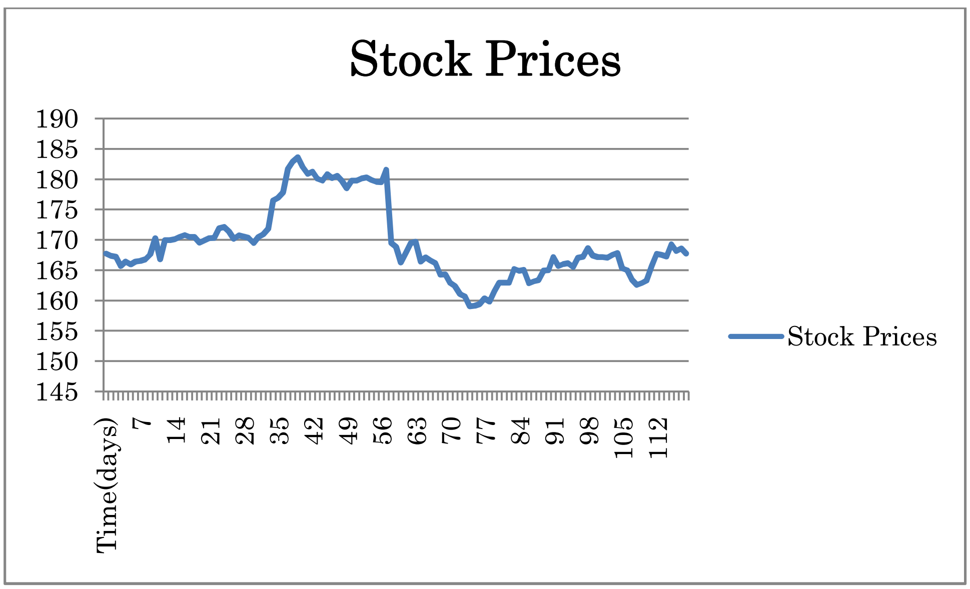


Figure 2: Daily Stock prices

It will be clearer if we graph only the major turning points and the corresponding dates. That is what is shown on the next figure. The results interpretation is based on the figure 3 to make the turning points more visible.

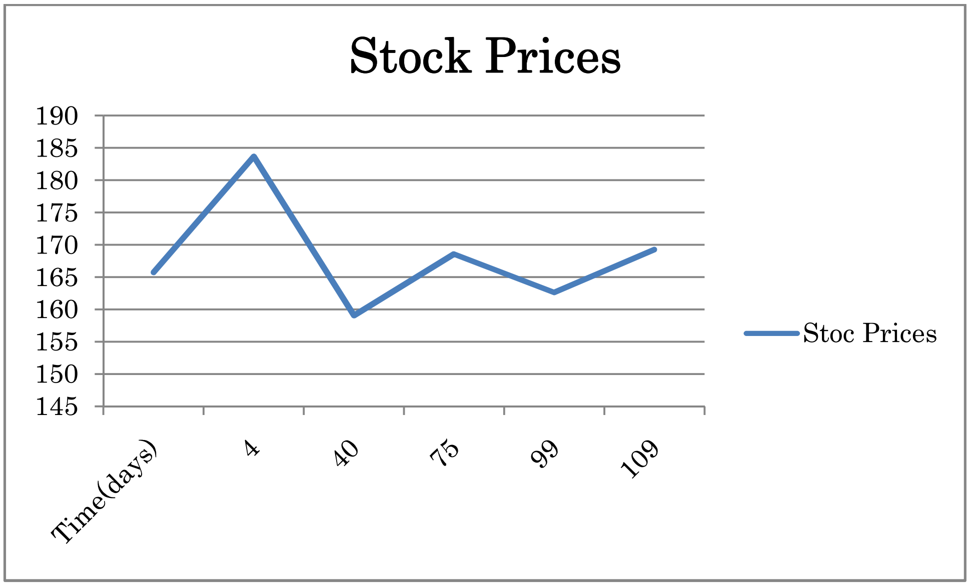


Figure 3 : Major turning points

The first upward trend represents a 17.92 increase from the starting point. It lasted 36 days from the fourth to the fortieth. From the fortieth day, it retraces back until the seventy fifth day. That long decline is a 137.16% retracement from the previous peak.

The retracement ratio is obtained by dividing the length of the decline by that of the previous bull phase. It is given by:

$$|165.72 - 183.64|/|183.64 - 159.06|$$

The 17.92 is obtained by deducting the first trough from the first peak, namely:

$$183.64 - 165.72$$

They are the prices for the fourth and the fortieth days respectively.

My aim is to compare that retracement ratio to the closest Fibonacci ratio to see if they are within 0.025 from each other.

The price experienced a 137.16% retracement from the precedent peak.

These are the Fibonacci ratios sequence we will be looking at:

0.236, 0.382, 0.5, 0.618, 1, 1.382, 1.618

0.5 is not a Fibonacci ratio but it is usually included in the sequence, because it is usually frequently found in technical analysis. So, the Fibonacci ratio which is the closest to the 137.16% retracement ratio is the 1.382 ratio. The difference between the two ratios is 0.0104:

$$1.382 - 1.3716 = 0.0104$$

I can see that the retracement ratio is very close to the corresponding Fibonacci ratio. Our hypothesis is verified for this ratio that the two ratios are within 0.025 from each other.

Now I will look at the projection ratio. After the long decline from the fortieth to the seventy fifth day, the stock experienced a reversal and started to go up again. It went into a bull phase from the seventy fifth to the ninety ninth day which lasted 24 days. The corresponding prices are respectively 159.06 and 168.65.

Therefore, the projection ratio which is calculated as in the previous case is equal to:

$$|159.06 - 168.55|:|183.64 - 168.55| = 0.3860$$

The projection ration will be compared to \$0.382\$ which represents the closest Fibonacci ratio to the one obtained from the previous equation.

If we deduct 0.0.382 from the projection ratio that is 0.382, we get 0.004. Again the difference between the market ratio and the Fibonacci ratio is less than 0.025 and we can conclude that our hypothesis is again verified.

So now we are going to look at the following trend and check if we will get to the same conclusion

As we can see from figure 4, at the ninetieth day, there is a local peak that is followed by a bear phase that has lasted until the hundred and ninth day. The price at that day is 162.6. We are going to apply the same algorithm as in the previous steps to compare the retracement ratio to the closest Fibonacci ratio and check if they are close in the sense given above.

The length of the retracement from the peak realized at the ninety ninth day is 5.95.

It is obtained by just deducting from the precedent that took place at the ninety ninth day the price level at the following trough.

$$168.55 - 162.6 = 5.95$$

The length of that downtrend divided by that of the previous uptrend will be the retracement ratio. That ratio will be compared to its closest Fibonacci ratio to see how far close they are from each other and draw a conclusion.

So the retracement is equal to:

$$5.95 / (168.55 - 159.06) = 0.6269$$

0.618 is the Fibonacci ratio that we are going to compare with the retracement ratio we have got from the preceding equation. The difference between them is:

$$0.6269 - 0.618 = 0.009$$

Here again we can see how close are the two ratios from each other. There is much less than our predetermined error. Therefore, they can be considered close enough and confirm our hypothesis.

After day hundred nine, the stock price experienced another reversal and started to go up again. That bull phase lasted until day hundred and sixteen. The stock prices for day hundred nine and day hundred sixteen are respectively 162.6 and 169.27

So the length of the uptrend is equal to:

$$169.27 - 162.6 = 6.67$$

Having the length of the bull phase, I can get the relative projection ratio. It is obtained by dividing 6.67, the length of the actual upward trend by 5.95 that the length of the previous downtrend.

Now I can calculate the corresponding projection ratio. It is equal to:

$$6.67 / 5.95 = 1.121$$

1 is the Fibonacci ratio that is the closest to that ratio we have found. The difference between them is greater than the error term. It is equal to:

$$1.121 - 1 = 0.12$$

In this last case, the two ratios are not close enough and the hypothesis is rejected. The reversal didn't happen around a Fibonacci ratio.

However, except from this last case; in all others the reversal took place around the Fibonacci ratios. It is possible therefore conclude from the analysis that the statistical test is backed by the results. So our hypothesis that the Fibonacci ratios can be found in the African market cannot be rejected.

However none of the Fibonacci ratios can be considered as the most frequent from the analysis. Even though, the reversal happens around Fibonacci ratios, it could be at any of them without any clear repeating ratios patterns.

The Fibonacci trading rules can be a very useful tool for traders in the African market, but it should applied with care or associated with others technical analysis tools. There is no trading rule that can be applied blindly and assure certain gains.

Further Research

A limitation of this research is the smallness of the range of data I looked upon in our analysis. A four month data can hardly be representative of what is going on in that has been opened many years ago. I think the results would be stronger if we extend the analysis period to at least a couple of years. I plan also to test if there is any logic between the dates at which the reversal occurs. The possible to apply the Fibonacci trading rules when we do not only look at prices but also the period of time between turning points should also be tested.

5.4 Conclusion

The applicability of Fibonacci trading in the stock market has been the center of intense debates for many years. Some scholars have argued to have pinpointed some logic in the turning points patterns in the stock market. The Elliot wave theory is one of the most famous supports of that belief. However Batchlor and Ramyar (2006) in a very critical paper claimed to have enough evidence to dismiss it. So, in this study we tried to check if there is any logic in the turning points patterns in the African market. I looked upon a range of four months data from September 7 2010 to February 7 2011 to from the African regional stock exchange in Abidjan in Cote d' Ivoire to know where stand between the two standpoints. I considered the closing stock prices of the BRVM10 for the analysis.

I first found the different turning points during the study period using the Pagan and Sossounov (2003) algorithm modified a bit to account for the shorter timeframe of our study. We used the mathematical software Scilab to find the different turning points and then calculated the retracement and projection ratios.

The K.S test was then used to test the extent at which the Fibonacci ratios sequence and the different ratios found in the market. The test revealed that the null hypothesis that the two sequences are the same could not be rejected.

Afterwards, I conducted a results analysis that proved that the market prices almost always reversed around Fibonacci ratios. However it was not possible to clearly identify which Fibonacci ratios was the most frequent.

My conclusion from this analysis is that the Fibonacci trading can be a very useful tool for traders in the African stock exchange.

The conclusion is based on a range of data not so long; therefore I plan to extend it in my further research to get stronger results.

6 General Conclusion

Forecasting is one the most important issues facing companies. Improving forecasting will always remain a difficult task because the future will always remain uncertain.

The aim of the paper was to assess the applicability of the Fibonacci trading rules in the African market. It is used to forecast future buying and selling points in the stock market. Those turning points are believed to occur at Fibonacci ratios.

The analysis has proved that the market prices almost always reversed around Fibonacci ratios. However it was not possible to clearly identify which Fibonacci ratios was the most frequent. My conclusion from this analysis is that the Fibonacci trading can be a very useful tool for traders in the African stock exchange.

The conclusion is based on a range of data not so long; therefore I plan to extend it in our further research to get stronger results. In this paper, I also tried to get a forecasting model to improve the objectivity of the loss reserves that need to be put aside to face up future losses. For the sake of simplicity, we proposed to modify the BF method which is of course deterministic to make it more objective. That modified BF method addressed many of the flaws faced by the classical one. The main problem with the classical BF method stems from the non-objectivity in the choice of the loss ratio that needs to be included in the formulae. Therefore, we suggested estimating the loss ratio using a Bayesian model.

As in the classical Bf method managerial judgment can be used in the new one not in terms of guessing or thinking but it will be scientifically backed. It can be used as a prior distribution and tested to effectively fit the assumed the distribution before being used.

I intend to determine in further research, in which cases this method is the most suitable.

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