On the moduli space of marked cubic surfaces from viewpoint of root system of type $3A_2$

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When observing the smooth cubic surfaces together with markings, their moduli space has a natural action of the Weyl group $W(E_6)$ of type E_6. As a $W(E_6)$-equivariant compactification of this moduli space, we have the cross ratio variety constructed by Isao Naruki. In connection with the information of the automorphism groups of cubic surfaces, it is advantageous to observe the action of $W(E_6)$ on this variety.

There are known several models of the moduli on which the action of some maximal subgroup in $W(E_6)$ is naturally visible. For a maximal subgroup of index 45, it is performed from the adjoint torus of type D_4, and for index 36, from the viewpoint of the description of a smooth cubic surface as the blowing up of the projective plane with center six points in a general position. Recently, by using a Cartan subalgebra of type D_5, there appeared a description for a maximal subgroup of index 27. However, the description for index 40 does not appear yet.

In this thesis, we perform a description for this kind subgroup, that is, the main objective is to construct a model of the moduli on which the action of a maximal subgroup of type $3A_2$ is clearly observed. In the first three sections, as preliminaries to our main discussion, we summarize some of the basic and important facts about the marked cubic surfaces. The main discussion begin from Section 4, and we first select, under the viewpoint of $3A_2$, essential 45 cross ratio representatives for Cayley's 270 cross ratios which are used on the construction of the cross ratio variety. We explain relations between them by using a graphical object called lattice-cube. In Section 5 as the last section, by using the special 18 of the 45 cross ratio representatives, we construct one variety as our model. In order to observe the structure of this variety, we further construct three varieties as a kind of determinantal varieties, and describe how the cross ratio varieties is obtained from these three varieties. As a result, we see that our model has only 27 isolated singular points, and by resolving these singular points to non-singular rational curves, we obtain exactly the cross ratio variety.