Divergence-free WKB theory

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Divergence-free WKB theory is a new semiclassical theory, which is firstly presented by the joint research with Satoshi Adachi (Tokyo Institute Technology)[T. Hyouguchi, S. Adachi and M. Ueda, Phys. Rev. Lett, vol 88, p 170404 (2002)].Conventionally, the WKB theory is constructed upon a trajectory that obeys the bare classical dynamics expressed by the Hamilton-Jacobi equation,namely a quadratic equation in momentum space. The WKB theory bridges the two most fundamental realms of theoretical physics, the theories of quantum mechanics and classical mechanics. It allows us to describe quantum-mechanical quantities by using the language of classical mechanics, which has been familiar for us. However, the WKB theory breaks down around the classical turning point;the wavefunction described by the WKB theory diverges there.

Is there any variant of the WKB method that does not exhibit any divergence of the wavefunction and does produce some semiclassical interpretation? This is the main question that the divergence-free WKB theory addresses. This theory is based on a higher-order algebraic equation in momentum space, which represents ¥emph{a dressed classical dynamics}. More precisely, this higher-order algebraic equation is obtained by including quantum corrections to the quadratic equation, which is the bare classical limit. An additional solution of the higher-order algebraic equation enables us to remove the notorious divergence of the wavefunction from the WKB theory. Moreover, our theory is able to produce wavefunctions and eigenenergies more accurate than those given by the traditional WKB theory. This paper presents full formalism of this theory and its several applications concerning wavefunction and eigenenergy to show that our theory is a natural extension of the traditional WKB theory that incorporates nonperturbative quantum corrections.