Uniqueness results and stochastic flows of SDEs with non-Lipschtzian coefficients driven by multi-dimensional symmetric α stable processes

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The aim of the present paper is to investigate some essential properties of solutions of stochastic differential equations driven by symmetric α stable processes (symmetric α stable SDEs), such as pathwise uniqueness conditions, the non- confluent property of paths ,the continuity problem of paths with respect to initial data and also such as the construction of stochastic flows. We focus on the case, where coefficients of equations are non-Lipschitzian, all through our study. The paper consists of two parts.

The Part I is devoted to the uniqueness problem of solutions of stochastic differential equations(SDEs). First of all, Uniqueness results in the case of SDEs driven by Brownian motions (Ito processes) are reviewed. It is remarkable that the best possible condition for solutions of Ito processes is given differently corresponding to the following three cases; the case I where the dimensions, $d = 1$, the case II where, $d = 2$ and the case III where, $d \geqq 3$. In the case of symmetric αstable SDEs, T. Komatsu has obtained a best possible uniqueness condition for the case I ($d = 1$) (1982). Our main result in the part ,given in the section 2, is a common uniqueness condition for the case ,$d \geqq 2$. We also construct an example which shows that the condition is best possible in some sense. Uniqueness results for symmetric α stable SDEs contrast considerably with results in the case of Ito processes. By our discussion given in Section 3, these phenomena could be understood by the difference between the Newton potential corresponding to Brownian motions and the Riesz potential corresponding to symmetric, αstable processes.

The Part II is mainly devoted to construct stochastic flows in the case of symmetric αstable SDEs. In the section 2 of this part, we show the common uniqueness condition for, $d \geqq 2$, obtained in the first part, guarantees the non-confluent property for solutions of SDEs for, $d \geqq 1$. The continuity of paths with respect to initial data is investigated in the section 3. The result obtained here corresponds to that given by Fang and Zhang (2005) in the case of Ito processes.

Combining these results and using Jordan's curve theorem, we construct stochastic flows under non-Lipschitzian coefficients conditions. Many analytic tools concerning Fourier analysis such that Bessel functions, the Riesz potential operator and also hypergeometric functions play important roles in the paper.