主論文要旨

論文題名 Analysis of Wiener- and Poisson- space using representations of Lie algebras

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It is known that there are representations of the Heisenberg algebra on Brownian noises and Poisson noises. In particular, the action of Heisenberg algebra is inherited when we discretize (in time) the framework of Malliavin calculus. We will employ this property in our framework. Although it appears that our framework depends strongly on these nature and thus is restrictive, but it covers several important objects such as the Euler-Maruyama scheme for stochastic differential equations, and by taking the limiting, everything described by Brownian noises and Poisson noises. In this thesis, we give applications of the representations of the Heisenberg algebra. The study is divided into two parts. In Part 1, we study the change of variable formula on the classical Wiener space, which is called the Ramer-Kusuoka formula. We will see that the Ramer-Kusuoka formula can be described as a formula in the ring of formal power series with the coefficients in a (generalized) Heisenberg algebra. The formula would describe not only a Girsanov formula on the Wiener space, but also a Girsanov formula on the Poisson space. In that sense, our formula has to unify both the change of variable formulae on the Wiener and Poisson spaces. Part 2 is devoted to the study of a discrete version of Clark-Ocone formulae. The Clark-Ocone formula is a stochastic version of the fundamental theorem of calculus, which is also an explicit expression of the martingale representation theorem. It is an important problem to ask whether or not a given noise has martingale representation property, that is, whether it has a finite number of martingale basis. The Brownian noises and Poissonian noises have the martingale representation property. However, when we discretize the noises, this property fails. Because we are always in separable Hilbert spaces, we have at most countably many martingale basis, and in fact, our discrete Clark-Ocone formula will use these countable basis. After we establish the discrete Clark-Ocone formula, we will see how the superfluous bases tend to vanish, when we take infinitesimally small partitions of the time interval.