Article

Orbit Analysis of Leading-Following Relations among Multiple Variables[1)]

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Abstract

Orbit analysis is a statistical method of revealing leading-following relations between two variables in x-axis and y-axis; it is conducted by tracing coordinates of the variables in time-series in a scatter diagram, usually used in correlation analysis, and by identifying the direction of rotation of the orbit thereby depicted. The method is applicable to multiple variables and produces a set of their consistent leading-following relations in time-series. An important point is that those are different from temporal preceding-lagging relations. From this viewpoint, it is suggested that so-called Granger causality loses its validity. Orbit analysis is empirically applied to show the relations among short-term interest rates in US, UK, Germany, the Euro area and Japan during the period of 1995-2011, which form one global system of interest rates.

Keywords: Orbit analysis, leading-following relations, preceding-lagging relations, Granger causality, global system of interest rates

Contents

I. Differences between orbit analysis and correlation and regression analyses

Correlation analysis and regression analysis are both statistical methods for revealing linear relations between two variables. Results of their correlation analysis are expressed in a value between -1 and +1 inclusive: the more the value approaches -1, the more negative their correlation is; the more the value approaches +1, the more positive their correlation is; and the value 0 suggests no correlation. Regression analysis distinguishes variables into the independent variable x and the dependent variable y, and estimates parameters a and b in a linear equation $y = ax + b$ with the technique of least squares.

For example, the scatter diagram Fig. 1 shows a correlation coefficient to be 0.6319, suggesting that the two variables are in a positive correlation to that extent. Fig. 2 shows a regression function between x and y to be $y = 0.6498x + 0.2104$ with the coefficient of determination $R^2 = 0.3993$. The "ideal" situation in correlation and regression analyses would be that in which all dotted points are on a straight line; otherwise, points out of the line would be regarded as "errors" which decrease their correlation coefficient and coefficient of determination.

By contrast, orbit analysis proposed here does not recognize all points as a whole at one time, but rather, traces them along time, if they are time-series data, and attempts to extract certain statistical information out of the rotation of an *orbit* that the points linked together depict. It is not a high correlation coefficient or determination coefficient that matters; on the contrary, a low coefficient or even an "abnormal" value would convey an important statistical meaning to observers.

Look at Fig. 3, which is in fact the same as Fig. 1 and Fig. 2, depicting for the period of 1970-2004 US Federal Fund rates on the horizontal axis and Japan's call rates on the vertical axis. They are both short-term interest rates which FRB, the central bank of US, and the Bank of Japan, the Japanese counterpart, use as a measure of exerting their financial policies. As vividly shown in the figure, dots combined along time in the case of time-series data can produce a kind of *orbit*, which more often than not reveals regular rotating movements. We would like to pay a closest attention to the shape and pattern of the movements.

If we carefully observe Fig. 3, we could find an orbit rotating anticlockwise during the periods of 1970-79 and 1982-93. It would be worth asking what it means that the orbit rotated quite regularly for 10 years or more, without fluctuating clockwise or anticlockwise every few years. Anyone who studies international finance could notice the reason behind that FRB's financial policy leads the Bank of Japan's. The fact does not only suggest that the two countries' short-term interest rates *synchronized* with each other at correlation coefficient 0.63: *synchronization* would imply no distinction between the leader and the follower, but imply that they *move together* as if they were two variables in a simultaneous equation model, being determined altogether. The fact also suggests quite strongly that the two variables are in the relations between the leader and the follower.

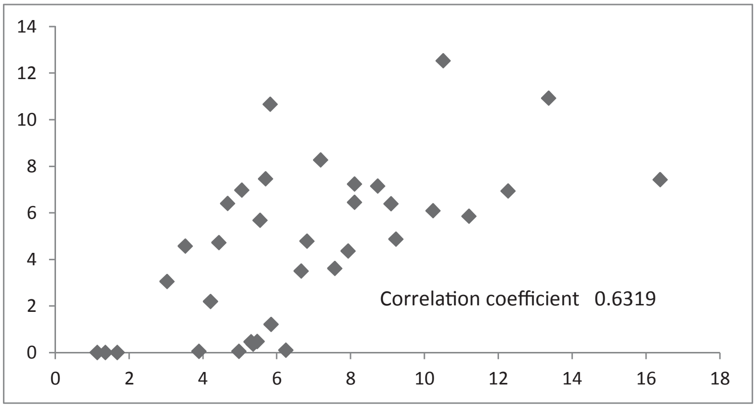

Fig. 1: Correlation analysis

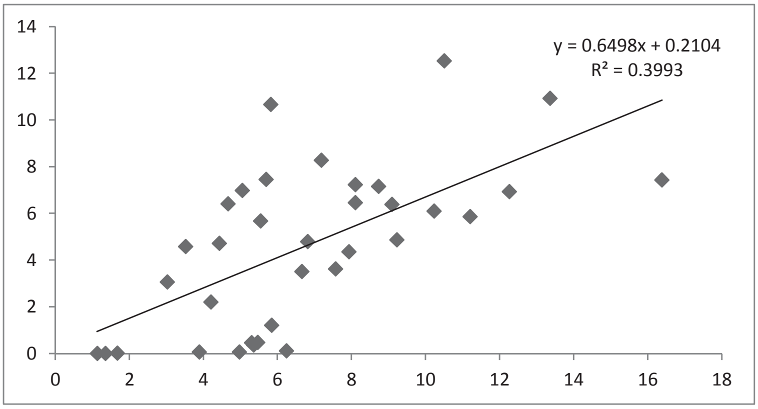

Fig. 2: Regression analysis

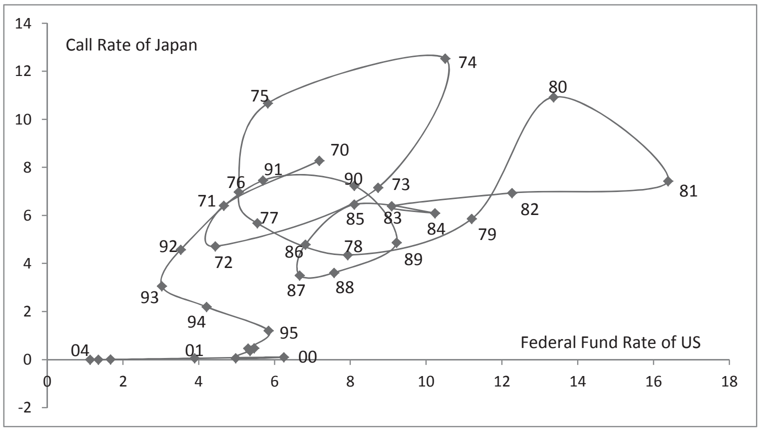

Fig. 3: Orbit analysis

II. Some basic principles of orbit analysis

Let us now clarify some basic principles to follow when we conduct orbit analysis, according to the figure in Supplementary Section 2, Chapter 3 in Itaki (2006) (see Fig. 4.).

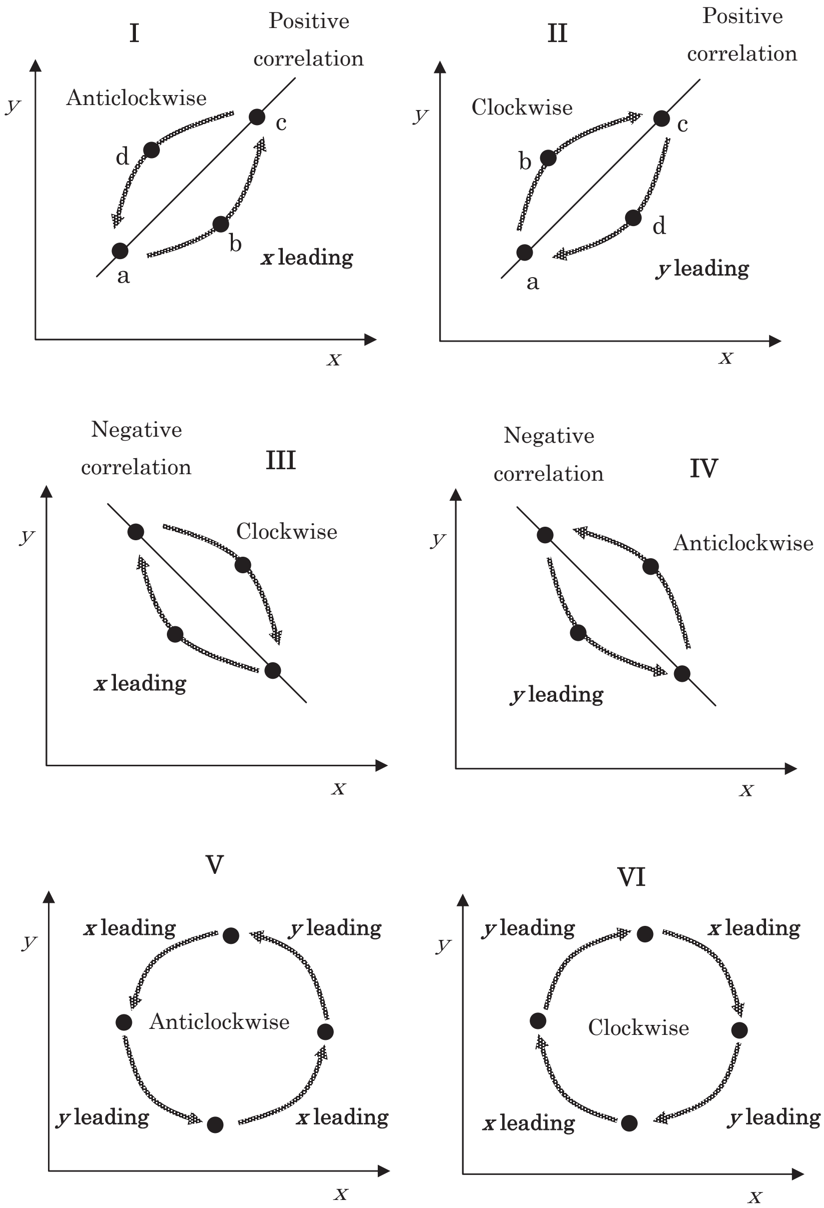

Fig. 4: Six Cases of Orbit Analysis

Trace coordinates of variables x and y for four periods, i.e. a, b, c and d, in the scatter diagram, you acquire the following six basic patterns, which are produced by associating positive correlation, negative correlation and a circular movement with clockwise and anticlockwise rotations:

Case Ⅰ: positive correlation and an anticlockwise rotation (x leading, y following)

Variables x and y exhibit an anticlockwise rotation rather than a straight movement from a to c, in which a does not directly go to c, but rises to c via b. And when c goes down to a, it first goes to d and then to c. We can judge that x leads y and y follows x.

This is because in the upward movement variable x rises first and variable y responds and rises with some time-lag and thus, the locus takes the form of an anticlockwise exponential curve via b. In the downward movement as well, variable x leads and first decreases, being followed by variable y slightly lagging behind and thus, the locus takes the form of an anticlockwise exponential downward curve. Therefore, we can judge that x leads y and y follows x.

Case Ⅱ: positive correlation and a clockwise rotation (y leading, x following)

Case Ⅲ: negative correlation and a clockwise rotation (x leading, y following)

Case Ⅳ: negative correlation and an anticlockwise rotation (y leading, x following)

These three cases can be understood by applying the basic principle of Case I.

Case Ⅴ: an anticlockwise circular movement

Case Ⅵ: a clockwise circular movement

In practice, we may well encounter some cases in which despite strong positive or negative correlation for years, clockwise or anticlockwise circular movements continue to exist for some years and the locus stays around the same position. It would be quite possible to strictly identify leading and following variables year by year as exemplified in Cases V and VI. We could assume that leading and following statuses actually alternate each other in turn, or that instability might prevail for the period concerned between the two variables and thus, no clear judgment could be made regarding leading-following relations. Thorough investigations into theoretical and historical conditions apart from the mere shape of an orbit should be introduced to make the final judgment in these cases[2].

There is another case in which although an orbit does not take a circular shape, it alternates directions of its rotation clockwise and anticlockwise in succession. One possibility is that leading-following relations between x and y actually alternate in succession; another possibility is that the whole bunch of those points shows a clockwise or anticlockwise rotation altogether, if we allow for their possible margin for error. In that case as well, the final judgment should be made on the basis of case by case while fully taking into consideration historical circumstances that surround the variables.

This is all for some basic principles of orbit analysis. In putting the principles into practice, we should be careful enough to choose an appropriate set of variables. They have to be variables that are reasonably expected to have close correlation or causality in theory. Otherwise, we would commit the same error as William S. Jevons (1835-82) did when he insisted that sunspots had a close correlation with business cycles on the earth.

III. An example of an orbit in the trigonometric function

Here we observe in a reverse manner to the previous exposition that two functions with one preceding and another lagging actually manifest themselves in an orbit on a scatter diagram. Fig. 5 depicts two sine curves with a time-lag of 0.2 periods (in radians). In the scatter diagram of Fig. 6, x-axis represents the preceding sine curve in a solid line, and y-axis represents the lagging sine curve in a broken line. They produce a clear anticlockwise orbit as expected. Next in Fig. 7, we increase the time-lag to 0.5 periods (in radians) and depict another scatter diagram with a new set of sine curves. A larger time-lag produces an elliptic-like orbit with a larger minor axis.

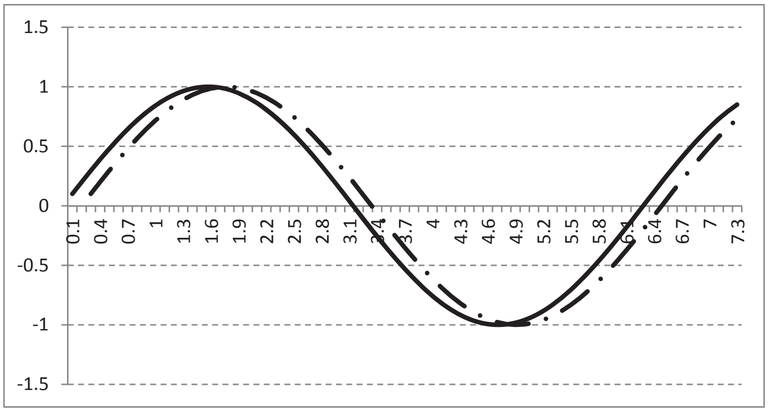

Fig. 5: Sine curves with 0.2-period time-lag

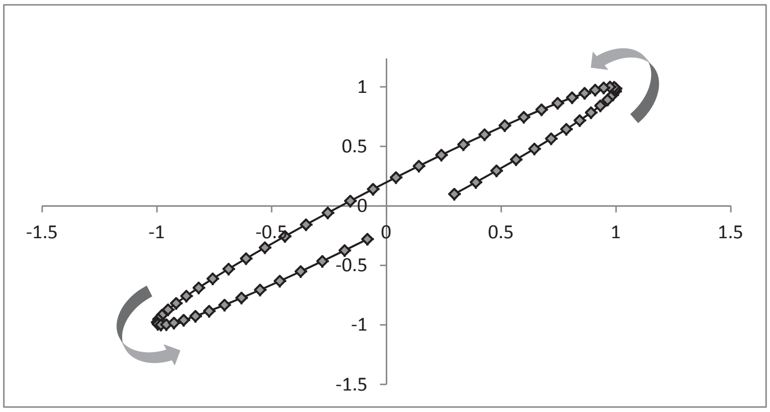

Fig. 6: Orbit of two sine curves with 0.2-period time-lag

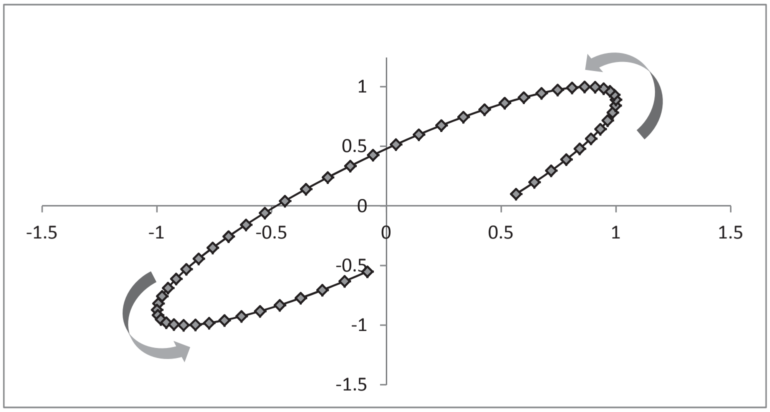

Fig. 7: Orbit of two sine curves with 0.5-period time-lag

IV. Cross correlation analysis and partial correlation analysis

Cross correlation analysis is a type of correlation analysis that takes into consideration preceding-lagging relations with time-lags between a pair of variables in a time-series. Its calculation is carried out by applying positive or negative time-lags (e.g. 1, 2 or 3 periods) to a set of sequences of one variable, calculating correlation coefficients for all time-lags and identifying a specific time-lag with the highest correlation coefficient (see Tanaka (2002) pp.65-67). However, cross correlation analysis has some problems for the sake of specifying detailed and flexible leading-following relations that we are pursuing here. And thus, we will not adopt cross correlation analysis here.

The problems are as follows:

Firstly, selection of time-lags is arbitrary.

Secondly, time-lags have to be positive or negative integral numbers and thus, do not take less than one period.

Thirdly, correlation coefficients are calculated for all the sequences of variables and thus, cross correlation analysis cannot deal with changes in preceding-lagging relations that may take place during the periods.

In order to extract direct correlation between a pair of variables, the effect of a third variable on them should be removed. Partial correlation coefficients are correlation coefficients between residuals of a pair of variables being performed regression analysis on against the third variable (see Tanaka (2002) pp.67-69).

For example, nominal GDP and money stock often reveal a very high correlation coefficient, which is a spurious correlation owing to a long-term upward trend of both variables. The common long-term upward trend is the third variable discussed above. In order to remove its effect, regression analysis is performed between nominal GDP and money stock, and correlation analysis is carried out between the residuals as a result of the regression analysis of the both variables. The coefficient thus produced is their partial correlation coefficient (see Tanaka (2002) pp.53-57).

Because extraction of accurate correlation is an essential premise for orbit analysis, it might be preferable to perform orbit analysis on residuals of variables. However, partial correlation analysis has the following problem for the sake of our analytical purpose here. Removing a common long-term trend such as that in the case of nominal GDP and money stock requires a whole set of sequences of residuals as a result of regression analysis performed over entire periods. But, in many cases of economic variables, a long-term trend may well suffer changes small or big, or sometimes break off. It suggests that partial correlation analysis has to assume *a priori* stability of a long-term trend; in other words, it would be most useful and effective under the theoretical guarantee of stability of a long-term trend.

V. Calculation of the direction of orbit rotation and leading-following relations

Observation of an orbit with one's bare eyes would make it quite possible to find the direction of its rotation and leading-following relations of variables, although it is much easier and more convenient if one uses spreadsheet software. Now we first clarify some basic principles of calculation. We need coordinates of at least three points in the plane in order to determine the direction of orbit rotation; let us assume them to be *a*, *b* and *c* in Fig. 8 for periods 1, 2 and 3 respectively. They are positively correlated with each other and rotate clockwise. Then we calculate the degree of angle *bac* with the help of arctangent function. If the angle is between 0 and π in radians (i.e. between 0 and 180 in degrees), the rotation is anticlockwise; if the angle is between 0 and $-\pi$ in radians (i.e. between 0 and -180 in degrees), the rotation is clockwise.

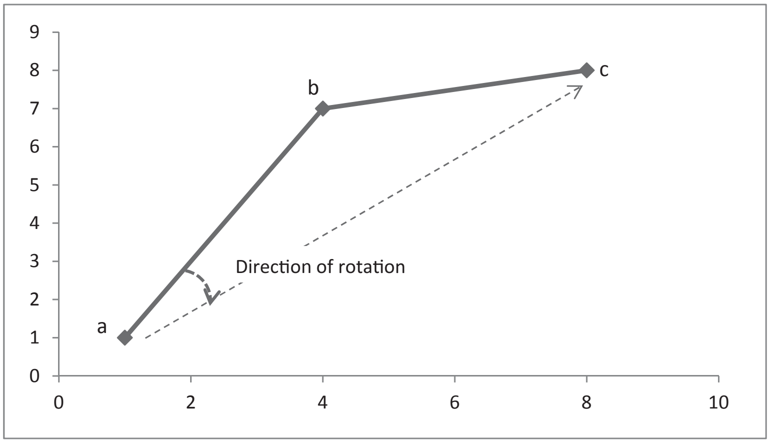

Fig. 8: Calculationof direction of orbit rotation

In practice, the calculation is conducted by parallel transport of triangle *abc* to the position with *a* at the origin; and, subtract the angle between *x*-axis and line segment *ab* from the angle between *x*-axis and line segment *ac*, and we get the degree of angle *bac*. Radians take a positive value less than π and a negative value more than $-\pi$; therefore, after parallel transport towards the origin, we need to distinguish a certain number of cases according to in which quadrant *b* and *c* are positioned, and also according to the result of subtraction whether being more than π or less than $-\pi$. This is why the functions below of "Dissolved functions for rotation" and "Rotation (in radians)" which sums up the former look so complicated.

Then the four cases are classified: firstly, an anticlockwise rotation in positive correlation suggests variable *x* leads; secondly, a clockwise rotation in positive correlation suggests variable *y* leads; thirdly, an anticlockwise rotation in negative correlation suggests variable *y* leads; and fourthly, a clockwise rotation in negative correlation suggests variable *x* leads.

For the sake of convenience of calculation, functions in Microsoft EXCEL are as follows. Table 1 is an example of variables *x* and *y* in the period of 1989-2000.

Slope（D3）=SLOPE(C2:C3,B2:B3)、4D =SLOPE(C3:C4,B3:B4)

Rotation (in radians)（E3）

```
=IF(AND(-PI()<=IF(0<=ATAN2(B4-B2,C4-C2),ATAN2(B4-B2,C4-
C2),2*PI()+ATAN2(B4-B2,C4-C2))-IF(0<=ATAN2(B3-B2,C3-C2),ATAN2(B3-B2,C3-
C2),2*PI()+ATAN2(B3-B2,C3-C2)),IF(0<=ATAN2(B4-B2,C4-C2),ATAN2(B4-B2,C4-
C2),2*PI()+ATAN2(B4-B2,C4-C2))-IF(0<=ATAN2(B3-B2,C3-C2),ATAN2(B3-B2,C3-
C2),2*PI()+ATAN2(B3-B2,C3-C2))<=PI()),IF(0<=ATAN2(B4-B2,C4-C2),ATAN2(B4-
B2,C4-C2),2*PI()+ATAN2(B4-B2,C4-C2))-IF(0<=ATAN2(B3-B2,C3-C2),ATAN2(B3-
B2,C3-C2),2*PI()+ATAN2(B3-B2,C3-C2)),IF(PI()<=IF(0<=ATAN2(B4-B2,C4-
C2),ATAN2(B4-B2,C4-C2),2*PI()+ATAN2(B4-B2,C4-C2))-IF(0<=ATAN2(B3-B2,C3-
C2),ATAN2(B3-B2,C3-C2),2*PI()+ATAN2(B3-B2,C3-C2)),IF(0<=ATAN2(B4-B2,C4-
C2),ATAN2(B4-B2,C4-C2),2*PI()+ATAN2(B4-B2,C4-C2))-IF(0<=ATAN2(B3-B2,C3-
C2),ATAN2(B3-B2,C3-C2),2*PI()+ATAN2(B3-B2,C3-C2))-
2*PI(),2*PI()+IF(0<=ATAN2(B4-B2,C4-C2),ATAN2(B4-B2,C4-C2),2*PI()+ATAN2(B4-
B2,C4-C2))-IF(0<=ATAN2(B3-B2,C3-C2),ATAN2(B3-B2,C3-C2),2*PI()+ATAN2(B3-
B2,C3-C2))))
```

Rotation (in degrees)（F3）=DEGREES(E3)

Leading or following（G3）

```
=IF(AND(0<=D3,0<F3),"X",IF(AND(0<=D3,F3<0),"YY",IF(AND(D3<=0,0<F3),"-YY",IF
(AND(D3<=0,F3<0),"-X","-"))))
```

Dissolved functions for rotation

H3 =B3-B2

H4 =B4-B2

I3 =C3-C2

I4 =C4-C2

J3 =ATAN2(H3,I3)

J4 =ATAN2(H4,I4)

K3 =IF(0<=J3,J3,2*PI()+J3)
K4 =IF(0<=J4,J4,2*PI()+J4)
L3=IF(AND(-PI()<=K4-K3,K4-K3<=PI()),K4-K3,IF(PI()<=K4-K3,K4-K3-2*PI(),2*PI()+K4-K3))

Table 1: Calculation of the direction of orbit rotation and leading-following relations in EXCEL

	A	B	C	D	E	F	G	H	I	J	K	L
1		Variable x	Variable y	Slope	Rotation (in radians)	Rotation (in degrees)	Leading or following	Dissolved functions for rotation				Rotation (in radians)
2	1989	18.63	9.21									
3	1990	20.78	5.75	−1.61	−1.28	−73.45	−X	2.15	−3.46	−1.01	5.27	−1.28
4	1991	12.39	2.18	0.43	−0.81	−46.45	YY	−6.24	−7.03	−2.30	3.99	
5	1992	16.04	7.80	1.54	1.59	90.82	X					
6	1993	4.42	7.19	0.05	−1.90	−108.97	YY					
7	1994	17.23	11.96	0.37	1.96	112.46	X					
8	1995	−1.00	13.02	−0.06	2.78	159.18	−YY					
9	1996	28.65	6.84	−0.21	0.04	2.16	−YY					
10	1997	17.53	9.88	−0.27	0.47	26.98	−YY					
11	1998	6.09	2.18	0.67	−0.45	−25.97	YY					
12	1999	11.83	9.08	1.20	−0.13	−7.41	YY					
13	2000	21.34	16.32	0.76								
14												
15	Notes:	1. Positive radians or degrees signify an anticlockwise rotation, and negative ones signify a clockwise rotation.										
16		2. *X* signifies that variable *x* leads, and *YY* signifies that variable *y* leads, while negative means that slope is negative.										

Observe again Fig. 3 and the result of the calculation in Table 2, and we clearly see the initiative taken by FRB over the Bank of Japan's call rates for the most periods of 1970-2004. However, it is not the whole story; if you look into the details of the orbit, you would find reasons for the exceptional clockwise rotation early in the 1980s and for the extraordinary situation after 1994. The former was an exceptional clockwise rotation that was brought about by the hyper interest rate policy of Chairman P. Volker of FRB that sharply raised Federal Fund rates and abruptly dragged the orbit rightwards. The latter, by contrast, was a consequence of paralyzed short-term financial market of Japan that had drifted towards the zero-interest-rate policy after the collapse of an economic babble. In both cases correlation coefficients and determination coefficients lower as a result, but they are statistically meaningful and accurately represented in the rotation of the orbit. Therefore, orbit analysis can identify temporary reversals in their leading-following relations between two variables in specific years or periods, as well as general tendencies of their leading-following relations.

Table 2: Leading-following relations between US Federal Fund Rate and Japan's Call Rate

	Federal Fund Rate	Call Rate	Slope	Rotation (in radians)	Rotation (in degrees)	Leading or following
1970	7.18	8.28				
1971	4.66	6.41	0.74	0.27	15.74	X
1972	4.43	4.72	7.35	1.89	108.19	X
1973	8.73	7.16	0.57	0.39	22.61	X
1974	10.50	12.54	3.04	1.01	57.87	X
1975	5.82	10.67	0.40	0.42	23.79	X
1976	5.05	6.98	4.79	0.15	8.58	X
1977	5.54	5.68	-2.65	0.47	27.05	-YY
1978	7.93	4.36	-0.55	0.54	30.73	-YY
1979	11.20	5.86	0.46	0.45	25.79	X
1980	13.36	10.93	2.35	-0.87	-50.06	YY
1981	16.38	7.43	-1.16	-0.98	-56.20	-X
1982	12.26	6.94	0.12	0.02	1.34	X
1983	9.09	6.39	0.17	0.22	12.64	X
1984	10.23	6.10	-0.25	-2.96	-169.77	-X
1985	8.10	6.46	-0.17	0.53	30.55	-YY
1986	6.81	4.79	1.29	0.20	11.67	X
1987	6.66	3.51	8.53	0.69	39.69	X
1988	7.57	3.62	0.12	0.37	21.09	X
1989	9.22	4.87	0.76	0.78	44.52	X
1990	8.10	7.24	-2.12	0.50	28.44	-YY
1991	5.69	7.46	-0.09	0.62	35.36	-YY
1992	3.52	4.58	1.33	0.10	5.75	X
1993	3.02	3.06	3.04	0.60	34.15	X
1994	4.20	2.20	-0.73	0.05	2.82	-YY
1995	5.84	1.21	-0.60	-0.46	-26.43	-X
1996	5.30	0.47	1.37	0.15	8.62	X
1997	5.46	0.48	0.06	-1.17	-67.01	YY
1998	5.35	0.37	1.00	-0.08	-4.40	YY
1999	4.97	0.06	0.82	2.17	124.51	X
2000	6.24	0.11	0.04	3.10	177.75	X
2001	3.89	0.06	0.02	0.00	0.03	X
2002	1.67	0.01	0.02	0.00	-0.04	YY
2003	1.13	0.00	0.02	0.01	0.73	X
2004	1.35	0.00				

Source: IMF, *International Financial Statistics.*

VI. Leading-following relations among multiple variables and their hierarchy

Now we apply orbit analysis between two variables to that among multiple variables and construct a hierarchy of leading-following relations among them. Short-term interest rates of US, UK, Germany, the Euro area and Japan during the period of 1994-2012 are used as an example (Table 3). The analytical procedures are as follows:

（1） Ten cases of leading-following relations are calculated among US, UK, Germany, the

Euro area and Japan.

(2) The ranking order of leading-following relations among those 5 countries/ region is determined according to the results of those ten cases.

(3) Ranking points are given to each rank: 4 points to the first, 3 points to the second, 2 points to the third, 1 point to the fourth and 0 point to the fifth.

(4) 5-year moving averages of ranking points are calculated in time-series, figures of which are depicted.

Table 3 : A hierarchy of leading-following relations

	Money market interest rates					X-axis	US	US	US	UK	UK	Germany	US	UK	Euro area	Euro area
	US	UK	Germany	Euro area	Japan	Y-axis	UK	Germany	Japan	Germany	Japan	Japan	Euro area	Euro area	Germany	Japan
1994	4.20	4.88	5.35	6.53	2.20											
1995	5.84	6.08	4.50	6.82	1.21		X	-X	-X	-X	-X	YY	YY	YY	-X	-X
1996	5.30	5.96	3.27	5.09	0.47		YY	X	X	X	X	YY	X	X	YY	YY
1997	5.46	6.61	3.18	4.38	0.48		X	-YY	YY	-YY	YY	-YY	-X	-YY	YY	-YY
1998	5.35	7.21	3.41	3.96	0.37		-YY	-YY	YY	X	-X	-X	YY	-X	-YY	X
1999	4.97	5.20	2.73	2.96	0.06		X	X	X	YY	X	X	X	YY	YY	X
2000	6.24	5.77	4.11	4.39	0.11		X	X	X	X	X	YY	X	X	X	YY
2001	3.89	5.07	4.37	4.26	0.06		X	-YY	YY	-YY	YY	-X	X	X	-YY	YY
2002	1.67	3.89	3.28	3.26	0.01		X	X	YY	X	YY	YY	X	X	YY	YY
2003	1.13	3.59	2.32	2.26	0.00		YY	X	X	X	X	YY	X	X	X	YY
2004	1.35	4.29	2.05	2.05	0.00		YY	-YY	-YY	-YY	-YY	YY	-YY	-YY	X	YY
2005	3.21	4.70	2.09	2.12	0.00		YY	X	X	X	X	X	X	X	X	X
2006	4.96	4.77	2.84	3.01	0.12		X	X	X	YY	YY	X	X	YY	X	X
2007	5.02	5.67	3.86	3.98	0.47		X	X	X	X	X	X	X	X	X	X
2008	1.93	4.68	3.82	3.78	0.46		X	X	X	X	X	YY	X	X	X	X
2009	0.16	0.53	0.63	0.70	0.11		X	X	X	X	X	YY	X	X	X	YY
2010	0.18	0.48	0.38	0.48	0.09		-X	-X	-X	YY	X	X	-X	YY	YY	X
2011	0.10	0.52	0.81	0.82	0.08		-YY	-YY	X	YY	-X	-X	-YY	YY	X	-X
2012	0.14	0.48	0.26	0.06	0.08											

	Ranking points					Five-year moving average of ranking points				
	US	UK	Germany	Euro area	Japan	US	UK	Germany	Euro area	Japan
1994										
1995	3	2	0	4	1	2.7	2.0	1.3	1.7	2.3
1996	3	4	1	0	2	2.0	2.5	1.8	1.8	2.0
1997	2	0	3	1	4	2.4	2.2	2.0	1.8	1.6
1998	0	4	3	2	1	2.6	2.4	2.0	1.2	1.8
1999	4	1	3	2	0	2.4	1.8	2.6	1.2	2.0
2000	4	3	0	1	2	2.6	2.2	2.2	1.0	2.0
2001	2	1	4	0	3	3.2	2.2	1.6	0.8	2.2
2002	3	2	1	0	4	2.4	2.2	1.4	1.0	3.0
2003	3	4	0	1	2	2.2	2.4	1.6	1.2	2.6
2004	0	1	2	3	4	2.6	2.2	1.2	1.8	2.2
2005	3	4	1	2	0	2.8	2.4	1.2	2.2	1.4
2006	4	0	2	3	1	3.0	2.2	1.2	2.4	1.2
2007	4	3	1	2	0	3.8	2.6	0.8	2.0	0.8
2008	4	3	0	2	1	4.0	2.0	1.2	2.0	0.8
2009	4	3	0	1	2	3.4	2.4	1.4	2.2	0.6
2010	4	1	3	2	0	3.3	2.3	1.5	2.3	0.8
2011	1	2	3	4	0	3.0	2.0	2.0	2.3	0.7

Notes: 1. In ranking points, the first receives 4 points, the second 3 points, the third 2 points, the fourth 1 point and the fifth 0 point.

2. In five-year moving average of ranking points, 1995 and 2011 are three-year moving averages and 1996 and 2010 are four-year moving averages.

Source: IMF, *International Financial Statistics*.

The hierarchy thereby calculated shows consistent leading-following relations among variables: e.g. Germany follows UK, UK follows US and thus, Germany definitely follows US. Therefore, the application of orbit analysis allows us to extract a global system of short-term interest rates and to have insights into its mechanism. Although this paper is not expected to comprehensively analyze a global system of short-term interest rates *per se*, let us pick up some interesting features of the hierarchy so as to exemplify fruitful possibilities of orbit analysis.

Fig. 9 makes it clear that over the period of 1995-2011 US Federal Fund rates played the role of the kick-starter of the global interest rates, which is highly remarkable when the IT (i.e. information technology) bubble collapsed in 2000-2001 and again when the subprime loan crisis and the Lehman Brothers shock shook the world in 2007-2008.

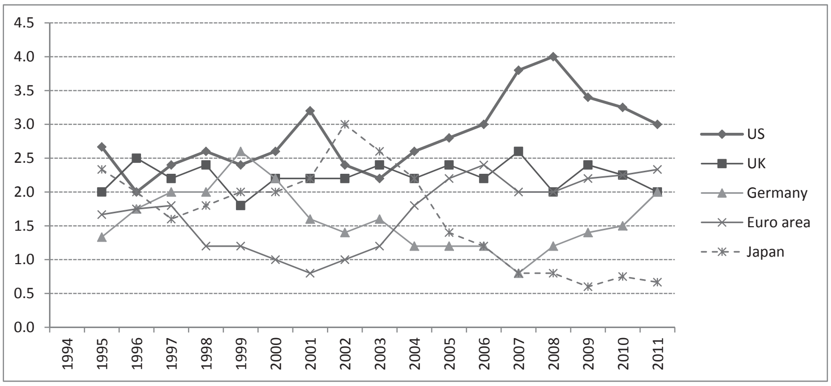

Fig. 9: Five-year moving average of ranking points

As for the former, the waves of interest rate fluctuations that stemmed from US first reached Japan and UK, and spread into the Euro area via Germany. As for the latter, the serious influences of FRB's financial policy that eventually led to the subprime loan crisis first reached UK and the Euro area, followed by Germany. Japan followed furthest behind all of them.

Japan, since its adoption of the zero-interest-rate policy, has fallen from the leader of the global interest rate system to the lowest rank and least responsive economy to its changes. The Euro area, until around 2003, had been least responsive to changes in the global interest rates, but since then has conspicuously raised its rank and now almost synchronizes with UK. The Euro area and UK join Germany, and form the global system of short-term interest rates in the order of US → EU → Japan.

It would be worth asking why economic booms and busts take place almost simultaneously as global phenomena. Elucidating their synchronization mechanism and transfer mechanism requires a thorough investigation into leading-following relations that hide under the global interest rate system. An examination of the hierarchy of short-term interest rates in 1995-2011 would provide a useful clue for that purpose.

However, extraction of global interest rate system is not so easy. The leading-following relations during the period of 1995-2011 were relatively stable. By a sharp contrast, in the 1970s and further back in the 1950s, Japan, the Netherlands and France played the role of the kick-starter for certain periods; e.g. Japan had been the kick-starter for as long as nearly 10 years until the mid-1960s, which would be totally beyond common sense of international economics. Further theoretical and empirical investigations are necessary for a coherent understanding.

Let us pay an attention to the fact that orbit analysis on variables in leading-following relations will certainly detect even a smallest "time-lag" and produce a consistent hierarchy of the variables, but not *vice versa*: a consistent or seemingly consistent hierarchy does not necessarily guarantee the existence of leading-following relations among variables concerned. Table 4 exemplifies this point with preceding-lagging relations among three random numbers, of which Fig.10 is a graph. The correlation coefficients of those random numbers are as low as - 0.298, 0.026 and 0.052 as shown in the table; random number 3, however, precedes random number 2 during six consecutive periods from periods 10 to 15. Furthermore, Fig. 11 reveals quite plausible 5-year moving averages of their ranking points. It should serve as another proof that if we attempt to withdraw some empirical proposition from an analysis of hierarchy, there must be theoretical backing that guarantees leading-following relations among variables. And at the same time, it is necessary to develop a statistical method that enables us to distinguish between "true leading-following relations" and "spurious leading-following relations, i.e. mere preceding-lagging relations".

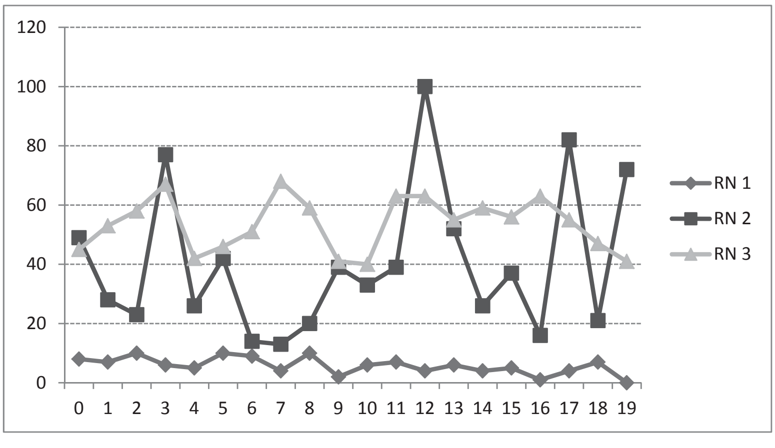

Fig. 10: 3 random numbers

Table 4: “Leading-following relations” among 3 random numbers

	Random numbers (RN)			Individual leading-following relations			Leading-following ranks			Five-year moving average of ranking points		
	RN 1	RN 2	RN 3	RN 1 RN 2	RN 1 RN 3	RN 2 RN 3	1	2	3	RN 1	RN 2	RN 3
0	8	49	45									
1	7	28	53	X	-X	-X	RN 1	RN 1	RN 2	1.0	1.3	0.7
2	10	23	58	-YY	X	-X	RN 2	RN 1	RN 2	1.3	1.3	0.5
3	6	77	67	-YY	-YY	YY	RN 2	RN 3	RN 3	1.2	1.4	0.4
4	5	26	42	X	X	X	RN 1	RN 1	RN 2	0.8	1.6	0.6
5	10	42	46	YY	X	X	RN 2	RN 1	RN 2	0.8	1.6	0.6
6	9	14	51	YY	-YY	-X	RN 2	RN 3	RN 2	1.2	1.4	0.4
7	4	13	68	YY	-X	-X	RN 2	RN 1	RN 2	1.2	1.4	0.4
8	10	20	59	X	-X	-YY	RN 1	RN 1	RN 3	1.0	1.2	0.8
9	2	39	41	-X	X	-X	RN 1	RN 1	RN 2	1.4	0.8	0.8
10	6	33	40	-YY	-YY	YY	RN 2	RN 3	RN 3	1.4	0.6	1.0
11	7	39	63	X	X	YY	RN 1	RN 1	RN 3	1.4	0.6	1.0
12	4	100	63	-YY	X	YY	RN 2	RN 1	RN 3	1.2	0.4	1.4
13	6	52	55	-X	-X	YY	RN 1	RN 1	RN 3	1.6	0.2	1.2
14	4	26	59	X	-YY	-YY	RN 1	RN 3	RN 3	1.2	0.6	1.2
15	5	37	56	X	-X	-YY	RN 1	RN 1	RN 3	1.0	0.8	1.2
16	1	16	63	YY	-YY	-X	RN 2	RN 3	RN 2	1.0	1.0	1.0
17	4	82	55	YY	-	-X	RN 2	RN 3	RN 2	1.0	1.3	0.8
18	7	21	47	-X	-X	X	RN 1	RN 1	RN 2	0.7	1.7	0.7
19	0	72	41									

Correlation coefficients
-0.298 0.026 0.052

Notes 1. In ranking points, the first receives 2 points, the second 1 point and the third 0 point.
2. In five-year moving average of ranking points, period 1 and period 18 are three-year moving averages and period 2 and period 17 are four-year moving averages.

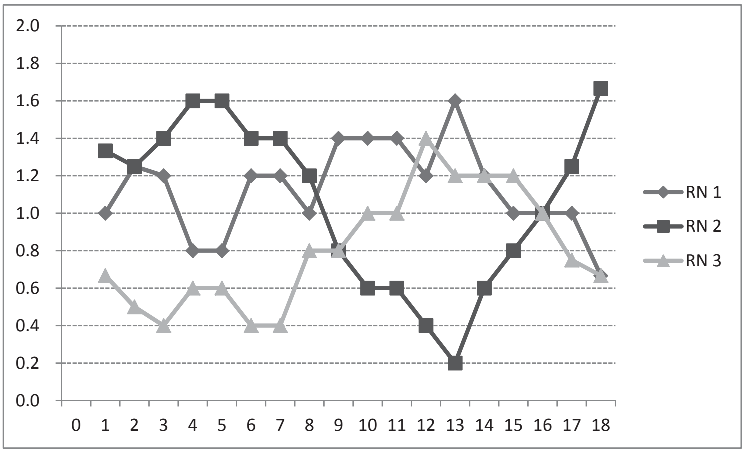

Fig. 11: Five-year moving average of ranking points of 3 random numbers

VII. Conceptual framework of orbit analysis

We have so far examined some features, principles and calculation of orbit analysis. Next, on the basis of these examinations, we will summarize some basic concepts regarding orbit analysis, in which we draw examples from a marathon race in order to make a close linkage between concepts and reality.

(1) Provisional definition of leading-following relations
Two variables in a certain correlation that are observed to be in *temporal preceding-lagging relations* are, for the present, defined to be in leading-following relations when the preceding-lagging relations are caused by one variable's *quantitative traction* of the other. The leading variable and the following variable must have internal necessity and internal/external conditions that enable them to lead or follow the other. The definition above will be replaced by a more accurate one after the following examination.

Let us here use a marathon race as an example of exposition. A graph of a function between time and distance of a bunch of marathon runners would probably show near straight lines, whose correlation coefficients should be almost +1. Another graph of a function between time and velocity among them would also be characterized by a set of very high correlation coefficients. Let us then divide the bunch of runners into pairs, draw a scatter diagram for each pair with velocity in both axes and trace plotted dots along time. We now produce a set of orbits with many complicated rotations in them. The shape of an orbit represents tactics of two runners. They spurt at a certain time in the race and leave others behind, or they catch up with and overtake others. Those tactics evidently manifest themselves in changes in rotation.

For example, a runner in x-axis spurts and another runner in y-axis is left behind, but catches up soon. That is expressed in a rising anticlockwise rotation of an orbit (Fig. 4 case I). Later the runner in y-axis overtakes the runner in x-axis and slows down his/her pace so as to see the consequences. That produces rising and falling clockwise rotations of the orbit (Fig. 4 case II). Observe all cases of the runners and put them in order, and we determine leading-following relations of the runners and their changes along time.

The reason why those marathon runners' movements are leading-following relations rather than preceding-lagging relations is that they have internal necessity of overtaking others, i.e. competition spirit, and they have running capability to make it possible. This case is completely different from another case in which a car happens to run along a *Nozomi* express that is approaching the Tokyo Station, and looks as if it were intentionally catching up with the train.

If we draw a scatter diagram with the velocity of the *Nozomi* in x-axis and the velocity of the car in y-axis, it certainly shows negative correlation of a rather high correlation coefficient. If the *Nozomi* puts on the brakes sooner, the *Nozomi* precedes and the car lags, resulting in a rising clockwise rotation (Fig. 4 case III). If, by contrast, the car steps on the accelerator sooner, a rising anticlockwise rotation appears (Fig. 4 case IV). They are both preceding-lagging relations instead of leading-following relations defined here. This is because firstly, there is no quantitative traction between the *Nozomi* and the car and secondly, there is no internal necessity of leading or following in either variable despite their capability of running along.

(2) Analytical unit of orbit analysis

Analytical unit of orbit analysis is leading-following relations between two variables during a single period. Leading-following relations are judged by the shape of an orbit which two variables x and y draw while they pass three coordinates $a(x_1, y_1)$, $b(x_2, y_2)$ and $c(x_3, y_3)$ over two periods, which are assigned to be the leading-following relations in the first of the two periods. Combine analytical units in time and space, and we distinguish among "individual leading-following relations", "successive leading-following relations", "leading-following relations of a group" and "a hierarchy of leading-following relations". They also signify analytical steps of orbit analysis.

(3) Individual leading-following relations

Individual leading-following relations are those between two variables during a single period, i.e. a minimum analytical unit of orbit analysis, which set the starting point of the whole analysis of leading-following relations. Despite being a minimum analytical unit, or exactly because of being so, should the existence of quantitative tractional relations between two variables, their internal necessity of traction and internal/ external conditions that make the traction possible be elucidated here in the analysis of individual leading-following relations. In other words, the general analysis of variables *per se*, namely the premise of orbit analysis, will be conducted here.

As exemplified in the case of random numbers, preceding-lagging relations are always observable for any variables during certain periods; or to the contrary, true leading-following relations may witness a reversal of their preceding-lagging relations according to certain contingencies that surround the variables and are specific to that single period. Those facts suggest inherent analytical limits that the individual form under strong influence of contingencies allows us to abstract only *potential* leading-following relations between variables concerned.

We assume a unit of period for observing a marathon race to be a very short time during which two runners perform some maneuvers. As already said, a marathon is a good example of true leading-following relations with runners' competition spirit as their internal necessity. In a moment of a single period, however, a runner may accidentally stumble over a pebble or suffer from a sudden stomachache and slow down, which might lead them to a situation very similar to leading-following relations. It is certain that, even in that moment, runners' competition spirit prevails, and that delicate maneuvering as an expression of the spirit takes place; observers of the race have to accurately analyze all of them. However, the leading-following relations thereby examined still remain *potential* in the individual form: only if and when we expand our analytical perspective from a two-runner case to an all-runner case and from a single period to the whole periods, we can thoroughly clarify all accounts of leading-following relations, internal necessity that drives them and internal/ external conditions that make them possible.

Another important point in individual leading-following relations is that action-reaction relations exist between a leading variable and a following variable. Suppose that a runner leads another with a string between them. One is pulling the other with a certain force and then, the leader feels reaction from the follower as if the leader were pulled back by the follower. If the follower obediently follows the leader, the reaction would be a weak one; to the contrary, if the follower somewhat resists, the reaction would be a strong one.

In this case, action-reaction relations are, by means of a string, inseparable dynamic relations; by contrast in the case of a marathon above, we can understand that action-reaction relations are psychological ones. The two runners have run alongside until now and runner *A* begins to lead runner *B* and then, *B* will start to catch up in a short time-lag. Psychological influences, if not visible, will be exerted on *A* in accordance with, for example, the time-lag and velocity of *B*'s catch-up. Maneuvering on the basis of psychological interactions between the two runners will take place from now on.

It is important, therefore, that leading-following relations are not unilateral relations. If the moment that one had pulled the string the other had pulled back more strongly, their leading-following relations would have been reversed. If immediately after *A*'s spurt *B* had accelerated more than *A* did, their leading-following relations would also have been reversed. Note that even after taking these reactions into consideration, *a posteriori* results that one took the initiative of generating a change over the other are statistically reflected on their leading-following relations.

(4) Successive leading-following relations

Successive leading-following relations are those between two variables during multiple periods. An observation on leading-following relations, accidental or not, during just one period will be replaced by successive observations on an orbit that would manifest various regularities and tendencies between the two variables. Internal necessity, being still potential in the individual leading-following relations, and internal/ external conditions that make them possible will gradually manifest themselves, which may take the form of continuum of certain leading-following relations or their continuous/ sporadic changes. This form and stage of analysis, however, does not allow us to grasp general laws of leading-following relations between the two variables concerned.

The tasks of this special form of analysis are observations on steady leading-following relations during certain multiple periods or their continuous/ sporadic changes: for example, observations on the way in which two runners compete each other in the first 10 kilometers after the start, in the first half until the turning point or in the last 5 kilometers that finally decides their competition. Those observations shed light on special tactics that are characteristic to respective periods: for example, one of the runners spurts so as to make the other exhausted; one preserves stamina by means of running behind and avoiding wind; or one stubbornly keeps leading the other.

Firstly, an observation on successive leading-following relations enables us to remove accidental leading-following relations from our analysis: temporary disturbances in relations, such as a slow-down due to stumbling over a pebble or a sudden stomachache, are removed as accidental phenomena when being placed in certain successive periods.

Secondly, concrete contents of internal necessity that drives two runners, i.e. competition spirit, gradually manifest themselves by observing special tactics: for example, marking one's best record, overtaking a rival regardless of a record in order to be qualified for the next Olympic Games, and working out before the coming season. In addition, temperature, weather or injuries in recovery that are among examples of indispensable internal/ external conditions for realizing runners' competition spirit will gradually manifest their meanings to the race as a whole.

Thirdly, having said so, such an observation on successive leading-following relations does not allow us to understand the ultimate strategy that runners try to realize during the entire race. The general, comprehensive strategy is a synthesis of all the special tactics and/but what transcends their mere total: a sudden spurt on that slope and running intentionally behind for avoiding that strong wind would be all meaningful only by placing them properly in the context of comprehensive strategy. Furthermore, they are only two runners in the whole bunch of fiercely competing runners and thus, have to be positioned in all the runners' highly complicated maneuvering.

Successive leading-following relations overcome analytical limits inherent in individual leading-following relations and further seek the next "leading-following relations of a group" and "a hierarchy of leading-following relations".

(5) Leading-following relations of a group

Leading-following relations of a group are those in a single period among a number of variables in a specific group. The task assigned to this special form of analysis is to perform orbit analysis on all the combinations of variables in a group and to acquire a series of consistent leading-following relations. The analysis reveals that leading-following relations are consistently established among all the variables of a group, rather than between a specific pair of variables: in other words, the totality of leading-following relations. It reveals, therefore, that internal necessity and internal/ external conditions that make them possible hold true not only to a pair of variables picked up by chance, but also to all the variables of a group. This special form of analysis, though, has its own inherent limits that the totality thereby abstracted may accidentally appear owing to the singularity of period, and thus that it does not allow us to appreciate the continuity and/or continuous changes of leading-following relations.

As a marathon race proceeds, runners are likely to be divided into a number of groups. Suppose that, in the second half of the race, the top group and other groups are clearly separated away from one another, and that the lagging groups have lost will and possibility of catching up with the top group again. Even in this case, however, if we choose one runner out of the top group and another runner out of a lagging group, and perform correlation analysis between the two runners on distance and time and on velocity and time, we would get high correlation coefficients in either case. This is because the runner of a lagging group has not completely given up the race and hence, is still in moderate leading-following relations with the top-group runner. It is undeniable, though, that runners' maneuvering is virtually concentrated within the top group and within each of the lagging groups. Those double relations should be reflected on the shape of the orbit. The velocities of the two runners probably depict a strange orbit: a spurt may be quickly followed by another spurt, or a spurt of one runner may be followed by a slow-down of another. In a word, unlikely marathon tactics are found on the orbit. In fact, their correlation coefficients are expected to be lower. If we choose, for comparison, two runners from the same group, they would produce higher correlation coefficients and their maneuvering should be coherent with marathon tactics. Those two examples suggest the need of taking into account the possibility of difference in internal necessity and internal/ external conditions among groups, and the need of carefully observing correlation coefficients and the shapes of orbits in order to distinguish discrete groups and their separation from one another.

Here we examine the resultant force effect that is characteristic to leading-following relations of a group. It is an effect in which traction from a leader to a follower is multiplied step by step as follows: the first leading variable transfers traction to the first following variable; it turns into the second leading variable; it transfers traction to the second following variable; and so on and so forth. Multiplication occurs because the first leading variable transfers its traction not only to the first following variable, but also to the second, third, etc. following variables at the same time and thus, when the first following variable turns into the second leading variable and transfers its own traction to the second following variable, a set of traction of the first and second leading variables jointly works as a resultant force.

Suppose that a specific runner in a group now attempts a spurt. Another runner who responds most quickly and begins to catch up is the first follower. The successive spurts of those two runners should strongly affect other runners as psychological traction: the second follower and the third follower appear in turn; the group would expand forwards and backwards a little bit; and they begin spurts altogether in a bunch and enter the second period.

The same effect can happen in a system of global interest rates. A rise in a specific country's interest rate works as a pressure to raise those in countries in close economic ties with the country. Unless they follow, they may suffer from import inflation owing to a depreciation of their foreign exchange rates or they may undergo a loss of international currency reserves owing to an outflow of capital. If, under those circumstances, countries begin to follow one after another, the pressure exponentially strengthens and then, it will be almost unavoidable to follow the wave of interest rate hikes.

As suggested in the examples above, the first runner who attempted a spurt or the first country that raised an interest rate may have become the first leader thanks to various individual conditions, and the first traction may have been a very weak one. And the first follower and the second follower may have taken those positions just by chance. However, regardless of who took which position, as ranks went down, the reaction of following *per se* should have turned to be unavoidable and many runners and countries followed like an avalanche. In other words, *a resultant force* of contingencies transformed the action of following *per se* into a necessity.

> Of course, there must be some runners and countries in a group which do not follow. They may have accidentally belonged to the group in pursuit of improving their own personal records or they may have preferred a depreciation of their currencies because their economies were in a dire recession. Necessity of following does not exclude individual contingencies such as those. Necessity penetrates as *a general trend* through some exceptions and contingencies against the general trend. We may say that a resultant force is an intermediate that synthesizes many contingencies with regard to ranks, timing, strength and others, and transforms them into a necessity of a set of consistent leading-following relations: it is a necessity that others must follow the leader once their relations start, regardless of which variable leads which, when it leads or how strongly it leads, etc.

Next we examine the resultant force effect of reactions. We have already defined a reaction to be reverse traction of a following variable against traction of a leading variable (i.e. an action). Extended to leading-following relations of a group, reactions are a set of traction exerted to the first leading variable by the first, second, third, etc. following variables respectively with a certain time-lag. We understand it to be a resultant force of reactions in the sense that all the following variables respond and send back their reactions. Although a resultant force effect of actions work at the same time, that of reactions is exerted one by one with time-lag. We should pay enough attention to the fact that, depending on the working of the effect, the very existence of leading-following relations of a group may well be denied altogether.

> A spurt of a specific runner during a specific period quickly spreads over and influences other runners as a resultant force effect; at the same time, reactions from all the other runners are sent back one by one to the first one as a resultant force effect. The preceding runner cannot predict in advance either the pattern of reactions or their strength: i.e. whether others react to the spurt with their own spurts or just ignore it, whether or not others accelerate more than the preceding runner did, and so on. Note that those reactions occur within one period of observation and analysis and thus, they are neither observable nor objects of analysis. Therefore, it would be quite possible that, depending on patterns, strength and timing of reactions, leading-following relations might be reversed.
>
> > It is certain that runner A first spurted at the start of the period, but runner D may have counter-spurted so fiercely that, at the end of the period, D and A may well be observed to be the leader and the follower respectively. We observe during one period as the minimum unit and compare runners' situations at the start of the period with those at the end, disregarding what happens within the period. Such reversals between A and D could take place to any runners in a group. Who becomes the first leader, the first follower and the rest is determined by chance, but also once a race starts leading-following relations can be reversed and reversed

again by various contingencies as time goes on and thus, the results remain undetermined until the last moment.

Under the situation in which only a resultant force effect of actions works, *a resultant force effect of contingencies* transforms leading-following relations *per se* to a necessity: i.e. with the effect strengthening step by step, following in a specific direction would turn out unavoidable. However, with a resultant force effect of reactions taken into account, the very existence of leading-following relations should remain accidental: one does not have to follow the preceding one; on the contrary, one can force it to be its follower at the end of the period.

Norbert Wiener's concepts of positive feedback and negative feedback in cybernetics (Wiener (1948, 1961) (1954)) would be a useful conceptual framework in order to take into account the possibility of such reversals. Reactions that strengthen the leadership of the first leading variable are positive feedbacks and those that weaken the leadership are negative feedbacks. However, when the leadership of the first leading variable is reversed and another first leading variable appears, the positive feedbacks so far are also reversed to negative feedbacks. Therefore, clear distinction between positive and negative feedbacks remains undetermined until the end of the period. It suggests that those concepts are useful as long as they show instantaneous directions of reactions.

We lastly examine actual leading-following relations in which resultant force effects of actions and reactions are both working. On the one hand, cumulative forces work for constructing a set of consistent leading-following relations owing to resultant force effects of actions; on the other hand, wavelike forces work for possibly destroying leading-following relations so far constructed owing to resultant force effects of reactions. The one is the force that builds up necessity out of contingencies and the other is the force that dissolves the necessity again into contingencies. They are in a conflict in which all the variables are competing for higher ranks in leading-following relations. How can we understand those situations?

No actions or reactions within a single period are observable or objects of analysis: they are, as it were, in a black box in which we compare coordinates at the start of a period before actions and reactions have not yet been exerted with coordinates at the end of the period after all actions and reactions have been exerted, and we calculate consistent leading-following relations of all variables in a group. Hence, *a posteriori* at the end of a period, all actions and reactions, positive feedbacks and negative feedbacks, are synthesized together and finally determine a set of leading-following relations. We understand, in other words, that they are *a posteriori* determined relations after incorporating all internal/ external contingencies about and interactions among all variables of a group. This is how necessity of leading-following relations that penetrates by means of the resultant force effect of actions is synthesized with contingencies that affect all variables of a group.

If an observer uncritically accepts results of calculation at the end of a period, he/she presumes in a dichotomous manner that *A* first moved by chance of internal/ external conditions, *B* followed second by chance, *C* followed third also by chance, etc., although, as ranks went down along *A*, *B*, *C*, etc., force of necessity worked to produce consistent leading-following relations. However, incorporation of interactions between actions and reactions into the logic enables the observer to have a completely different understanding. Namely, it does not matter at all whether *A* preceded in time or *B* lagged in time. Beyond temporal preceding-lagging relations, all variables of a group deployed complicated relations of quantitative traction with one another: sometimes previous temporal relations had been reversed and were reversed again, and finally at the end of a period all results manifested themselves as if they could be likened to consistent temporal preceding-lagging relations. Therefore, we should not presume that *A* became the first leading variable by chance, *B* became the first following variable by chance, or *C* became the second following variable by chance. But rather, given internal/ external accidental conditions exerted on *A*, *B*, *C*, etc., *A* ought to have been the first leading variable, *B* ought to have been the first following variable, *C* also ought to have been the second following variable, etc. as a result of their interactions.

> In our example of a marathon race, the situation above can be concretely exemplified as follows. Suppose that runner *A* spurts at a climax of the race after being fully prepared for the opportunity. Although *A* does not have to be the top of the group, his sudden acceleration is conspicuous to other runners. *B*, *C*, *D*, etc. begin to follow *A* in succession with a certain time-lag respectively. They do not passively follow in response to *A*'s spurt; on the contrary, they have eagerly waited for this very moment. *D* in particular, inferior to others, has kept his stamina until this moment under his unfavorable conditions of rain and low temperature; *D* almost overtakes not only *A* but also *B* and *C* who look surprised by *A*'s spurt.
>
> Above is a detailed account of what happened in the period. Although *A* spurted first, *D* was recorded as the first leader at the end of the period. A commentator would explain, "*D* is evidently inferior in power, and there is no other opportunity but now to make a spurt with guts." However, in fact, that was not the whole story of the race. *D* could not survive the rest of the race and *C*, who pretended to be totally surprised by *A*'s spurt and passively followed *A* and *D*, finally finished first in the group. All in all in the race, *C* is the first leader. As long as the specific period was concerned, *A* may have spurted just by chance. By contrast, *D* ought to have been the first leader and *C* ought to have been its follower. And as long as the whole race was concerned, *C* with a good reputation under his/her favorable conditions of rain and low temperature ought to have got the group's first place in the race.
>
> The marathon example above clearly suggests that there are three types and/ or levels of necessity: first, necessity of a group as a whole that once a spurt begins everyone has to follow; second, necessity of *D* who has to overtake others now: and third, necessity of *C* who finally wins the race on the basis of his/her running capability. Those three types and/or levels of necessity penetrate numerous contingencies and realize themselves, although the third one has to wait the next logical step, i.e. a hierarchy of leading-following relations, to be elucidated.

The distinction between those two modes of inference is not for logical rigidity. When we face actual data and need to interpret the meaning of a set of leading-following relations, it appears as a critical distinction. We often witness in orbit analysis that, as in the example above, an inferior runner leads a group of runners for some time or even for a long time. We also have analytical results that short-term interest rates of Japan during the 1950s to 1960s, when Japan could not be called an advanced economy in terms of its scale, led the global short-term interest rates for almost 10 years. How can we properly interpret those cases?

We would infer that, according to the former mode, a change in velocity that was accidentally caused by the runner transferred to others in an accidental order and finally as a result, a certain change in velocity of the group as a whole took place. According to the latter mode of inference, by contrast, the runner or Japan did not necessarily *preceded* in a temporal sense other runners or countries; however, complicated interactions in the period ought to have made the runner be the leader despite his/her lack of running capability and Japan be the leader of the global interest rates despite the small scale of its economy. According to the former, the leading-following relations in the period might be dealt with as mere *errors*; according to the latter, we have to answer the questions on why an inferior runner could and should lead others specifically in that period, and why Japan could and should lead the others despite much less development of its financial market under the fixed foreign exchange rate system at that time. They are many times as difficult questions as those if we regard them just as *errors*, which would stand out and demand answers only when we adopt the latter standpoint.

In leading-following relations of a group that we have so far discussed, however, the question on why variables *ought to* lead or follow cannot be given a decisive answer. The totality of leading-following relations has been elucidated: i.e. they do not happen accidentally between specific two variables, but do happen consistently throughout all variables. They still leave the possibility of the totality being accidental owing to the singularity of period, and have the limits that continuity or continuous changes over time cannot be properly analyzed. Therefore, the pursuit of necessity would next seek the analysis of a hierarchy of leading-following relations.

(6) A hierarchy of leading-following relations

A hierarchy of leading-following relations is a system of those among all variables during all periods, which contains a number of groups of leading-following relations. In a hierarchy, ranks of all variables show a series of continuity or continuous changes, which, for the first time, completely reveal internal necessity and internal/ external conditions of leading-following relations and thus, make it possible to grasp the general law of the relations. In practice, there are three steps of analysis: a hierarchy in a single period, a hierarchy during a number of periods and a hierarchy during all periods.

The top variable of a hierarchy is called the kick-starter: it possesses outstanding internal necessity of leading and internal/ external conditions that make its leading possible. As already discussed in details, a kick-starter acquires its status not because of its temporal *preceding*; a specific variable takes the status of a kick-starter during a specific period or periods because of comprehensive resultant force effects of actions and reactions among all variables and also because of those between positive feedbacks and negative feedbacks. A large absolute value or rate of change of a variable does not guarantee the status of a kick-starter. An external contingency of a special kind does not pick up a certain variable as a kick-starter, either. Given all external conditions, complicated interactions of internal traction among variables will abstract a specific variable as a kick-starter. And the kick-starter leads entire changes of all variables, which empirically and statistically manifests themselves as temporal *preceding-lagging relations*.

The first following variable is that which receives the change of the kick-starter, strengthens it, pulls all the other variables and promotes their changes. It is not in the position, in which most quickly to follow the kick-starter, but rather in which to encourage changes of all variables but the kick-starter, although empirically and statistically recorded as "second in rank" in terms of time. A set of leading-following relations is formed along the time line after the first following variable, including a number of bunched groups. Upper ranks would be occupied by variables that produce quantitative changes more autonomously, and lower ranks by those whose changes are more passively determined by upper-ranked variables.

Determination of all ranks over all periods makes it possible to abstract necessity on a higher level regarding leading-following relations. A hierarchy in a single period represents all variables' chain relations of "ought to be so" in space, while hierarchies over all periods represent their chain relations of "ought to be shifting so" in time.

Our observation and analysis is divided into individual periods, which are then combined again for the analysis of certain periods or the entire periods. It is certainly a proper analytical method, but may mislead us to believe that leading-following relations are determined independently period by period. In fact, however, the actions and reactions in period t are continuously connected with those in period t+1: the actions and reactions in period t abstract a kick-starter and determine other ranks in period t; at the same time they are continuously involved in the determination of a kick-starter and other ranks in period t+1. Leading-following relations during the entire periods are in inseparable chain relations in space and time. Given internal/ external contingencies regarding all variables, the whole set of leading-following relations appears in front of an observer and an analyst as "ought to be so" and "ought to be shifting so".

In practice, leading-following relations, which are precisely calculated each period, are likely to produce rather unstable ranking, as seen in "ranking points" of Table 3 for a hierarchy of short-term interest rates. The countries and region undergo frequent ups and downs just like a roller coaster from the kick-starter to the bottom of the ranking. A country's short-term interest rates reflect literally all the economic factors of the country, from its real economy to stock market, and thus, are exposed to many internal/ external contingencies outside their global interactions. It is certainly a difficult business, therefore, to extract fundamental necessity of dynamic system of global interest rates. Here in this paper, the simplest method of all, i.e. five-year moving average, is adopted to extract a stable long-term trend.

It seems, on the other hand, assuming *a priori* validity of the empirical results, big annual fluctuations themselves raise quite interesting questions: in particular, huge changes in leading-following relations around 2000 when the IT babble collapsed and around 2008 when the Lehman Brothers Shock occurred could be a research theme of great importance. An application of orbit analysis requires a pursuit of both a long-term trend and short-term irregularities.

Now then, what is necessity on a higher level that is abstracted from the analysis of hierarchies? All variables in hierarchies during the entire periods are in inseparable chain relations in space and time, in which a chain in space in period t (i.e. the kick-starter in period t → following variables in period t) is linked with a chain in space in period t+1 (i.e. the kick-starter in period t+1 → following variables in period t+1), etc. and they all produce a chain in time, i.e. the kick-starter in period t → following variables in period t → the kick-starter in period t+1 → following variables in period t+1 → the kick-starter in period t+2, etc. In such a flow in space and time, unilaterality in leading-following relations is replaced by its bilaterality. A kick-starter is certainly followed by other following variables, which then, as a bunch, lead the next kick-starter. It is a continuous bilaterality in which a leader follows its followers and followers lead their leader. A kick-starter actively initiates a change and other variables passively follow, to which responding passively the next kick-starter initiates a new change. This is what we call necessity on a higher level.

Leadership and followership turn out to be relative in dynamically transforming hierarchies. We may express it as mutual penetration between opposing moments: leading is also following and following is also leading; no leader could only lead or no follower could only follow. Although being relative, a leader is a leader and a follower is a follower. While the face of a leader may well shift, a leader does actively initiate a change and, passively responding to its followers' changes, does actively initiate a new change again.

Let us examine the point at issue with the same marathon example. Suppose, as before, that runner A spurts at the beginning of period t and runners B, C and D follow in succession with a certain time-lag. They do not passively follow in response to A's acceleration; D in particular counter-spurts in full blast and almost overtakes even A as well as B and C. At the end of period t, D is recorded as the kick-starter, followed by A, B and C regardless of their order. Note that the leading-following relations between the kick-starter and others are not necessarily the same as their actual ranks in period t.

Maneuvering, however, does not finish yet. Suppose that, from around the end of period t to the beginning of period t+1, D further accelerates as a reaction to fierce catching up of A, B and C. As a result, D is still recorded as the kick-starter at the end of period t+1 and A, B, and C as its followers. Then, suppose that in period t+2 C begins to spurt, noticing D's exhaustion after two consecutive spurts, overtakes others and finishes the race in the first place. It is needless to say that C is the kick-starter of period t+2, passively followed by A, B and D. The final ranks are determined accordingly.

We can sum up the chain relations in space and time as follows: the preceding variable in period t (A) → the kick-starter in period t (D) → the following variables in period t (A, B and C) → the kick-starter in period t+1 (D) → the following variables in period t+1 (A, B and C) → the kick-starter in period t+2 (C) → the following variables in period t+2 (A, B and D).

Next we explain other concepts than the kick-starter with respect to a hierarchy of leading-following relations. *Mutual leading-following relations* are those in which a number of variables go up and down around similar ranks and alternate leading and following for certain periods. Those variables intertwine one another, frequently changing their ranks; and thus, they mutually exert negative feedbacks as reactions to one another. They negate actions exerted by temporally preceding variables and counteract in a view to acquiring a leading status. By contrast, they share the same stable leading-following relations to other variables. It is suggested, therefore, that variables in mutual leading-following relations are in constant conflicts between actively initiating changes and making others follow on the one hand, and passively following others' active changes on the other hand. We can understand that the substances represented by those variables have a similar structure in terms of response to changes. A situation in which a number of runners, rivaling each other in terms of running capability and competition spirit, are running together in a bunch and frequently alternating their ranks would serve as a good example.

There is another kind of relations in which some variables are far apart and keep stable in ranks for certain periods. An often observed example is those between the kick-starter and the variable at the bottom, which keep stable at the both ends of a spectrum though, in the middle, other variables are in mutual leading-following relations that are unstable. Those variables near or at the bottom often form such stable relations, which do not exert negative feedbacks to changes generated by the kick-starter and rather, passively respond to movements of the hierarchy as a whole.

A turning point is a period or periods in which concentrated reversals of leading-following relations among a large number of variables take place. A certain kind of structural transformation is expected to occur in a part of a hierarchy or an entire hierarchy. *A stable period* is one in which an entire hierarchy of leading-following relations is stable and few reversals in ranks take place, while *an unstable period* is one in which a number of variables undergo reversals for certain periods but a large number of reversals do not take place. Combine turning points, stable periods and unstable periods, we can make a basic chronology of the whole periods.

Lastly, let us refer to resultant force effects in a hierarchy as a whole. As discussed in details in leading-following relations of a group, an absolute amount of the starting power of a kick-starter is usually not very large. If, however, resultant force effects of positive feedbacks gradually strengthen them more than being weakened by negative feedbacks, so-called "The tail wags the dog" phenomenon may be brought about. A tiny amount of starting power at first may turn into a kind of shock wave at last, which is transferred beyond space and time[3]. Furthermore, such a shock wave does not work unilaterally, but rather, being reflected just like a boomerang, repeatedly expands and shrinks, and then encompass the entire hierarchy.

The determination mechanism of national income serves as a good example for our purpose. Components of national income Y, i.e. private consumption C, investment I, government expenditure G and trade balance $X - M$, form leading-following relations: either ΔC, ΔI, ΔG or $\Delta(X-M)$ plays the role of a kick-starter that leads others, and they altogether cause a change in national income ΔY. A tiny change at first, repeating leading and following for certain periods, can bring about a large change in national income. It is different, however, from the mechanism of the multiplier effect. Investment's multiplier effect is cumulative effects of investment on private consumption (i.e. $\Delta I \rightarrow \Delta C$) that are unilateral and work just once; and government expenditure's multiplier effect is also cumulative effects of government expenditure on private consumption (i.e. $\Delta G \rightarrow \Delta C$) that are unilateral and work just once. In addition, the effects gradually decrease and ultimately converge to zero as leakage of savings diminishes the effects. By a sharp contrast, the determination mechanism of national income by means of leading-following relations stems from continuous inducement effects on the basis of complicated interactions among ΔC, ΔI, ΔG and $\Delta(X-M)$, which carry out chain effects over the entire periods and thus, cause a large change, positive or negative, of national income. A kick-starter may keep with the same demand item or be replaced by another. Unilateral causality as Keynes assumed from investment to private consumption (i.e. $\Delta I \rightarrow \Delta C$) is not the whole story; $\Delta C \rightarrow \Delta I$ and $\Delta C \rightarrow \Delta G$ are both quite possible to work, either in positive feedbacks or negative feedbacks. In fact, orbit analyses of some countries reveal a variety of special patterns in leading-following relations in the time of booms, repressions and crises respectively.

VIII. Leading-following relations and the Granger causality

Finally, let us make a supplementary reference to similarities and differences between leading-following relations and so-called Granger causality. It would probably be needless to say that pursuit for causality is one of the most important objectives of social sciences. Nowadays, causality is understood in the context of the *Granger causality* in the mainstream economics. According to him, two conditions must be satisfied for causality to be established (Granger (2003)):

1. The cause occurs before the effect, and
2. The cause contains information about the effect that is unique, and is in no other variable.

These two conditions originally appeared in his following remarks: “The theory … will rely entirely on the assumption that the future cannot cause the past” and “We say that Y_t is causing X_t if we are better able to predict X_t using all available information than if the information apart from Y_t had been used” (Granger (1969) p.428.). As for the first condition, which seems to be self-evident, he stated that “The flow of time clearly plays a central role in these definitions. In the author's opinion there is little use in the practice of attempting to discuss causality without introducing time, although philosophers have tried to do so."（Granger (1969) p.430.）

So-called Granger causality defined as such is evidently "pragmatic" (Granger (2003) p.366.), has only "predictability" (Granger (1969) p.430.) as its standard and is anything but "true causation" (Granger (2003) p.366.). He actually asked some people to tell him what the definition of true causation is, but nobody answered to his question (Granger (2003) p.366.). It seems, though, that he probably asked wrong people.

We can safely say that causality is one of the most important concepts in methodology of social sciences and that it is also a very difficult concept to appreciate. According to G. W. F. Hegel, we must not stay in the realm of causality, but must go to the understanding of relationship of reciprocal actions and further, to the synthetic conceptual understanding. Leave it to be discussed later, let us examine the validity of the "Granger causality" along his own arguments.

Granger's argument is summarized as follows: variable X precedes variable Y and you can predict variable Y more accurately if you use variable X and thus, variable X is a cause of variable Y in the sense of Granger causality. We do not question the second half of his argument, but the first half that seems to be a self-evident necessary condition. The question is what "precede" means.

Even if he is thoroughly pragmatic and regards anything as a cause only if it slightly serves for predictability, he would probably not commit a mistake that William S. Jevons did. He would never call sunspots as a cause of business cycles on the earth if the former preceded the latter for some periods. He in fact laments that "I got plenty of citations. Of course, many ridiculous papers appeared" (Granger (2003) p.366.). His real intention seems to be that there must be a certain kind of "true causation" between variables X and Y, but it is impossible to directly grasp it and hence, by setting the standard of the "Granger causality" we can verify their relations. If expressed in our concept, it means that a certain kind of quantitative traction must actually work between variables X and Y, although it may be a psychological one as in the case of a marathon race.

Now, let us ask whether there are such matters that are in the relations of quantitative traction, but do not show any interactions between actions and reactions, in other words, relations of matters in which only positive feedbacks work and thus, temporal preceding-lagging relations turn out to be the same as leading-following relations. Those would be matters that could exist only under mechanical causality in the world of engineering and could hardly exist as a social phenomenon.

If the reasoning above holds true, we have to ask how we can observe that variable X *precedes* variable Y. As reiterated some times, we conduct our observation and analysis during a single period, in which we can certainly compare variables' coordinates at the beginning of the period with those at the end of the period, but cannot know what actions and reactions occur within that minimum period. Therefore, it would be appropriate to expect that what we observe with respect to variables X and Y might be different from temporal preceding-lagging relations between them. However, Granger says "one might suggest that in many economic situations an apparent instantaneous causality would disappear if the economic variables were recorded at more frequent time intervals"

(Granger (1969) p.430.). Namely, you could observe preceding-lagging relations only if you shortened annual intervals to quarterly, monthly or daily intervals. Unfortunately, there still remain two problems:

1. However short a single period is set, temporal preceding-lagging relations within that period cannot be observed.
2. Apart from the observability problem, variables X and Y are in chain relations of actions and reactions in space and time and thus, no distinction can be made between a preceding variable and a lagging variable.

It follows that the Granger causality cannot be established because in principle no distinction is made between preceding and lagging variables. Distinction that we can actually observe and measure is that between a variable that actively leads changes and a variable that passively follows the changes as a result of complicated actions and reactions (that include positive feedbacks and negative feedbacks), which is observed and recorded as if being distinction between temporal preceding and lagging.

Now we know that philosophers' understanding of causality that Granger rejected is relevant. Hegel's insight is appropriate that acknowledgement of unilateral causality from cause to effect is superficial and that we should further go to the next stage of acknowledgement of reciprocal actions among variables. Let us listen to what a philosopher says:

"The cause is not only the cause of another, but also the effect of itself. So, the finitude of things consists in the fact that, although cause and effect are conceptually identical, the two forms occur separated in just *this* way: that although the cause is indeed an effect too and the effect is also a cause, nevertheless, the cause is not an effect in the same relation in which it is cause, and the effect is not a cause in the same relation in which it is an effect. This then gives us once again an infinite progression in the shape of an endless series of causes, which exhibits itself at the same time as an endless series of effects." (Hegel [1817] 153, p.229.)

"As a result causality has passed over into the relationship of *reciprocal action*. Although causality is not yet posited in its genuine determination, the progress, as an infinite progress from causes to effects, is truly sublated as progress in reciprocal action, because the rectilinear progression from causes to effects and from effects to causes is *curved* and *bent back* upon itself." (Hegel [1817] 154, p.230.)

Conclusions

Lastly here in conclusions, let us give the definition of leading-following relations on the basis of our examinations above. You may feel doubtful why in conclusions the definition of the very fundamental concept ought to be given again. But, a definition is "a provisional assumption that has been transformed into a definition" (Hegel [1817] 10, p.34.) and "is only supposed to be a *characteristic*" (Hegel [1817] 229, p.297.). Hence, the definitive *definition* would be offered as a result of all examinations on the matter[4].

Suppose that two variables are in relations of pulling and being pulled in a quantifiable manner and exert actions and reactions on each other. Those variables are not necessarily in temporal preceding-lagging relations, but can be observed and recorded as such every period. The relations are defined as leading-following relations if one of those variables has internal necessity of actively leading a quantitative change of the other or passively following the change of the other, and both have internal/ external conditions to make that possible.

Next we attempt a provisional positioning of orbit analysis in a big framework of social science methodology, though it remains yet a hypothetical, preliminary one.

To understand static structure and dynamic transformation of a matter is always an objective of science. If positioned in such a big framework, orbit analysis is a method that allows us to sense a herald of changes during a minimum observable period and to construct an acknowledgement of dynamic transformation of a matter by means of chaining them in space and time.

Orbit analysis is expected to be applicable to trade relations among countries and regions, demand inducement effects in input-output analysis, a spiral phenomenon between nominal wage rate and inflation rate, an inventory cycle, correlation between profit rate and accumulation rate, ripple effects of an economic babble, etc. in addition to the global system of interest rates and national income that have been already mentioned here.

NOTES

1） The Japanese translation of this paper is available in *Working Paper Series*, IR2014-1, "多変数間の先導・追従関係に関する軌道分析について", published in April 2014, College of International Relations, Ritsumeikan University (http://www.ritsumei.ac.jp/acd/cg/ir/college/bulletin/workingpaper/IR2014-1.pdf).

2） A typical example of an orbit in circular movements is an inventory cycle. We are likely to observe a clear anticlockwise orbit over several years with volume of production in *x*-axis and volume of inventory in *y*-axis. A set of leading-following relations is as follows: from the lower right, production increase leading inventory increase → inventory increase leading production decrease → production decrease leading inventory decrease → inventory decrease leading production increase. For a detailed analysis of quantitative adjustment processes of production by means of an inventory cycle, though not from the viewpoint of orbit analysis, see Morioka (2005).

3） The phenomenon has been mentioned as "a butterfly effect" in the discussion on *complexity*: in a bit sensational manner to put it, flapping of a butterfly in Beijing may eventually cause a hurricane in the Gulf of Mexico. See Waldrop (1992), Kauffman (1995) and Itaki (2000) (2001).

4） "This is how it happens that the Idea of speculative philosophy is simply kept fixed in its abstract definition; -- in the opinion that a definition must appear to be clear and definitive on its own account, and must have its methodic rule and touchstone only in presupposed notions; or at least without knowing that the sense of the definition, like its necessary proof, lies in its development alone – and precisely in its emergence as the result of the development." (Hegel [1817] p.7.) Replace "speculative philosophy" with "the mainstream economics".

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Orbit Analysis of Leading-Following Relations among Multiple Variables（ITAKI）

多変数間の先導・追従関係に関する軌道分析について

相関分析で用いられる散布図の座標を時系列に沿って連結することによって描かれる「軌道」の回転方向から，x 軸，y 軸二つの変数の間の「先導・追従関係」を明らかにしようとするものが，軌道分析である。この手法を多変数間の時系列に応用することによって，一連の首尾一貫した先導・追従関係を析出することができる。重要な点は，これが時間的な「先行・遅行関係」とは異なるという点である。この観点から，いわゆるグレンジャー因果関係の成立根拠が失われることが示される。また，軌道分析の例証として，1995 年―2011 年の短期金融市場金利（アメリカ，イギリス，ドイツ，ユーロ圏，日本の 5 カ国・地域）が一つの世界金利体系を形成していたことが示される。

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