# Non-Price Competition and Strategic Trade Policy under Duopoly

Kazunori Tanigaki


## Abstract

We take non-price competition such as marketing, making sales network, sellers, advertising and government promotion into consideration in the theory of trade policy under Bertrand and Cournot duopoly. In the third market models, we examine the effects of non-price parameter on strategic variables and analyze the optimal policies for non-price activity and export. Under Bertrand duopoly, I show that, (1) the results depend on the cross partial effect of the non-price activity on another country's strategic price ; (2) the optimal policy to export is not always tax ; and (3) the advertising or the export promotion of government is not always welfare-improving.


Key words : non-price activity, Bertrand and Cournot-Nash equilibrium, trade policy
JEL Classification Numbers : D43, F12, and F13

## 1. Introduction

Almost firms compete not only through quantity and price but also non-price activity which includes marketing, making sales network, sellers, advertising, government promotion and even bribe. A little attention to non-price activity has been paid in international trade theory. This paper tries to focus on the non-price activity in international trade theory. Taking the activity into consideration in the model of Eaton and Grossman (1986) and Brander and Spencer (1985), we consider Nash equilibriums for strategic variables of non-price inputs, prices and quantities.

In our model, non-price activity is added to the strategic variables. The most characteristic is that the non-price activity of one firm decreases demand or price of another firm. This causes significant difference in results especially under Bertrand duopoly. We find the optimal policy for welfare maximization is not always tax in the third market model unlike the results of Eaton and Grossman. We could interpret that advertising or the export promotion of governments is subsidy to non-price activity. In our paper, we show that the subsidy to

* An earlier version of this paper was presented at a Conference (2002) at Kobe University. Thanks for comments are due to participants. The author is grateful to Masayuki Okawa for helpful comments.

non-price activity of the domestic firm is not always welfare-improving. The optimal policy depends on the cross effect of non-price activity on the other country's price. Under Cournot duopoly, the optimal policy to export and non-price activity become subsidy.

A number of papers have focused on the profit-shifting motive for trade policy under oligopoly[1]. On the other hand, the analysis with non-price activity has been hardly ever considered in international trade theory. As an exceptional paper related to our paper, there is a paper of Yamawaki and Audretsch (1988) and Lahiri and Ono (1999). Yamawaki and Audretsch seek to identify the determinants of the Japanese import share in the U. S. market. The import share is considered as Cournot-Nash equilibrium. They show that the share depends on strategic variables such as advertising, R & D, capital and wages. Lahiri and Ono consider the optimal tariff in the presence of sellers who exist between producers and consumers. They show that the sign of the optimal tariff may depend on the nature of the producer-seller relationship.

In section 2 we analyze the role of non-price activity under Bertrand duopoly. In subsection 2.1, we describe a model in which strategic variables are price and non-price activity. In subsection 2.2, we examine the effects of non-price parameter on strategic variables. The optimal policy of tax (subsidy) to non-price activity is considered in subsection 2.3. The optimal policy to the domestic export is considered in subsection 2.4. In section 3 we analyze the role of non-price activity under Cournot duopoly. The effects of non-price parameter and the optimal policies are examined.

## 2. Bertrand Competition

### 2.1 The model

There are only two firms. The one is domestic and the other is foreign. Each firm produces one goods which is differentiated. The firms compete in a third-country market. We adopt the so called third market model. The output (export) of each firm is a function of domestic non-price input $S$ and foreign input $S^*$ as well as the prices of both firms. We assume that domestic non-price activity decreases the output of foreign firm.

The each demand in the third market is shown by

$$d=d(p,\ p^*,\ S,\ S^*)=1-p+\alpha p^*+A(S)-B(S^*) \tag{1}$$

$$d^*=d^*(p,\ p^*,\ S,\ S^*)=1-p^*+\alpha^* p+A^*(S^*)-B^*(S), \tag{2}$$

where d is the demand of domestic firm, $d^*$ is foreign, $S$ is non-price activity of domestic and $S^*$ is that of foreign. In the following, the asterisk denotes the variables associated with the foreign firm. The parameter $\alpha$ is the price effect of the rival firm and we assume that $0<\alpha$, $\alpha^*<1$. $A(S)$ is the effect of non-price activity on the domestic firm price. It is assumed that the function is concave, that is, $a(=dA/dS)>0$, $a''(=A'')<0$. The other functions which are related to non-price activity are similar. The terms $B(S^*)$ and $B^*(S)$ are the negative cross effects of foreign and domestic non-price inputs on demand. $B(S^*)$ and $B^*(S)$ are also assumed concave. For $b=B'$ and $b^*=B^{*\prime}$, we assume $(a-b^*)>0$, $(a^*-b)>0$. These assumptions mean that non-price activity raises the total demand, that is, $\partial(d+d^*)/\partial S>0$ and $\partial(d+d^*)/\partial S^*>0$.

The profit functions of domestic and foreign firm are respectively given by

$$\pi=R(p,\ p^*,\ S,\ S^*)-c(d,\ S)=(p-c_d)d-c_S S, \tag{3}$$
$$\pi^*=R^*(p,\ p^*,\ S,\ S^*)-c^*(d^*,\ S^*)=(p^*-c_d^*)d^*-c_{S*}S^*, \tag{4}$$

where $R$ and $c$ are respectively revenue function and cost function, $c_d$ and $c_S$ are marginal costs of production and non-price input. In our model, we assume these marginal costs are constant.

### 2.2 The first order conditions and comparative statistics

The Bertrand-Nash equilibriums are determined by the first-order conditions:

$$\pi_p=(p-c_d)(-1)+d=0, \tag{5}$$
$$\pi_{p*}^*=(p^*-c_d^*)(-1)+d^*=0, \tag{6}$$
$$\pi_S=(p-c_d)a-c_S=0, \tag{7}$$
$$\pi_{S*}^*=(p^*-c_d^*)a^*-c_{S*}^*=0, \tag{8}$$

where derivatives are denoted by subscripts. The second derivatives are respectively given by

$$\begin{aligned}&\pi_{pp}=-2,\ \pi_{pp*}=d_{p*}=\alpha,\ \pi_{p*p}^*=d_p^*=\alpha^*,\ \pi_{p*p*}^*=2d_{p*}^*=-2,\\ &\pi_{SS}=(p-c_d)a''<0,\ \pi_{SS*}=0,\ \pi_{S*S}^*=0,\ \pi_{S*S*}^*=(p^*-c_d^*)a^{*\prime\prime}<0,\\ &\pi_{pS}=d_S=a>0,\ \pi_{p*S}^*=d_S^*=-b^*<0,\ \pi_{pS*}=d_{S*}=-b<0,\ \pi_{p*S*}^*=a^*>0,\\ &\pi_{Sp}=a,\ \pi_{S*p}^*=0,\ \pi_{Sp*}=0,\ \pi_{S*p*}^*=a^*.\end{aligned} \tag{9}$$

#### 2.2.1 The effect of non-price parameter on strategic variables

In this section we examine the effects of non-price parameter. For example, when the efficiency of non-price activity is increased, does the domestic firm raises non-price activity or the strategic price? What are the reactions of the rival firm?

Analyzing it, we rewrite (1) as

$$d=d(p,\ p^*,\ S,\ S^*)=1-p+\alpha p^*+\beta A(S)-\gamma B(S^*), \tag{1'}$$

where $\beta$ and $\gamma$ are parameters for non-price activities. It is initially assumed that $\beta$ and $\gamma$ are normalized to 1 for simplicity. The profit condition for $S$, (7), is also rewritten as

$$\pi_S=(p-c_d)\beta a-c_S=0. \tag{7'}$$

**(1) The effect of $\beta$**

What is the effect of parameter $\beta$ on non-price activity and prices? From (1)', (5) and (7)', we obtain

$\pi_{p\beta}=A,\ \ \pi_{S\beta}=a(p-c_d)=ad$

Using the above equations and totally differentiation of (5) - (8) give

$$\begin{pmatrix}-2 & \alpha\\ \alpha^* & -2\end{pmatrix}\begin{pmatrix}dp\\ dp^*\end{pmatrix}+\begin{pmatrix}a & -b\\ -b^* & a^*\end{pmatrix}\begin{pmatrix}dS\\ dS^*\end{pmatrix}=\begin{pmatrix}-A\\ 0\end{pmatrix}d\beta, \tag{10}$$

$$\begin{pmatrix}a & 0\\ 0 & a^*\end{pmatrix}\begin{pmatrix}dp\\ dp^*\end{pmatrix}+\begin{pmatrix}(p-c_d)a'' & 0\\ 0 & (p^*-c_d^*)a^{*''}\end{pmatrix}\begin{pmatrix}dS\\ dS^*\end{pmatrix}=\begin{pmatrix}-ad\\ 0\end{pmatrix}d\beta. \tag{11}$$

From (10), we have

$$(4-\alpha\alpha^*)\begin{pmatrix}dp\\ dp^*\end{pmatrix}=\begin{pmatrix}2a-\alpha b^* & a^*\alpha-2b\\ a\alpha^*-2b^* & 2a^*-\alpha^* b\end{pmatrix}\begin{pmatrix}dS\\ dS^*\end{pmatrix}+\begin{pmatrix}2A\\ A\alpha^*\end{pmatrix}d\beta. \tag{12}$$

By substituting (12) into (11), we can derive[2)]

$$\begin{pmatrix}dS\\ dS^*\end{pmatrix}=\begin{pmatrix}a^*(2a^*-\alpha^* b)+a^{*''}(4-\alpha\alpha^*)(p^*-c_d^*) & -a(a^*\alpha-2b)\\ -a^*(a\alpha^*-2b^*) & a(2a-\alpha b^*)+a''(4-\alpha\alpha^*)(p-c_d)\end{pmatrix}$$

$$(1/\Delta)\begin{pmatrix}-ad(4-\alpha\alpha^*)-2A\\ -a^*\alpha^* A\end{pmatrix}d\beta. \tag{13}$$

Thus, we have

$$\Delta S_\beta=-\{a^*(2a^*-\alpha^* b)+a^{*''}(4-\alpha\alpha^*)(p^*-c_d^*)\}\{da(4-\alpha\alpha^*)+2A\}$$
$$+a(a^*\alpha-2b)a^*\alpha^* A$$
$$\Delta S_\beta^*=a^*(a\alpha^*-2b^*)\{ad(4-\alpha\alpha^*)+2A\}-\{a(2a-\alpha b^*)+a''(4-\alpha\alpha^*)(p-c_d)\}a^*\alpha^* A$$

The terms, $\{a^*(2a^*-\alpha^* b)+a^{*''}(4-\alpha\alpha^*)(p^*-c_d^*)\}$ and $\{a(2a-\alpha b^*)+a''(4-\alpha\alpha^*)(p-c_d)\}$, are respectively negative from the stability conditions.[3)]

Then, we obtain

$$(a\alpha^*-2b^*)>0 \text{ and } (a^*\alpha-2b)>0 \rightarrow S_\beta>0 \text{ and } S_\beta^*>0.$$

Similarly we can obtain

$$(a\alpha^*-2b^*)>0 \text{ and } (a^*\alpha-2b)>0 \rightarrow p_\beta>0 \text{ and } p_\beta^*>0.$$

What are the conditions of $(a\alpha^*-2b^*)>0$ and $(a^*\alpha-2b)>0$? From (10), we find that $(a\alpha^*-2b^*)=(4-\alpha\alpha^*)\partial p^*/\partial S$ or $(a^*\alpha-2b)=(4-\alpha\alpha^*)\partial p/\partial S^*$.[4)] The term $(a\alpha^*-2b^*)$ means the effect of exogenous domestic non-price activity to the foreign price under $dS^*=0$. We can divide the effect, $\partial p^*/\partial S$, to $a\alpha^*=(4-\alpha\alpha^*)\partial p^*/\partial S|\pi^*_{p*S=0}$ and $(-2b^*)=(4-\alpha\alpha^*)\partial p^*/\partial S|\pi_{pS=0}$.[5)] We interpret $(a\alpha^*)$ as the effect through the profit function of domestic firm and $(-2b^*)$ through the profit function of foreign firm.

We call $(a\alpha^*)$ indirect price effect of domestic non-price activity since this is through the domestic price optimization ($\pi_p=0$). We also call $(-2b^*)$ direct external price effect since this through the foreign ($\pi^*_{p*}=0$). Similarly we can interpret $(a^*\alpha-2b)$. Since these are obtained by the first order conditions for prices, we say that these terms, $(\partial p^*/\partial S)$ and $(\partial p/\partial S^*)$, are cross effects of non-price activity on another country's strategic price. If the term is positive (negative), we can also say that the strategic price is complement (substitute) for the non-price activity of the other country.

### (2) The negative external effect, $\gamma$

What is the effect of parameter $\gamma$ on non-price activity and prices? From (1)' and (3)', we obtain

$$\pi_{p\gamma}=-1,\quad \pi_{S\gamma}=\pi^*_{S*\gamma}=0$$

Since the right-side hand of (13) is rewritten to $(1/\Delta)\begin{pmatrix}2\\ a^*\alpha^*\end{pmatrix}d\gamma$, we can derive the effects of $\gamma$, that is,

$$\Delta S_\gamma=-\{a^*(2a^*-\alpha^*b)+a^{*''}(4-\alpha\alpha^*)(p^*-c^*_d)\}2-(a^*\alpha-2b)a^*\alpha_*,$$
$$\Delta S^*_\gamma=-a^*(a\alpha^*-2b^*)2+\{a(2a-\alpha b^*)+a''(4-\alpha\alpha^*)(p-c_d)\}a^*\alpha^*.$$

Thus, under the conditions of $(a\alpha^*-2b^*)>0$ and $(a^*\alpha-2b)>0$, we obtain

$$S_\gamma<0,\quad S^*_\gamma<0,\quad p_\gamma<0 \text{ and } p^*_\gamma<0.$$

From the above results related to $\beta$ and $\gamma$, we can state

*Proposition 1. If the two cross effects of non-price activity on strategic prices are positive, that is,* $(a\alpha^*-2b^*)>0$ *and* $(a^*\alpha-2b)>0$, *the increase in the efficiency of non-price activity* $(d\beta)$, *raises all the strategic variables and the negative external effect* $(d\gamma)$, *reduces all the variables.*

Intuitively, we may interpret the results as follows. When the efficiency for $S$ is improved, $S$ and $p$ are initially raised from each first order conditions and $\pi_{p\beta}>0$, $\pi_{S\beta}>0$. Under the condition of $(\partial p^*/\partial S)>0$, the foreign price increased and it immediately $dS^*>0$ from (11). If $(\partial p/\partial S^*)>0$, it raises $p$ and these changes becomes consistent. For $\gamma$, we can state similarly.

## 2.3 Optimal tax or subsidy for non-price activity

In this section, we consider the optimal tax or subsidy for non-price activity. The domestic profit function is rewritten as

$$\pi=R(p,\ p^*,\ S,\ S^*,\ t_S)-C(d,\ S)=(p-c_d)d-c_SS-t_SS, \tag{3}''$$

where $t_S$ is tax $(t_S>0)$ or subsidy $(t_S<0)$ to non-price activity.
Eq. (7) becomes

$$\pi_S=(p-c_d)a-c_S-t_S=0. \tag{7}''$$

From $\pi_{St_S}=-1$, (10) - (13) can be rewritten respectively as

$$\begin{pmatrix}-2 & \alpha\\ \alpha^* & -2\end{pmatrix}\begin{pmatrix}dp\\ dp^*\end{pmatrix}+\begin{pmatrix}a & -b\\ -b^* & a^*\end{pmatrix}\begin{pmatrix}dS\\ dS^*\end{pmatrix}=0,$$

$$\begin{pmatrix}a & 0\\ 0 & a^*\end{pmatrix}\begin{pmatrix}dp\\ dp^*\end{pmatrix}+\begin{pmatrix}(p-c_d)a'' & 0\\ 0 & (p^*-c_d^*)a^{*''}\end{pmatrix}\begin{pmatrix}dS\\ dS^*\end{pmatrix}=\begin{pmatrix}1\\ 0\end{pmatrix}dt_S,$$

$$(4-\alpha\alpha^*)\begin{pmatrix}dp\\ dp^*\end{pmatrix}=\begin{pmatrix}2a-\alpha b^* & a^*\alpha-2b\\ a\alpha^*-2b^* & 2a^*-\alpha^* b\end{pmatrix}\begin{pmatrix}dS\\ dS^*\end{pmatrix},$$

$$\begin{pmatrix}dS\\ dS^*\end{pmatrix}=\begin{pmatrix}a^*(2a^*-\alpha^* b)+a^{*''}(4-\alpha\alpha^*)(p^*-c_d^*) & -a(a^*\alpha-2b)\\ -a^*(a\alpha^*-2b^*) & a(2a-\alpha b^*)+a''(4-\alpha\alpha^*)(p-c_d)\end{pmatrix}$$

$$(1/\Delta)\begin{pmatrix}(4-\alpha\alpha^*)\\ 0\end{pmatrix}dt_S.$$

Then the effects on strategic variables are respectively given by

$$\Delta S_{tS}=\{a^*(2a^*-\alpha^* b)+a^{*''}(4-\alpha\alpha^*)(p^*-c_d^*)\}(4-\alpha\alpha^*)<0,$$
$$\Delta S_{tS}^*=-(4-\alpha\alpha^*)a^*(a\alpha^*-2b^*),$$
$$(4-\alpha\alpha^*)\Delta p_{tS}=(2a-\alpha b^*)S_{tS}+(a^*\alpha-2b)S_{tS}^*,$$
$$(4-\alpha\alpha^*)\Delta p_{tS}^*=(a\alpha^*-2b^*)S_{tS}+(2a^*-\alpha^* b)S_{tS}^*.$$

The domestic non-price activity is reduced by the tax. The foreign activity depends only on the sign of cross effects of non-price activity. If and only if $(\partial p^*/\partial S)(=a\alpha^*-2b^*)>0$, $S_{tS}^*$ is negative. When the cross effects of non-price activity are positive, that is, $(a\alpha^*-2b^*)>0$ and $(a^*\alpha-2b)>0$, we find that $p_{tS}$ and $p_{tS}^*$ are both negative. Then we can state

*Proposition 2. The domestic non-price activity is always decreased by the tax and the foreign depends only on the sign of $(\partial p^*/\partial S)$. All the strategic variables are reduced by the tax if the both cross effects of non-price activity are positive. The foreign strategic variables, $S^*$ and $p^*$, are increased by tax if $(\partial p^*/\partial S)<0$.*

**Welfare 1**

Since our model is one of the third market models, $W^1$ (welfare 1) is defined as the profit of domestic firm plus tax revenue for non-price activity, that is,

$$W^1=\pi+t_S S.$$

By differentiating $W^1$ totally and using first order conditions (3)' and (7)', we get

$$\begin{aligned}W_{tS}^1&=\pi_p p_{tS}+\pi_{p*}p_{tS}^*+\pi_S S_{tS}+\pi_{S*}S_{tS}^*+\pi_{tS}+S+t_S S_{tS}\\ &=\pi_{p*}p_{tS}^*+\pi_{S*}S_{tS}^*+t_S S_{tS}\\ &=(p-c_d)(\alpha p_{tS}^*+(-b)S_{tS}^*)+t_S S_{tS}.\end{aligned} \tag{14}$$

Since $\{\alpha p^*_{tS}+(-b)S^*_{tS}\}=\{-a^{*\prime\prime}\alpha(p^*-c^*_d)/a^*-b\}S^*_{tS}$ from (11), we get

$$W^1_{tS}=(p-c_d)\{-a^{*\prime\prime}\alpha(p^*-c^*_d)/a^*-b\}S^*_{tS}+t_S S_{tS}. \tag{15}$$

Using the stability condition, $\{a^*(2a^*-\alpha^* b)+a^{*\prime\prime}(4-\alpha\alpha^*)(p^*-c^*_d)\}<0$, we have $\{-a^{*\prime\prime}\alpha(p^*-c^*_d)-a^*b\}>\alpha a^*(2a^*-\alpha^* b)/(4-\alpha\alpha^*)-a^*b=2(a^*\alpha-2b)/(4-\alpha\alpha^*)$.
Thus, we get

$$a^*\alpha-2b>0 \rightarrow \{-a^{*\prime\prime}\alpha(p^*-c^*_d)/a^*-b\}>0. \tag{16}$$

The optimal tax to non-price activity is given by

$$t_S=[-(p-c_d)/S_{tS}]\{-a^{*\prime\prime}\alpha(p^*-c^*_d)/a^*-b\}S^*_{tS}.$$

Then, under the condition of $(a^*\alpha-2b)>0$, we get

sign $t_S$=sign $S^*_{tS}$=sign $(-a\alpha^*+2b^*)$.

The above result is stated as follows

*Proposition 3. Under the condition of* $(\partial p/\partial S^*)>0$, *the sign of optimal policy to the domestic non-price activity depends only on the cross effect of* $S$ on $p^*$. *Thus, for example, the government export promotion (that is, subsidy) can be welfare worsening when the cross effect is asymmetric, that is,* $(\partial p/\partial S^*)>0$ *and* $(\partial p^*/\partial S)<0$.

The subsidy raises the domestic non-price activity but it also increases that of foreign if $(\partial p/\partial S^*)>0$ and $(\partial p^*/\partial S)<0$. From (15) and the above analysis, we find that it reduces the domestic profit and increases the government expenditure so that the subsidy becomes welfare-worsening.

### 2.4 Optimal tax or subsidy for export

In this section we consider the optimal tax or subsidy to the export of the domestic firm. When firms compete under Bertrand duopoly, it is known that the optimal trade policy becomes subsidy as Eaton and Grossman (1986) have shown. What is the optimal trade policy in our model with non-price activity ? The profit function of domestic is given by

$$\pi=(p-c_d-t)d-c_S S, \tag{3)'''$$

where $t$ is tax $(t>0)$ or subsidy $(t<0)$ to the export of the domestic firm. Using $\pi_{SS}=(p-c_d-t)a''<0$, $\pi_{St}=-a$, $\pi_{pt}=1$, $\pi^*_{p^*t}=0$,[6] (10) - (13) can be rewritten, respectively as

$$\begin{pmatrix}-2 & \alpha\\ \alpha^* & -2\end{pmatrix}\begin{pmatrix}dp\\ dp^*\end{pmatrix}+\begin{pmatrix}a & -b\\ -b^* & a^*\end{pmatrix}\begin{pmatrix}dS\\ dS^*\end{pmatrix}=\begin{pmatrix}-1\\ 0\end{pmatrix}dt,$$

$$\begin{pmatrix}a & 0\\ 0 & a^*\end{pmatrix}\begin{pmatrix}dp\\ dp^*\end{pmatrix}+\begin{pmatrix}(p-c_d-t)a'' & 0\\ 0 & (p^*-c^*_d)a^{*\prime\prime}\end{pmatrix}\begin{pmatrix}dS\\ dS^*\end{pmatrix}=\begin{pmatrix}a\\ 0\end{pmatrix}dt,$$

$$(4-\alpha\alpha^*)\begin{pmatrix} dp \\ dp^* \end{pmatrix}=\begin{pmatrix} 2a-\alpha b^* & a^*\alpha-2b \\ a\alpha^*-2b^* & 2a^*-\alpha^* b \end{pmatrix}\begin{pmatrix} dS \\ dS^* \end{pmatrix}+\begin{pmatrix} 2 \\ \alpha^* \end{pmatrix}dt,$$

$$\begin{pmatrix} dS \\ dS^* \end{pmatrix}=\begin{pmatrix} a^*(2a^*-\alpha^* b)+a^{*''}(4-\alpha\alpha^*)(p^*-c_d^*) & -a(a^*\alpha-2b) \\ -a^*(a\alpha^*-2b^*) & a(2a-\alpha b^*)+a''(4-\alpha\alpha^*)(p-c_d-t) \end{pmatrix}$$

$$(1/\Delta)\begin{pmatrix} a(2-\alpha\alpha^*) \\ -a^*\alpha^* \end{pmatrix}dt.$$

Then we have

$$\Delta S_t=\{a^*(2a^*-\alpha^* b)+a^{*''}(4-\alpha\alpha^*)(p^*-c_d^*)\}a(2-\alpha\alpha^*)+a(a^*\alpha-2b)a^*\alpha^*, \tag{17}$$

$$\Delta S_t^*=-a(2-\alpha\alpha^*)a^*(a\alpha^*-2b^*)-\{a(2a-\alpha b^*)+a''(4-\alpha\alpha^*)(p-c_d-t)\}a^*\alpha^*. \tag{18}$$

From the stability conditions, $\{a^*(2a^*-\alpha^* b)+a^{*''}(4-\alpha\alpha^*)(p^*-c_d^*)\}<0$, $\{a(2a-\alpha b^*)+a''(4-\alpha\alpha^*)(p-c_d-t)\}<0$, we find that $S_t<0$ if $(a^*\alpha-2b)<0$, and that $S_t^*>0$ if $(a\alpha^*-2b^*)>0$. The sign of $p_t^*$ is similar to $S_t^*$ but the sign of $p_t^*$ is more complex.

**Welfare 2**

Welfare is defined as the profit of the domestic firm plus the tax revenue, that is,

$$W^2=\pi+td.$$

By differentiating $W^2$ totally, we get

$$\begin{aligned} W_t^2 &=\pi_p p_t+\pi_{p*}p_t^*+\pi_S S_t+\pi_{S*}S_t^*+\pi_t+d+td_t \\ &=(p-c_d-t)(\alpha p_t^*+(-b)S_t^*)+td_t. \end{aligned} \tag{19}$$

Similarly to (15), the optimal tax is given by,

$$\begin{aligned} t&=-(p-c_d)(\alpha p_t^*+(-b)S_t^*)/d_t \\ &=-(p-c_d)\{-a^{*''}\alpha(p^*-c_d^*)/a^*-b\}S_t^*/d_t. \end{aligned}$$

From (16) and (18), we find that

$$(a\alpha^*-2b^*)<0 \text{ and } (a^*\alpha-2b)>0 \rightarrow \alpha p_t^*+(-b)S_t^*=\{-a^{*''}\alpha(p^*-c_d^*)/a^*-b\}S_t^*>0.$$

Although we can't derive the sufficient conditions for the sign of $d_t$, we obtain sufficient conditions for the optimal policy. That is, from (19), we can get

$$(\partial p^*/\partial S)<0 \text{ and } (\partial p/\partial S^*)>0 \rightarrow \text{sign }(t)=\text{sign } d_t$$

From the above results, we can state

*Proposition 4. Under the conditions of* $(\partial p^*/\partial S)<0$ *and* $(\partial p/\partial S^*)>0$, *the optimal policy depends on the effect of the tax to the domestic export. If the export is increased (reduced), the optimal policy becomes tax (subsidy).*

## 3. Cournot Competition

In section 3, we consider the Cournot competition. In the third market model with Cournot competition, it is known that the optimal policy for the export is subsidy (Brander and Spencer (1985)). In our model with non-price competition, what is the optimal policy and how different are the results as compared with Bertrand?

### 3.1 The model

Let $q$ denote the export of the domestic firm. Then the price is the function of strategic variables, $q$, $q^*$, $S$, $S^*$, that is,

$$p=1-q-\alpha q^*+\beta A(S)-\gamma B(S^*) \tag{20}$$
$$p^*=1-q^*-\alpha^* q+A^*(S^*)-B^*(S), \tag{21}$$

The profit functions are given by

$$\pi=(p-c_d-t)d-c_S S-t_S S \tag{22}$$
$$\pi^*=(p^*-c_d^*)d^*-c_{S*}S^*, \tag{23}$$

where $t$ is tax ($t>0$) or subsidy ($t<0$) to the export of the domestic firm and $t_S$ is tax ($t_S>0$) or subsidy ($t_S<0$) to non-price activity. The Cournot-Nash equilibriums are determined by the first-order conditions:

$$\pi_q=(p-c_q-t)-q=0,$$
$$\pi^*_{q*}=(p^*-c_q^*)-q^*=0,$$
$$\pi_S=\beta aq-c_S-t_S=0,$$
$$\pi^*_{S*}=a^*q^*-c^*_{S*}=0.$$

The second derivatives are respectively given by

$$\pi_{qq}=-2,\ \pi_{qq*}=-\alpha,\ \pi^*_{q*q}=-\alpha^*,\ \pi^*_{q*q*}=-2,$$
$$\pi_{SS}=qa''<0,\ \pi_{SS*}=0,\ \pi^*_{S*S}=0,\ \pi^*_{S*S*}=q^*a^{*''}<0,$$
$$\pi_{qS}=a,\ \pi^*_{q*S}=b^*<0,\ \pi_{qS*}=-b>0,\ \pi^*_{q*S*}=a^*<0,$$
$$\pi_{Sq}=a',\ \pi^*_{S*q}=0,\ \pi_{Sq*}=0,\ \pi^*_{S*q*}=a^*,$$
$$\pi_{St}=0,\ \pi_{qt}=-1,\ \pi^*_{q*t}=0,\ \pi_{q\beta}=A,\ \pi_{S\beta}=aq,\ \pi_{q\gamma}=0,\ \pi_{St_S}=-1.$$

Using the above equations and totally differentiation give

$$\begin{pmatrix}\pi_{qq} & \pi_{qq*}\\ \pi^*_{q*q} & \pi^*_{q*q*}\end{pmatrix}\begin{pmatrix}dq\\ dq^*\end{pmatrix}+\begin{pmatrix}\pi_{qS} & \pi_{qS*}\\ \pi^*_{q*S} & \pi^*_{q*S*}\end{pmatrix}\begin{pmatrix}dS\\ dS^*\end{pmatrix}=\begin{pmatrix}-\pi_{qi}\\ -\pi^*_{q*i}\end{pmatrix}di,$$
$$\begin{pmatrix}\pi_{Sq} & \pi_{Sq*}\\ \pi^*_{S*q} & \pi^*_{S*q*}\end{pmatrix}\begin{pmatrix}dq\\ dq^*\end{pmatrix}+\begin{pmatrix}\pi_{SS} & \pi_{SS*}\\ \pi^*_{S*S} & \pi^*_{S*S*}\end{pmatrix}\begin{pmatrix}dS\\ dS^*\end{pmatrix}=\begin{pmatrix}-\pi_{Si}\\ -\pi^*_{S*i}\end{pmatrix}di.\quad (i=t,\ \beta,\ \gamma,\ t_S)$$

In this case, (10) - (13) can be rewritten, respectively as

$$\begin{pmatrix} -2 & -\alpha \\ -\alpha^* & -2 \end{pmatrix}\begin{pmatrix} dq \\ dq^* \end{pmatrix}+\begin{pmatrix} a & -b \\ -b^* & a^* \end{pmatrix}\begin{pmatrix} dS \\ dS^* \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \end{pmatrix}dt+\begin{pmatrix} -1 \\ 0 \end{pmatrix}Ad\beta+\begin{pmatrix} 1 \\ 0 \end{pmatrix}Bd\gamma,$$

$$\begin{pmatrix} a & 0 \\ 0 & a^* \end{pmatrix}\begin{pmatrix} dq \\ dq^* \end{pmatrix}+\begin{pmatrix} qa'' & 0 \\ 0 & q^*a^{*''} \end{pmatrix}\begin{pmatrix} dS \\ dS^* \end{pmatrix}=\begin{pmatrix} -aq \\ 0 \end{pmatrix}d\beta+\begin{pmatrix} 1 \\ 0 \end{pmatrix}dts,$$

$$(4-\alpha\alpha^*)\begin{pmatrix} dq \\ dq^* \end{pmatrix}=\begin{pmatrix} 2a+\alpha b^* & -a^*\alpha-2b \\ -a\alpha^*-2b^* & 2a^*+\alpha^* b \end{pmatrix}\begin{pmatrix} dS \\ dS^* \end{pmatrix}+\left(\begin{pmatrix} -2 \\ \alpha^* \end{pmatrix}dt+\begin{pmatrix} 2 \\ -\alpha^* \end{pmatrix}Ad\beta+\begin{pmatrix} -2 \\ \alpha^* \end{pmatrix}Bd\gamma\right)$$

$$\begin{pmatrix} dS \\ dS^* \end{pmatrix}=\begin{pmatrix} a^*(2a^*+\alpha^* b)+a^{*''}(4-\alpha\alpha^*)q^* & a(a^*\alpha+2b) \\ a^*(a\alpha^*+2b^*) & a(2a+\alpha b^*)+a''(4-\alpha\alpha^*)q \end{pmatrix}$$

$$(1/\Delta)\left(\begin{pmatrix} 2a \\ -a^*\alpha^* \end{pmatrix}dt+\begin{pmatrix} -2aA-aq(4-\alpha\alpha^*) \\ a\alpha^* \end{pmatrix}d\beta+\begin{pmatrix} -2a \\ a^*\alpha^* \end{pmatrix}Bd\gamma+\begin{pmatrix} 4-\alpha\alpha^* \\ 0 \end{pmatrix}dts\right).$$

Note that we find that the cross effects are always negative. Then, we have

$$(1/\Delta)S_t=a[\{a^*(2a^*+\alpha^* b)+a^{*''}(4-\alpha\alpha^*)q^*\}2-(a^*\alpha+2b)a^*\alpha^*]<0,$$
$$(1/\Delta)S_t^*=a^*[2a(a\alpha^*+2b^*)-\{a(2a-\alpha b^*)+a''(4-\alpha\alpha^*)q\}\alpha^*]>0,$$
$$q_t<0,\quad q_{t*}^*>0.$$

Similarly we can derive

$$S_\beta>0,\quad S_\beta^*<0,\quad q_\beta^*<0,$$
$$S_\gamma<0,\quad S_\gamma^*>0,\quad q_\gamma<0,\quad q_{\gamma*}^*>0,$$
$$S_{tS}<0,\quad S_{tS}^*>0,\quad q_{tS}^*>0.$$

From the above results, we can state

*Proposition 5. In the Cournot competition, the tax to export and non-price activity restrains the domestic strategic variables and expand that of foreign firm. The increase in the efficiency of non-price activity raises its own activity and reduces the rival activity.*

**Welfare 3**

We consider the optimal policy to the non-price activity. $W^3$ is defined as $(\pi+t_S S)$. By using $\pi_{q*}=-\alpha q$ and $\pi_{S*}=-bq$, the welfare effect is given by

$$\begin{aligned} W_{tS}^4&=\pi_q q_{tS}+\pi_{q*}q_{tS}^*+\pi_S S_{tS}+\pi_{S*}S_{tS}^*+\pi_{tS}+S+tS_{tS},\\ &=\pi_{q*}q_{tS}^*+\pi_{S*}S_{tS}^*+t_S S_{tS},\\ &=-q(\alpha q_{tS}^*+bS_{tS}^*)+tS_{tS}.\end{aligned}$$

From $q_{tS}^*>0$, $S_{tS}^*>0$, $S_{tS}<0$, the optimal policy is always subsidy.

**Welfare 4**

In the policy of tax or subsidy to the domestic export, $W^4$ is defined as the profit of domestic firm plus tax revenue or subsidy, that is,

$$W^4 = \pi + tq$$

By differentiating $W^3$ totally and using first order conditions, we get

$$\begin{aligned} W_t^3 &= \pi_q q_t + \pi_{q*} q_t^* + \pi_S S_t + \pi_{S*} S_t^* + \pi_t + q + tq_t, \\ &= \pi_{q*} q_t^* + \pi_{S*} S_t^* + tq_t, \\ &= q(-\alpha q_t^* - bS_t^*) + tq_t. \end{aligned}$$

From $q_t^* > 0$, $S_t^* > 0$, $q_t < 0$, $S_t < 0$, the optimal tax (subsidy) is given by

$$t = q(\alpha q_t^* + bS_t^*)/q_t < 0.$$

Thus, the optimal policy is always subsidy.
The above results are summarized as follows

*Proposition 6. The optimal policies to the export and the non-price activity are always subsidy. In this sense, the results of the Brander and Spenser are remained in our model with non-price strategic variables.*

## 4. Concluding Remarks

Almost firms have not only the production sector but also its sales department. The firms compete through advertising, image, marketing, bride, free gift, besides price and quantity. In the literature of international trade, this activity is not much considered. Strategic variables are not only price and quantities.

In this paper we tried to examine the role of these sectors. The results between Bertrand and Cournot duopoly are different. In the Cournot, the results are unambiguous and subsidy is optimal. But in the case of Bertrand they are ambiguous and depend on the cross effect of non-price activity.

Why are the results different and not symmetric in our two models ? The reason is that our two models, Bertrand and Cournot, are not symmetric. If there is no non-price activity, we find from (1), (2), (20) and (21) that the reaction functions become symmetric for $p$ and $q$. However in our models with non-price activity, the added terms, $\{A(S) - B(S^*)\}$ and $\{A^*(S^*) - B^*(S)\}$ in Bertrand, are not symmetric to the terms, $\{\beta A(S) - \gamma B(S^*)\}$ and $\{A^*(S^*) - B^*(S)\}$ in Cournot. They are identical basically so that the two results become not symmetric.

From our analysis, we can say theoretically that policy makers or firms, who consider strategies under international trade, should take non-price activity into consideration. Our analysis which introduces non-price activity has important implications for the nature of the optimal tax-subsidy policy. Besides Bertrand and Cournot duopolies in the third market, there may be potentially many models or themes to which we can apply the non-price activity.

## Notes

1) As recent papers, Kemp and Shimomura (2001) and Raff (2000) analyzed under Cournot competition, and David (1998) under Bertrand. As a survey paper, Brander (1996) examined.

2) Where $\Delta$ is the $4\times4$ determinant for the left-side of (10) and (11). It is positive from stability condition.

3) These are $3\times3$ determinants for the left-side of (10) and (11).

4) From (10), these effects are derived from

$$\begin{pmatrix}-2 & \alpha\\ \alpha^* & -2\end{pmatrix}\begin{pmatrix}dp\\ dp^*\end{pmatrix}+\begin{pmatrix}a & -b\\ -b^* & a^*\end{pmatrix}\begin{pmatrix}dS\\ 0\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix} \qquad ①$$

5) From ① in footnote 4, these effects are derived from

$$(4-\alpha\alpha^*)\begin{pmatrix}dp\\ dp^*\end{pmatrix}=\begin{pmatrix}-2 & -\alpha^*\\ -\alpha & -2\end{pmatrix}\begin{pmatrix}-a\\ 0\end{pmatrix}dS+\begin{pmatrix}-2 & -\alpha^*\\ -\alpha & -2\end{pmatrix}\begin{pmatrix}0\\ b^*\end{pmatrix}dS$$

6) The first-order conditions are given by

$$\pi_p=(p-c_d-t)(-1)+d=0 \qquad (5)'$$

$$\pi^*_{p*}=(p^*-c^*_d)(-1)+d^*=0 \qquad (6)$$

$$\pi_S=(p-c_d-t)a-c_S=0 \qquad (7)'$$

$$\pi^*_{S*}=(p^*-c^*_d)a^*-c^*_{S*}=0, \qquad (8)$$

## References


Brander, J. A. and B. J. Spencer, 1985, *Export Subsidies and International Market Share Rivalry*, Journal of International Economics 18, 83-100.

Brander, J. A., 1995, *Strategic Trade Policy*, in : G. Grossman and K. Rogoff ed., Handbook of International Economics Volume 3 (Elsevier).

Collie, D. A., 1998, *Trade Policy under Bertrand Duopoly with Integrated Markets : The Pure Strategy Equilibrium*, Economics Letters 60, 179-183.

Eaton, J. E. and G. E. Grossman, 1968, *Optimal trade and industrial policy under oligopoly*, Quarterly Journal of Economics 101, 381-406.

Lahiri, S. and Y. Ono, 1999, *Optimal tariffs in the presence of middleman*, Canadian Journal of Economics 32, 55-70.

Yamawaki, H. and D. B. Audretsch, 1988, *Import share under International Oligopoly with Differentiated Products : Japanese Imports in U. S. Manufacturing*, The Review of Economics and Statistics 70, 567-579.