Note

The dual equation and the net economic resource

ITAKI, Masahiko

Introduction

An economy is a social organism that reproduces and develops itself over the long term. In the process of reproduction, the human groups, first of all, that constitute the society must be physiologically reproduced. And in order to realize it, the productive forces that drives the society must be reproduced and developed period by period. Furthermore, the stability of the society can be maintained only if the social relationships among human groups are reproduced in each period.

Human beings continuously sustain their lives through consumption activities in each household as a unit. To that end, various means of consumption are supplied from industries to households; numerous means of production are mutually input and output among industrial sectors; and labor is provided for those industrial sectors by households. Consumption and production activities are such a complex network and, in other words, the indispensable two wheels of reproduction processes.

The specific contents of household consumption greatly vary depending on what social status the household occupies in society. In a capitalist society, it is broadly divided into wage-workers' households and other households. The latter includes, for example, those of capitalists, landlords and civil servants. Income distributed as wages, profits, land rents and taxes will be more or less used for consumption, and the rest will be reserved for savings to support investment and growth of an economy.

In this research note, we construct a four-sector model that is a modified version of von Neumann [1938] and describe as simply as possible the reproduction and development processes of productive forces, human groups and their social relationships. By doing so, we would like to observe closely intertwined relationship between resource allocation (i.e. growth and consumption) and income distribution (i.e. profits and wages). In our argument, a new concept, *the net economic resource*, plays the pivotal role for the analysis. It would enable us to derive *the dual equation* between resource allocation and income distribution. The dual equation, while taking a surprisingly simple form, succeeds in extracting the essential features of complex reproduction processes among industrial sectors. Furthermore, the net economic resource per unit of labor makes it possible to correctly and comprehensively measure the results of technological development and increased productivity.

This article still remains as a tentative examination of the research theme described above in the sense that it does not mention similarities to and differences from recent economic schools. However, the position the article occupies in the "classical" school in its broad sense, from Smith, Ricardo, Malthus, Marx, to Sraffa and Pasinetti, is clear. And its approach is also very "modern" in that it proposes a simple method for macroscopically measuring the ultimate results of technological innovation. It also substitutes Keynes's national income model with a new model without relying on the propensity to save as a crucial element of analysis. In addition, bridging the two systems of quantity and price by means of von Neumann's duality provides a new answer to some post-Keynesian issues. Therefore, aside from trivial similarities to or differences from recent economic schools, the fundamental contrast with them seems to be obvious.

I. The quantity and price systems of the modified von Neumann type

Von Neumann's paper "A model of general economic equilibrium", first reported in a mathematics seminar at Princeton University in the winter of 1932 and first published in German in 1938, had the following two characteristics (Neumann [1938] p.1.)[1]:

(1) Goods are produced not only from "natural factors of production", but in the first place from each other in the circular production processes.

(2) There may be more technically possible processes of production than goods. The problem is rather to establish which processes will actually be used and which not (being "unprofitable").

When dealing with them, von Neumann freely "idealises" the following conditions which he regards are "irrelevant" to the nature of the problems (*Ibid*. pp.2-3):

(a) It is possible that the number of production processes $m >$ the number of goods n.

(b) Constant returns to scale.

(c) The natural factors of production, including labor, can be expanded in unlimited quantities.

(d) Consumption of goods takes place only through the processes of production which include necessities of life consumed by workers and employees. In other words, it is assumed that all income in excess of necessities of life will be reinvested.
(e) Fixed capital goods are inputs and outputs at the same time in production processes. Fixed capital goods as outputs are treated as different fixed capital goods in different stages of depreciation
(f) Production spans a single production period and a long production period can be divided into a number of unit production periods.
(g) Joint production is allowed.

Under these assumptions, von Neumann's conclusion was that there was a "remarkable duality" between the price and quantity systems. In other words, the interest rate (or the profit rate) is identical with the growth rate, and the two are uniquely determined by the production processes that are technically possible and most profitable (*Ibid.* p. 8)[2)].

The reason why the model constructed in this paper is named "modified von Neumann" is that it shares the above feature (1) in particular. It is completely consistent with the basic structure of Leontief's input-output system (Leontief 1951) and Sraffa's, *Production of Commodities by Means of Commodities*, (Sraffa 1960). And the duality between the price system and the quantity system is also shared by both the original and modified von-Neumann models. However, some assumptions that von Neumann considered non-essential are replaced by different assumptions in the modified von Neumann model:

(a) The number of production processes is equal to that of goods produced.
(b) Constant returns to scale.
(c) Natural resources are secured without problems during the period of analysis. As for wage workers, there are a sufficient number of unemployed and thus, the necessary number of wage workers can also be secured without any problems during the time in response to capital accumulation (or investment) in industrial sectors. The number of employees is determined passively in response to the amount of capital accumulation (or investment), and the household sector that reproduces labor force does not expand autonomously seeking a certain growth rate.
(d) Household consumption of wage workers and capitalists takes place as part of production processes. The wage worker's consumption is not necessarily fixed to the minimum subsistent level and is set in general to its scalar multiple. The real wage rate is usually higher than the subsistent wage rate and thus, wage workers also may well save. Capitalists do not spend all their income on investments and thus, consume as well as workers do. In addition, the household sector is a normal household sector, neither accumulating capital, investing nor demanding a certain profit rate for producing labor force.
(e) Means of production is limited to fluid means of production.
(f) Production spans a single production period.
(g) Joint production is not allowed.

In the following sections, we would like to proceed our analysis using a four-sector model, which is the minimum version of the modified von Neumann model.

II. The basic four-sector quantity and price systems

Although being essential for setting a general framework of analysis, a multi-sector model is difficult to handle for producing concrete and specific results. We adopt a minimum four-sector model, i.e. three industrial sectors and a household sector, to acquire some important implications from it. The basic quantity system is as follows:

$\boldsymbol{Q} = (\boldsymbol{I} + \boldsymbol{G}') \boldsymbol{A}_c \boldsymbol{Q}$

in which $\boldsymbol{Q}$ is a column vector (4×1) of output, including labor force $\boldsymbol{E}$, $\boldsymbol{A}_c$ is a square matrix (4×4) of input coefficients per unit of output, $\boldsymbol{G}'$ is a diagonal matrix (4×4) of the balanced growth rate g with element (4, 4) being zero and $\boldsymbol{I}$ is a unit matrix (4×4) as follows:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ E \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} g & 0 & 0 & 0 \\ 0 & g & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right) \begin{bmatrix} 0 & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & 0 & 0 & c \\ l_1 & l_2 & l_3 & 0 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ E \end{bmatrix} \tag{1}$$

in which Sector I, II and III are the materials sector, the parts-and-components sector and the means-of-consumption sector, respectively. Sector I outputs materials into Sectors II and III, but not into Sector I itself. Sector II outputs parts and components into Sectors I, II and III. Sector III outputs means of consumption only into Sector IV, i.e. the household sector. These input-output relations assume that Sector I is the primary sector which abstracts materials from the nature only with the help of parts and components, and that Sector II is the secondary sector which processes materials and produces parts and components for all three industrial sectors. No materials input into Sector I itself clearly characterizes and distinguishes between Sectors I and II[3]. All in all, Sector III allows reproduction of human groups with its means of consumption; Sector II provides parts and components and thus, allows reproduction of means of consumption; and Sector I provides materials and allows reproduction of means of production themselves. Therefore, the social and physical reproduction system is logically completed with these industrial sectors, the minimum number of which is no more or no less than three.

The household sector IV does not seek to grow by itself and its homemaking labor l_4 is omitted for the sake of simplicity. The minimum quantity of means of consumption that would be necessary to reproduce one unit of labor force is assigned to be the quantity numeraire and measures Q_3 and c, i.e. the real consumption rate per unit of labor. Therefore, c is a scalar multiple of the quantity numeraire.

Here we need some more clarification about the precise meaning of the quantity numeraire that is made of the minimum subsistent amount of means of consumption, which serves also as the price numeraire as we see soon. In reality, though, people consume a variety of means of consumption and thus, the numeraire has to be a composite set of the *minimum subsistent* quantities of goods and services that are *physiologically and socially necessary* for reproducing one unit of labor force. Their basket enables us to correctly measure the real wage rate and the real consumption rate by its scalar multiples, respectively.

Note that the real consumption rate c includes those goods and services per unit of labor that are consumed by capitalists, landlords, rentiers and bureaucrats as well as workers. It is physiologically and socially necessary for maintaining and reproducing a society and all its members, whether being productive or unproductive. Workers are *socially reproduced* by the consumption of all the members in the society, c times as much as the subsistent quantities. The total output over total consumption turns out to be social savings, regardless of its allocation in the society among workers, capitalists and others. No distinction among social classes regarding quantities and composition of consumption goods and services may limit the explanatory power of our model and ask for a careful treatment when applied to an analysis of class societies[4]. It should be remembered that Keynes shares exactly the same limit and that von Neumann adopts the same framework of social consumption per unit of labor[5].

The basic four-sector quantity model is expressed in equations as follows:

$$Q_1 = (1+g)(a_{12}Q_2 + a_{13}Q_3)$$
$$Q_2 = (1+g)(a_{21}Q_1 + a_{22}Q_2 + a_{23}Q_3)$$
$$Q_3 = (1+g)cE$$
$$E = l_1Q_1 + l_2Q_2 + l_3Q_3 \tag{2}$$

We solve the quantity equations in terms of G, provided that $G = 1 + g$ and standardizing it by $Q_3 = 1$:

$$Q_1 = \frac{G^2a_{12}a_{23} + a_{13}G(1 - Ga_{22})}{1 - G^2a_{12}a_{21} - Ga_{22}} \tag{3}$$

$$Q_2 = \frac{G^2a_{13}a_{21} + Ga_{23}}{1 - G^2a_{12}a_{21} - Ga_{22}} \tag{4}$$

$$E = \left[\frac{G^2a_{12}a_{23} + a_{13}G(1 - Ga_{22})}{1 - G^2a_{12}a_{21} - Ga_{22}}\right] l_1 + \left[\frac{G^2a_{13}a_{21} + Ga_{23}}{1 - G^2a_{12}a_{21} - Ga_{22}}\right] l_2 + l_3 \tag{5}$$

$$c = \frac{1}{\left[\frac{G^2a_{12}a_{23} + a_{13}G(1 - Ga_{22})}{1 - G^2a_{12}a_{21} - Ga_{22}}\right] Gl_1 + \left[\frac{G^2a_{13}a_{21} + Ga_{23}}{1 - G^2a_{12}a_{21} - Ga_{22}}\right] Gl_2 + Gl_3} \tag{6}$$

Next, our four-sector price system is as follows:

$\boldsymbol{P} = \boldsymbol{P}\boldsymbol{A_c}(\boldsymbol{I} + \boldsymbol{R'})$

in which $\boldsymbol{P}$ is a row vector (1×4) of prices with $P_3 = 1$ as the price numeraire and $P_4 = w$ as the real wage rate, $\boldsymbol{R'}$ is a diagonal matrix (4×4) of the unified profit rate r with element $(4, 4)$ being s, $\boldsymbol{A_c}$ is a square matrix (4×4) of input coefficients per unit of output and $\boldsymbol{I}$ is a unit matrix (4×4) as follows:

$$[P_1\ \ P_2\ \ 1\ \ w] = [P_1\ \ P_2\ \ 1\ \ w]\begin{bmatrix} 0 & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & 0 & 0 & c \\ l_1 & l_2 & l_3 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} r & 0 & 0 & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & s \end{bmatrix}\right) \tag{7}$$

The basic four-sector price model is expressed in equations as follows:

$$P_1 = (1+r)(a_{21}P_2 + l_1 w)$$
$$P_2 = (1+r)(a_{12}P_1 + a_{22}P_2 + l_2 w)$$
$$1 = (1+r)(a_{13}P_1 + a_{23}P_2 + l_3 w)$$
$$w = c(1+s) \tag{8}$$

in which the real wage rate w is expressed by a scalar multiple of the price numeraire (i.e. $P_3 = 1$). We need to discuss about s. Rewrite equation w above, and we acquire the following equation:

$$c = \left(\frac{1}{1+s}\right)w \tag{9}$$

The coefficient $1/(1 + s)$ looks as if being the propensity to consume of workers if c were the real consumption rate of workers only, although, if fact, c includes consumption of capitalists, state bureaucrats and others as well. As we later know, s takes a negative value and thus, $w < c$ holds. We would rather leave its precise definition and interpretation later when we introduce consumption of capitalists and others that is separated from c.

The price system is solved in terms of R, provided that $R = 1 + r$:

$$P_1 = \frac{(1-Ra_{22})l_1 + Ra_{21}l_2}{(1-R^2a_{12}a_{21}-Ra_{22})\left\{\left[\frac{R^2a_{12}a_{23}+a_{13}R(1-Ra_{22})}{1-R^2a_{12}a_{21}-Ra_{22}}\right]l_1 + \left[\frac{R^2a_{13}a_{21}+Ra_{23}}{1-R^2a_{12}a_{21}-Ra_{22}}\right]l_2 + l_3\right\}} \tag{10}$$

$$P_2 = \frac{Ra_{12}l_1 + l_2}{(1-R^2a_{12}a_{21}-Ra_{22})\left\{\left[\frac{R^2a_{12}a_{23}+a_{13}R(1-Ra_{22})}{1-R^2a_{12}a_{21}-Ra_{22}}\right]l_1 + \left[\frac{R^2a_{13}a_{21}+Ra_{23}}{1-R^2a_{12}a_{21}-Ra_{22}}\right]l_2 + l_3\right\}} \tag{11}$$

$$w = \frac{1}{\left[\frac{R^2a_{12}a_{23}+a_{13}R(1-Ra_{22})}{1-R^2a_{12}a_{21}-Ra_{22}}\right]Rl_1 + \left[\frac{R^2a_{13}a_{21}+Ra_{23}}{1-R^2a_{12}a_{21}-Ra_{22}}\right]Rl_2 + Rl_3} \tag{12}$$

Comparing equations c and w above, we see that the “remarkable duality” (von Neumann, [1938] p.8) is established in our modified von Neumann model as perfectly as in the original von Neumann model, although neither $R = G$ nor $w = c$ does hold as in the latter. The equality between w and c is *a priori* established in the von Neumann model, in which the wage rate is set at its subsistent level and capitalists consume nothing. This guarantees the complete duality between the two systems. Furthermore, $w = c = 1$ guarantees *a priori* the identity between the quantity numeraire and the price numeraire. By contrast, in our modified von Neumann model, only if the identity between the quantity numeraire and the price numeraire is intentionally and strictly established, the duality is guaranteed even if the real wage rate is over the subsistent level and capitalists and others also consume.

A condition for the quantity system to be sustainable is $0 < 1 - G^2 a_{12} a_{21} - G a_{22}$; and the equivalent condition for the price system is $0 < 1 - R^2 a_{12} a_{21} - R a_{22}$. The physical productivity condition is $0 < 1 - a_{12} a_{21} - a_{22}$, which means that production of one unit of parts and components must require directly and indirectly less than one unit of parts and components. Since $1 < 1 + r = R$ and $1 < 1 + g = G$, these three inequalities suggest that the sustainability conditions of a growing capitalist economy are much severer than the simple productivity condition of an economy without profit or growth. As we see in equations c and w above, G and R that satisfy $0 = 1 - G^2 a_{12} a_{21} - G a_{22}$ and $0 = 1 - R^2 a_{12} a_{21} - R a_{22}$, respectively are the maximum G with zero real consumption rate and the maximum R with zero real wage rate and thus, we know that $0 < 1 - G^2 a_{12} a_{21} - G a_{22}$ and $0 < 1 - R^2 a_{12} a_{21} - R a_{22}$ hold.

III. The net economic resource

From equations Q_3 and E in the quantity system above, we acquire the following equation:

$$Q_3 = GcE \tag{13}$$

$$\frac{Q_3}{E} = Gc \tag{14}$$

First, the left side of equation (14) denotes the amount of means of consumption produced by one unit of labor. Since the total labor E includes not only the labor input in the means-of-consumption sector but also that of the raw materials sector and the parts-and-components sector, the left side expresses *social average productivity of labor* of the entire economy in terms of means of consumption.

Labor productivity has to be physical productivity in its strict engineering sense: i.e. a certain amount of product produced per unit of labor. Therefore, it is an index that is measured only in each industrial sector and cannot measure social average in general. However, Q_3/E is an indicator that clears the contradiction of being both *physical* productivity and *social average* productivity at the same time.

It is because Q_3 denotes the quantity of a single means of consumption and thus, Q_3/E is physical productivity. In the case of the quantity numeraire consisting of a basket of means of consumption, Q_3/E denotes the number of the baskets produced per unit of labor, a different expression of physical productivity.

Physical labor productivity in its strict sense is determined by dividing output by working hours of workers directly employed in the sector. Since, however, all industrial sectors in one way or another output their intermediate products into the sector, its labor productivity can be measured more precisely by dividing its output by the *total* amount of labor, directly and indirectly needed. But this is not the social average productivity of labor of *all* sectors.

The social average productivity of labor of all sectors is obtained by dividing the *total* output of means of consumption by the *total* labor input of a country. But why does it represent the average labor productivity of *all* sectors? Is it not just a kind of indicator of the average labor productivity for producing means of consumption?

Let us understand this problem tentatively as follows for a while. An economy consists of such a variety of industries as agriculture, mining, steel, automobile, machinery, education, commerce, entertainment, etc. They are complicatedly intertwined with each other in the input-output relations and they invest, accumulate capital and expand their scale over time. Nevertheless, the ultimate purpose of economic activities exists in efficient and rich provision of means of consumption to its members, instead of growing quickly or further complicating roundabout production. Therefore, the ultimate indicator of productivity of a country as a whole is the *total* output of means of consumption divided by the *total* labor input in the country. We will later examine in detail that it is correctly functioning as the indicator of social average productivity.

Second, the left side of equation (14) also represents the upper limit of means of consumption that could be consumed per unit of labor. That is, the upper limit is $c = Q_3 / E$ when $G = 1$ and the growth rate r is zero. However in fact, as shown on the right side, Q_3 / E is divided into the real balanced growth rate G (> 1), which is common to all sectors, and the real social consumption rate c per unit of labor.

Third, if we simply interpret the left side as the output of means of consumption per unit of labor, and the right side as means of consumption consumed per unit of labor multiplied by the growth rate G, this formula turns out to be a simple definition formula: i.e. means of consumption produced in the third sector are allocated for consumption in the current period and investment for the next period. However, that is not the case. The meaning of the equation dramatically changes because G is the real balanced growth rate common to *all* sectors and c is the social real consumption rate common to *all* members of the society.

The right side Gc of equation (14) can be rewritten as $Gc = (1 + g)\,c = c + gc$: i.e. the sum of consumption per unit of labor and the growth of that consumption, net of intermediate inputs. Therefore, Q_3 / E can be interpreted as the net economic resource per unit of labor that can be devoted this period to consumption and growth in general, expressed specifically in terms of means of consumption. In other words, Q_3 / E reveals its double nature: i.e. being simply the output of means of consumption per unit of labor and simultaneously being a general indicator of available economic resources in a country per unit of labor. Let us call Q_3 / E with such double nature *the net economic resource* per unit of labor.

Some more characteristics of the net economic resource are revealed by differentiating equation (14) with respect to time:

$$\dot{Q}_3 - \dot{E} = \dot{G} + \dot{c} \tag{15}$$

The sum of the rate of change in the growth rate and that in the consumption rate on the right side is equal to the difference between the rate of change in output of means of consumption and that in total labor input on the left side. Thus, an increase in the growth rate and/or the consumption rate can be achieved by an increase in total output of means of consumption exceeding a change in total labor input. It is suggested that the *entire* fruits of productivity improvements in the whole economy are reflected in an increase in $\dot{Q}_3 - \dot{E}$, i.e. an increase in the country's net economic resource, and are *fully* allocated between growth and consumption

> The concept of the net economic resource may seem to be the same as or similar to net value added per capita (i.e. net national income per capita) in the Keynesian national income theory. However, while net value added is a nominal value aggregated in monetary units, the net economic resource is measured in terms of real and absolute physical units of means of consumption. In that sense, both are seemingly very similar but completely different concepts. However, it will be made clear later that the concept of net value added is included in the concept of the net economic resource, and not *vice versa*.

The net economic resource per unit of labor can be measured across time, space and modes of production and directly compared with each other. Let us examine the issue with the minimum consumption basket in an n-sector model, which is a generalization of our 4-sector model. The "baskets", whose contents vary in terms of quantity, quality, and composition due to differences in time, space and modes of production, are regarded as the equivalent price and quantity numeraires with each other. This is because, even if the physical contents of the "baskets" change, their economic function of reproducing one unit of labor force with minimum means of consumption, is maintained. Therefore, even if the price changes or the currency unit is different, we can measure and compare each net economic resource with the equivalent numeraire. This is also applicable when we measure and compare the real wage rate w and the real consumption rate c across time, space and modes of production[6]. The concept of the net economic resource as well as that of numeraire is one of the basic concepts of general economics.

Now we know that equation Q_3 is a unique formula that expresses production, consumption and resource allocation in an integrated manner, a feature not found in equations Q_1 or Q_2. We summarize it as follows:

The net economic resource per unit of labor = Social average productivity of labor
= The maximum consumption rate per unit of labor
= Resource allocation (growth × consumption)

Here, we look into some classical economists' long-term view of a capitalist economy, using our concept of the net economic resource. This scrutiny enables us to confirm the validity of the concept in the context of history of economic thought.

Malthus's claim in his famous population theory was as follows if put in a nutshell (Malthus [1798]). To begin with, Q_3 is read as foods output, and E as population. While foods output Q_3 increases only at an arithmetic rate, population E increases at a geometric rate. Therefore, it is unavoidable that sooner or later, the natural rate of increase in E overtakes the rate of increase in Q_3, causing a decrease in the consumption rate c, given a constant growth rate G. The only way to avoid the consequence is to artificially control the birth rate of the population. This is a conclusion that stems from, in his belief, the natural and universal law of population, focusing on one aspect of the double nature of Q_3 / E as means of consumption (foods).

Ricardo [1817], on the other hand, viewed Q_3 as agricultural output and believed that its rate of increase was constrained by the law of diminishing returns. Therefore, also in the case of Ricardo, the rate of increase in Q_3 would eventually drop to the natural rate of increase in population E, the growth rate G would fall and the whole economy would reach a stationary state. Since Ricardo thought that the real wage rate and the consumption rate of workers would stay at the subsistent level in the long term, with the help of Malthus's population theory, he focused on the problem of the declining growth rate, namely stagnant capital accumulation.

Smith [1776] contrasts sharply with Malthus and Ricardo. While the long-term view they drew was extremely dismal, the world depicted by Smith on the eve of the Industrial Revolution was very bright. For Smith, Q_3 denotes the total output, including not only the output of agriculture but also that of rising manufacturing industries. They both enormously improve productivity by applying his "division of labor". As a result, an economy grows and per capita consumption increases. And as population E grows, the country will become increasingly powerful. Smith's unshakable confidence in productive power of capitalism made a fundamental difference in his argument from Malthus and Ricardo.

Finally, Marx focused on the tragedy of class conflict caused by massive development of productive power in capitalism. Marx denied the natural and universal law of population that Malthus advocated. Instead, he replaced it with the capitalist law of population, i.e. the law of relative surplus population. Capitalism strongly promotes capital accumulation (i.e. higher G) through the compulsory law of competition and urges technological progress (i.e. higher Q_3/E). In the process of long-term development, labor force E increases during booming periods but decreases during the time of recessions and depressions, creating periodical cycles of unemployment. And in the long term, chronic industrial reserve army lingers at the bottom of society due to ever increasing organic composition of capital (i.e. capital intensity). Naturally, the consumption rate of wage workers remains almost at the subsistent level. For Marx, overpopulation does not occur because of the high natural fertility rate, but rather because population is excessive relative to capitalists' willingness to accumulate capital. In this way, the serious class conflict erodes capitalism, caused by massive accumulation of wealth and consumption of luxury goods due to the rise in G, Q_3 and Q_3/E on the one hand, and the accumulation of poverty and subsistent w and c among working people on the other hand due to stagnant E:

> "It follows therefore that in proportion as capital accumulates, the situation of the worker, be his payment high or low, must grow worse. Finally, the law which always holds the relative surplus population or industrial reserve army in equilibrium with the extent and energy of accumulation rivets the worker to capital more firmly than the wedges of Hephaestus held Prometheus to the rock. It makes an accumulation of misery a necessary condition, corresponding to the accumulation of wealth. Accumulation of wealth at one pole is, therefore, at the same time accumulation of misery, the torment of labor, slavery, ignorance, brutalization and moral degradation at the opposite pole, i.e. on the side of the class that produces its

own product as capital." (Marx [1867] p.799.)

IV. Social average productivity of labor

Here again, let us examine in more detail the concept of social average productivity of labor. From equations Q_3 and E in (2),

$$1 = Gc\left(\frac{l_1 Q_1}{Q_3} + \frac{l_2 Q_2}{Q_3} + l_3\right) \tag{16}$$

Substituting equation (14) here gives the following equation:

$$\frac{1}{\frac{Q_3}{E}} = \frac{l_1 Q_1}{Q_3} + \frac{l_2 Q_2}{Q_3} + l_3 \tag{17}$$

The left side of equation (17) is the reciprocal of social average productivity of labor (i.e. the net economic resource), meaning the total amount of labor that is socially required to produce one unit of means of consumption; therefore, an improvement in productivity decreases the left side. On the other hand, the right side shows the detailed composition of the socially required total labor. Equation (17), which is also a simple identity derived directly from equation (2), allows the following two interpretations for its right side:

Output of means of consumption Q_3, namely the ultimate purpose of the society's economic activities, requires output of materials Q_1, directly through $a_{13}Q_3$ or indirectly through $a_{12}Q_2$. The direct labor input into the production of Q_1 is l_1Q_1; and thus, the amount of direct labor for materials required to produce one unit of means of consumption is l_1Q_1 / Q_3. Similarly, the amount of direct labor for parts and components required to produce one unit of means of consumption is l_2Q_2 / Q_3, and obviously that for means of consumption is l_3. Therefore, the right side of equation (17) represents the total amount of direct labor required in a society as a whole to produce one unit of means of consumption. It follows that the only way to raise social average productivity of labor (i.e. the net economic resource) is to reduce direct labor input of each sector. This is the first interpretation.

On the other hand, the second interpretation comes from rewriting equation (17) as follows:

$$\frac{1}{\frac{Q_3}{E}} = l_1 \frac{Q_1}{Q_3} + l_2 \frac{Q_2}{Q_3} + l_3 \tag{18}$$

Q_1 / Q_3 in the equation is the ratio of raw materials output to means-of-consumption output, i.e. the output composition ratio of both sectors. However, it also represents the average ratio of raw materials / labor (i.e. raw materials intensity) of a country in a slightly different sense. Q_1 / Q_3 can be expanded as follows:

$$\frac{Q_1}{Q_3}=\frac{G(a_{12}Q_2+a_{13}Q_3)}{GcE}=\frac{1}{c}\left(\frac{a_{12}Q_2+a_{13}Q_3}{E}\right) \tag{19}$$

The sum $a_{12}Q_2 + a_{13}Q_3$ appearing in the numerator denotes the total input of raw materials in the economy. Therefore, dividing it by the total labor input E produces the average ratio of raw materials / labor (i.e. raw materials intensity) of the country in the usual sense. Similarly, Q_2 / Q_3 can be expanded as follows:

$$\frac{Q_2}{Q_3}=\frac{G(a_{21}Q_1+a_{22}Q_2+a_{23}Q_3)}{GcE}=\frac{1}{c}\left(\frac{a_{21}Q_1+a_{22}Q_2+a_{23}Q_3}{E}\right) \tag{20}$$

Q_2 / Q_3 is the ratio of parts-and-components output to means-of-consumption output, i.e. the composition ratio of both sectors. The parenthesis in the right side denotes the average ratio of parts-and-components / labor (i.e. parts-and-components intensity) in the country in the usual sense. This leads to the following two propositions:

The first one is, as revealed in the first interpretation above, the only way to increase social average productivity of labor (i.e. the net economic resource) is to reduce direct labor input in each sector, which is weighted in accordance with the composition ratio of each sector to the means-of-consumption sector. Therefore, an improvement in social average productivity of labor becomes greater as the amount of direct labor is reduced in a sector with a higher composition ratio. It should be noted that the size of direct labor inputs l_1, l_2, and l_3 has nothing to do with the sector composition[7].

Another proposition is that an increase in raw materials intensity and parts-and-components intensity *per se* reduces social average productivity of labor. Unless the increase in intensity replaces and reduces direct labor input in any sector, it worsens social average productivity of labor. This is especially the case for manufacturing industries in general. On the other hand, even if the raw materials and/or parts-and-components intensities decrease, social average productivity of labor may worsen if direct labor input increases. This is the case for the services industries in general.

The second proposition is subject to the important condition: raw materials intensity Q_1 / Q_3 and parts-and-components intensity Q_2 / Q_3 are both divided by the consumption rate c in equations (19) and (20). This suggests that an economy with a high consumption rate has higher social average productivity of labor and richer net economic resource than an economy with a low consumption rate, even if the input-output coefficients have not changed at all and thus, the technical conditions remain unchanged. It is a problem that in addition to technical conditions resource allocation is also involved in, instead of being separated from, measurement of productivity. However, in fact, this is an advantage rather than a flaw in the concept of social average productivity of labor.

The question is not about why an economy with a higher consumption rate c is richer in the net economic resource, but on the contrary, why an economy with a higher growth rate G, which is in an inverse relation to c, is poorer in the net economic resource. Differentiating Q_1 in equation (3) and Q_2 in equation (4) with respect to G would reveal a larger G shifts the industrial composition in the direction of $Q_3 < Q_1 < Q_2$. In other words, an economy with a higher growth rate G has to devote more resources to the production of raw materials and parts and components and thus, the means-of-consumption sector shrinks accordingly, given the unchanged technological level. It is an economy in which the people should be satisfied with a smaller consumption rate c. Such an economy cannot be regarded as rich in the net economic resource.

From equation (6), Gc can be expressed as follows:

$$Gc = \frac{1}{\left[\frac{G^2 a_{12} a_{23} + a_{13} G(1 - G a_{22})}{1 - G^2 a_{12} a_{21} - G a_{22}}\right] l_1 + \left[\frac{G^2 a_{13} a_{21} + G a_{23}}{1 - G^2 a_{12} a_{21} - G a_{22}}\right] l_2 + l_3} \tag{21}$$

The denominator on the right side is an increasing function of G, suggesting that, despite an increase in G both on the right and left sides, the net economic resource Gc decreases due to a sharp decrease in c. In other words, even a slight increase in G causes a large reduction in c. This seems to reflect the reality of a country in the early stage of high economic growth, as seen in Japan in the 1950s and 1960s, and in today's leading emerging economies such as China, in which a high growth rate would be compensated by a precipitous cut in consumption unless achieving significant improvement in labor productivity.

Therefore, incorporation of resource allocation into the determinant of social average productivity of labor and the net economic resource is rather an advantage as a theoretical model, in which abundant and efficient supply of means of consumption is treated as the correct indicator of high productivity and richness of economic resources.

V. The dual equation

Equation (14), which expresses social average productivity of labor and the product of the growth rate and the consumption rate, holds *for any combination* of G and c under the conditions $1 \leq G$ and $0 \leq c$. Note that the left side is not constant and varies depending on the combination of G and c. The G-c curve expressed in equation (6) and the R-w curve expressed in equation (12) are completely in duality. Therefore, the following equation is established, and we will call it *the dual equation*:

$$\frac{Q_3}{E} = Gc = Rw \tag{22}$$

When Q_3 / E is given by the quantity system in the short term in which the input coefficients are stable, the profit rate R and the real wage rate w are in a simple inverse relation[8]. Adding equation (22) to the price system closes and solves it, in which the profit rate R, the real wage rate w, and the prices P_1, P_2 are all uniquely determined[9].

Let us further examine the duality by inserting $G = 1 + g$ and $R = 1 + r$ into the dual equation above. First, the quantity equation is as follows:

$$\frac{Q_3}{E} = c + gc \tag{23}$$

It can be read as the price equation as well if we understand one unit of minimum means of consumption as the price numeraire instead of the quantity numeraire. The reason why such replacement is possible is that the left side and the right side are both expressed in terms of means of consumption.

By the way, in the real economy, is there an industrial sector whose product is means of consumption and means of production at the same time? Namely, it is an industrial sector in which "the product = means of consumption = means of production" is established in terms of use value. If there were an industry that could fit this condition, it should be the most primitive and fictitious food-gathering "industry", in which the bounty of nature is collected only by human labor without using machines or tools. And most of the collected foods is consumed directly, and the rest is accumulated and invested to increase the amount of labor for the next food-gathering work. If the minimum amount of foods required to reproduce one unit of labor is assigned to the quantity numeraire, we acquire the quantity equation above, and if it is assigned to the price numeraire, the same formula turns into the price equation. In this world, labor, products, accumulation and consumption, and profit and wages as we see soon, are all measured in terms of gathered foods.

Next, let us examine the price equation:

$$\frac{Q_3}{E} = w + rw \tag{24}$$

The right side of the equation can be interpreted as the value (i.e. price multiplied by output) of foods collected with only one unit of labor without using machines or tools. Since being divided into wages w and profits rw, it is equal to value added. However, if replacing the price numeraire with the quantity numeraire, both the left and right sides will be converted to the quantity equation.

By combining these two equations, an actual complex economy composed of input-output relations among multiple sectors can be reduced to a simple economy, in which a single means of consumption is produced only by human labor, and it is thoroughly distributed between wages and profits and allocated between consumption and growth. As a result, the inverse relations between the profit rate and the wage rate and between the growth rate and the consumption rate, given a certain level of social average productivity of labor, can be revealed simply and clearly.

The dual equation includes Keynes's national income theory as modified as follows:

$$c + gc = w + rw \tag{25}$$

The left side of the equation represents national income expended per unit of labor (i.e. consumption + investment), and the right side represents national income distributed (i.e. wages + profits), indicating that they are identical. However, although both the Keynesian system and the dual system look very similar in form, their theoretical contents are very different:

First, our dual equation expresses quantitative relations as well as real price relations and is a system in which both are united. In this respect, it differs from the Keynesian national income system which indispensably presumes the existence of exogenous prices, whether nominal or real[10].

Second, the Keynesian system is a gigantic one-sector model that completely abstracts input / output relations of intermediate goods and services, while by contrast our dual system was born from reducing complex input / output relations into a simple duality. Therefore, the dual system encompasses the Keynesian national income system, not *vice versa*.

VI. Resurrection of Ricardo's old corn-ratio theory

Our method of reduction into a primitive economic society has a striking resemblance to early Ricardo's old corn-ratio theory. It was an approach adopted before Ricardo later established the labor-embodied value theory, in which he attempted to determine income distribution between profits and wages by means of physical corn-ratio in agricultural production. Let us confirm its contents in Sraffa (1951):

"The rational foundation of the principle of the determining role of the profits of agriculture, which is never explicitly stated by Ricardo, is that in agriculture the same commodity, namely corn, forms both the capital (conceived as composed of the subsistence necessary for workers) and the product; so that the determination of profit by the difference between total product and capital advanced, and also the determination of the ratio of this profit to the capital, is done directly between quantities of corn without any question of valuation. It is obvious that only one trade can be in the special position of not employing the products of other trades while all the others must employ *its* product as capital. It follows that if there is to be a uniform rate of profit in all trades it is the exchangeable values of the products of *other* trades relatively to their own capitals (*i.e.* relatively to corn) that must be adjusted so as to yield the same rate of profit as has been established in the growing of corn; since in the latter no value changes can alter the ratio of product to capital, both consisting of the same commodity." (Sraffa 1951 "Introduction" p.xxxi.)

Sraffa evaluates the approach above as follows:

"The advantage of Ricardo's method of approach is that, at the cost of considerable simplification, it makes possible an understanding of how the rate of profit is determined without the need of a method for reducing to a common standard a heterogeneous collection of commodities." (*Ibid.* p.xxxii.)

In short, the significance of the old corn-ratio theory is that it provides a framework that makes it possible to discuss income distribution separately from value theory (*Ibid.* p.xxxiii). Ricardo completely abandoned it later when he established the labor-embodied value theory. However, it emerged again in a different fashion when he found "the curious effect which the rise of wages produces on the prices of those commodities which are chiefly obtained by the aid of machinery and fixed capital" (the letter to J. Mill on 14 October 1816 in Ricardo

(1952a p.82.). See Sraffa (1951) p.xlix) and attempted to overcome the problem with “an invariable measure of value”. It is a well-known fact in the history of economics that Sraffa, starting here, later constructed “the composite standard commodity”, i.e. “a method for reducing to a common standard a heterogeneous collection of commodities”.

A supplementary comment might be needed for Ricardo’s corn production sector: inputs into the sector have to include corn seeds as well as labor force. Therefore, capitalists demand profit for seeds that they invest as well as for wages, although the corn-ratio theory is still valid since seeds are also the same kind of corn. It should be noted, however, that Ricardo’s corn-ratio theory consists only of the price system without the quantity system.

In contrast, our dual system models the primitive food-gathering industry, not the corn industry, in which labor force is the unique input. In addition to the price system, it is also equipped with the quantity system, making it possible to analyze income distribution between profits and wages and resource allocation between growth and consumption simultaneously. Moreover, our system is substantially different from the corn-ratio model, assuming input of various means of production into the means-of-consumption sector.

In a nutshell, the dual equation can be viewed as modern economic “resurrection” of Ricardo’s old corn-ratio theory, in which the complex structure of an actual economy is reduced to a simple “food-gathering economy” and as a result, the inverse relations between the profit rate and the wage rate and between the growth rate and the consumption rate can be analyzed simply and clearly, given a certain level of social average productivity of labor.

We here have to examine Pasinetti’s “vertically integrated sector” in connection with the method of reducing the complex structure of an actual economy to a simple theoretical system (Pasinetti 1973 and 1988). He constructs a quantity system as follows (1988 p.126):

$$\boldsymbol{AX} + g\boldsymbol{AX} + \boldsymbol{C} = \boldsymbol{BX}$$
$$\boldsymbol{aX} = L$$
$$\boldsymbol{AX} = \boldsymbol{S}$$
$$(26)$$

in which $\boldsymbol{A}$ and $\boldsymbol{B}$ are the input and output coefficients matrices respectively: $\boldsymbol{a}$ is the direct labor input coefficient vector; $\boldsymbol{X}$ is the output vector; $\boldsymbol{C}$ is the means of consumption vector; $\boldsymbol{S}$ is the means of production vector; L is the total labor force (scalar); and g is the rate of growth of the labor force.

The following equation system can be obtained from the above for means of consumption i (C_i, i = 1, 2, ..., m) (*Ibid*. p. 127).

$$\boldsymbol{X}_i = [\boldsymbol{B} - (1 + g + r_i)\boldsymbol{A}]^{-1}\boldsymbol{C}_i$$
$$L_i = \boldsymbol{a}[\boldsymbol{B} - (1 + g + r_i)\boldsymbol{A}]^{-1}\boldsymbol{C}_i$$
$$\boldsymbol{S}_i = \boldsymbol{A}[\boldsymbol{B} - (1 + g + r_i)\boldsymbol{A}]^{-1}\boldsymbol{C}_i$$
$$\sum\boldsymbol{C}_i = \boldsymbol{C}, \Sigma\boldsymbol{X}_i = \boldsymbol{X}, \Sigma\boldsymbol{S}_i = \boldsymbol{S}, \Sigma L_i = L$$
$$i = 1, 2, \dots, m$$

(27)

in which $\boldsymbol{C}_i$, $\boldsymbol{X}_i$ and $\boldsymbol{S}_i$ are column vectors, the components of which are all zeros except the ith ones, which are the scalars C_i, X_i and S_i, i.e. the ith component of vectors $\boldsymbol{C}$, $\boldsymbol{X}$ and $\boldsymbol{S}$, respectively; L_i (scalar) is the labor force of sector i; and r_i, being positive or negative, is defined as the per capita rate of growth of consumption demand for each commodity i, therefore the rate of growth of consumption demand for each commodity i will be $(g + r_i)$, and in general will be different from one consumption good to another (i.e. $r_i \neq r_j$).

The vector $\boldsymbol{a}[\boldsymbol{B} - (1 + g + r_i)\boldsymbol{A}]^{-1}$ is called the vertically hyper-integrated labor coefficients for commodities 1, 2, …, m. And each row of the matrix $\boldsymbol{A}[\boldsymbol{B} - (1 + g + r_i)\boldsymbol{A}]^{-1}$ is called a unit of vertically hyper-integrated productive capacity for commodities 1, 2, …, m. They represent directly, indirectly and "hyper-indirectly" required labor force and means of production, respectively, for producing one unit of consumption goods 1, 2, …, m.

The remarkable feature of the Pasinetti model is that the quantity system that is complicated in mutual input-output relations is reduced to the two factors, i.e. the vertically integrated labor coefficients and vertically integrated productive capacity. Here, intermediate inputs are completely removed. Therefore, it is said that the effects of technological changes can be comprehensively measured as changes in these two factors.

As is evident from the above, Pasinetti's vertical integration model is basically a Leontief model, and the vertically integrated labor coefficients and vertically integrated productive capacity are variants of the Leontief inverse matrix. Although, in the case of Leontief, both investment and consumption are given exogenously, in the case of Pasinetti, only consumption is exogenously given and investment is treated as endogenous.

There are a few problems in his model. The first one lies in exogenous consumption: consumption of workers is given as the product of the consumption rate c and employment E, both of which ought to be endogenously determined variables in the quantity system. As discussed later, what can be treated as an exogenous variable is limited to consumption of non-working classes such as capitalists. Another problem is that the growth rate g is defined as that of labor force, rather than of output as usual. When capital intensity per unit of labor changes and so does labor productivity, the two growth rates are likely to diverge from each other. From these features, Pasinetti's vertically integrated system does not allow us to properly grasp the duality between the R-w curve and the G-c curve.

VII. Separation of consumption of non-working classes

The consumption rate c includes not only consumption of workers, but also that of capitalists, landowners and other non-working classes and the cost of maintaining a capitalist society and state, such as bureaucrats, police and military, all per unit of labor. Therefore, a more accurate description and analysis of a capitalist society requires to separate between consumption of workers paid from wages and other types of consumption.

In the case of consumption of workers paid from wages, there is a sufficient practical basis to assume that it is an increasing function of the real wage. Although their propensity to consume may not be so stable as the macroeconomic theory assumes, consumption undoubtedly increases as the real wage increases. On the other hand, the maintenance cost of the state apparatus can be considered to take a fixed value independent of output. In addition, consumption of landowners paid from rents is likely to be more or less fixed. Then, can we reasonably assume that consumption and savings of the capitalist class to be proportional to profits?

A significant portion of savings of the capitalist class is now accumulated within companies as retained profits. Therefore, savings are far from “residual” of profits minus capitalist consumption, the amount of which is basically determined by corporate investment demand. And, from the perspective of capitalist consumption, it seems to be fairly constant in order to maintain their social status, instead of increasing or decreasing in proportion to the amount of profits.

The consideration above will lead us to the assumption, as the first approach to the problem, that consumption of non-working classes and social maintenance costs are exogenously given and independent of the level of profits.

Let us start with von Neumann’s system with an extreme assumption in the opposite sense, in which workers’ wages are fixed at their subsistent level, capitalists consume nothing and all profits are invested. Therefore, applied to our modified von Neumann model, c is only workers’ consumption and is equal to w, which is then equal to unity. Substituting them into the dual equation $Gc = Rw$, $G = R$ holds. The growth rate is identical with the profit rate and takes its maximum value. This is the world of von Neumann.

Next, assuming that consumption of capitalists and other non-working classes is independent of profits and constant C, the quantity system is revised as follows, in which the consumption rate c_w in equation Q_3 denotes the consumption rate of workers only:

$$Q_1 = G(a_{12}Q_2 + a_{13}Q_3)$$
$$Q_2 = G(a_{21}Q_1 + a_{22}Q_2 + a_{23}Q_3)$$
$$Q_3 = Gc_wE + C$$
$$E = l_1Q_1 + l_2Q_2 + l_3Q_3 \tag{28}$$

When solved by standardizing Q_3 as unity, Q_1, Q_2 and E do not change at all, and only the real consumption rate changes as follows:

$$c_w = \frac{1 - C}{\left[\frac{G^2a_{12}a_{23} + a_{13}G(1 - Ga_{22})}{1 - G^2a_{12}a_{21} - Ga_{22}}\right]Gl_1 + \left[\frac{G^2a_{13}a_{21} + Ga_{23}}{1 - G^2a_{12}a_{21} - Ga_{22}}\right]Gl_2 + Gl_3} \tag{29}$$

It is identical with the previous equation c simply multiplied by $1 - C$. Since $Q_3 = 1$, C denotes the ratio of capitalist consumption to the total output of means of consumption. We now understand that consumption of capitalists, etc. is, given constant G, compensated by a decline in the consumption rate of workers without affecting the industrial structure and employment at all. Given the consumption rate of workers to be constant, G lowers and the industrial structure and employment change[11]. Consumption of capitalists and others is exactly unproductive consumption.

Malthus argues in Section IX "Of the distribution occasioned by unproductive consumers, considered as the means of increasing the exchangeable value of the whole produce", Chapter VII "On the immediate causes of the progress of wealth", *Principles of Political Economy*, as follows:

"The third main cause which tends to keep up and increase the value of produce by favouring its distribution is the employment of unproductive labour, or unproductive consumers. … It follows that, without supposing the productive classes to consume much more than they are found to do by experience, particularly when they are rapidly saving from revenue to add to their capitals, it is absolutely necessary that a country with great powers of production should possess a body of unproductive consumers." (Ricardo 1957 p.421.)

Malthus argues that the most important unproductive consumer class is landowners (*Ibid*. p. 424.), and mentions "the classes of unproductive labourers, which are supported by taxation". (*Ibid*. p.433.)[12]. And he argues that the general oversupply would occur if this condition were not met:

"the employment of a capital, too rapidly increased by parsimonious habits, may find a limit, and does, in fact, often find a limit, long before there is any real difficulty in procuring the means of subsistence; and that both capital and population may be at the same time, and for a period of great length, redundant, compared with the effective demand for produce." (*Ibid*. p.427.)

However, our quantity system shows that the claim of Malthus is wrong. Unproductive consumption of capitalists, landowners, bureaucrats, etc., given constant G, only leads to a decline in the rate of consumption of workers without affecting the industrial structure and employment. If given the constant consumption rate of workers, on the other hand, G will be lowered, which hinders capital accumulation and population growth.

However, the Malthusian causality may work under the following circumstances: the profit rate R is extremely high and thus, the real wage rate w is kept very low and so is the consumption rate of workers c_w; moreover, despite the high profit rate, capitalists are less willing to accumulate and the growth rate G is low. Under these circumstances, a large amount of unproductive consumption C may be indispensable to maintain economic balance. Increased government expenditure is another strong candidate. In addition, although not discussed in detail here, trade surplus may serve as another way to overcome the dire situation.

The following equation is now derived from equation $\boldsymbol{Q}_3$ (28):

$$\frac{Q_3 - C}{E} = G c_w \tag{30}$$

in which the left side denotes the net economic resource per unit of labor, net of capitalist consumption, and it is allocated to the growth rate and the consumption rate of workers on the right side. Therefore, unlike Malthus's claim, unproductive consumption leads to a reduction in the net economic resource in society.

This is a general formula for the net economic resource in a class society. It distinguishes between productive and non-productive classes, and clearly shows the conflicting relationship between them in the allocation of the net economic resource. It increases by increasing Q_3 for constant E, decreasing E for constant Q_3, or their combination. Additional net economic resource brought about by a reduction in employment rather than an increase in output gives workers the possibility of shortening their working hours and increasing leisure time while preserving their consumption level. However, in a capitalist society, a decline in E appears as an increase in unemployment, not an increase in workers' leisure time, in other words "forced leisure time", so the possibility remains only as an abstract possibility. However, an increase in the net economic resource per unit of labor will gradually and steadily expand the possibility of work-life balance in the future society, while embroiled in the contradiction of the accumulation of capital on the one hand and the accumulation of unemployment and poverty on the other.

Next, let us examine the price system, the partner of duality. The price system that includes consumption of capitalists, etc. is as follows, given the minimum necessary means of consumption as the price numeraire (i.e. $P3 = 1$):

$$P_1 = R(a_{21}P_2 + wl_1)$$
$$P_2 = R(a_{12}P_1 + a_{22}P_2 + wl_2)$$
$$1 = R(a_{13}P_1 + a_{23}P_2 + wl_3)$$
$$w = c_w(1 + s)$$

(31)

There is no change in the structure of the price system by introducing non-productive consumption of capitalists and others.

Here, we examine s: equation w is rewritten as $w = c_w + sc_w$ and thus, s can be regarded as a kind of workers' saving rate (i.e. workers' propensity to save) if we redefine s as a ratio to the consumption rate c_w, not as a ratio to the wage rate w as usual. We should recall that s is element $(n \times n)$ of the diagonal matrix $\boldsymbol{R}$' $(n \times n)$ of the unified profit rate in equation (7). Since the profit rate r means the income-claiming ratio of capitalists for their invested capital, so does s of workers for "outputting" one unit of labor force by "inputting" means of consumption c_w in workers' households.

The consumption function of workers is expressed in its simplest form $c_w = \beta_w w$, given exogenous β_w as workers' propensity to consume in Keynes or Kaldor (1955-56) that distinguishes between capitalists' consumption and workers' consumption, in which the right side determines the left side. In other words, it assumes that workers' households perform passive consumption activities, in which they receive wages as given and consume more with higher wages and less with lower wages, given a certain β_w.

By a sharp contrast, workers' households assumed in $w = c_w\,(1 + s)$ is completely different, in which the right side determines the left side. While enjoying consumption c_w that is a scalar multiple of the minimum necessary means of consumption, workers further demand the wage rate $1 + s$ times as much as c_w. This is a demand for partial cession of profits and for larger net economic resource. In other words, equation w assumes workers' households actively engaged in wage struggles, as are capitalists actively engaged in profit struggles.

Therefore, s is a variable that should be endogenously determined within the system, rather than a residual after consumption being given exogenously by socio-psychological factors. And thus, the four equations above in the price system maintain one degree of freedom even if the consumption rate of workers c_w is given exogenously from the quantity system.

VIII. Duality with consumption of non-working classes

Incorporating consumption of capitalists and others, our dual system is separated into two: i.e. the $R-w$ curve and the $G-c_w$ curve. These are expressed by the following two equations:

$$Gc_w = \frac{Q_3 - C}{E}$$

$$Rw = \frac{Q_3}{E} \tag{32}$$

The net economic resource Q_3 / E is divided into the profit rate R and the wage rate w regardless of capitalist consumption, while the net economic resource divided into the growth rate G and the consumption rate of workers c_w is $(Q_3 - C) / E$ after deducting capitalist consumption C.

From both equations, the following equation is obtained:

$$Gc_w = Rw - \frac{C}{E} \tag{33}$$

C / E denotes how much capitalist consumption must be borne per unit of labor. The $G-c_w$ curve is located below the $R-w$ curve by C / E. The equation can also be rewritten as follows:

$$G = \frac{w}{c_w}R - \frac{\frac{C}{c_w}}{E} \tag{34}$$

The ratio w / c_w is the inverse of workers' propensity to consume. C / c_w expresses the amount of labor input that corresponds to capitalists' consumption deducted into workers' consumption. Therefore, $(C / c_w) / E$ represents the proportion to the total labor input of unproductive consumption of capitalists converted into labor input: i.e. the level of social burden of unproductive consumption in terms of labor. It is subtracted from the maximum G without C.

Here, let us focus on "Introduction and plan of the work" in *The Wealth of Nations* by Adam Smith in relation to the dual equation that incorporates capitalist consumption.

He began the work with the following well-known paragraphs:

> "The annual labour of every nation is the fund which originally supplies it with all the necessaries and conveniencies of life which it annually consumes, and which consist always either in the immediate produce of that labour, or in what is purchased with that produce from other nations.
>
> According therefore, as this produce, or what is purchased with it, bears a greater or smaller proportion to the number of those who are to consume it, the nation will be better or worse supplied with all the necessaries and conveniencies for which it has occasion.
>
> But this proportion must in every nation be regulated by two different circumstances; first, by the skill, dexterity, and judgment with which its labour is generally applied; and, secondly, by the proportion between the number of those who are employed in useful labour, and that of those who are not so employed. Whatever be the soil, climate, or extent of territory of any particular nation, the abundance or scantiness of its annual supply must, in that particular situation, depend upon those two circumstances." (Smith [1776] pp.lvii-lviii.)

The opening part presents many interesting issues; first, the proposition only in a few dozen words completely destroyed the mercantile fallacy that the wealth of nations exists in "all the necessaries and conveniencies of life" and they are supplied only by "the annual labour of every nation". It is notable that Smith refers only to means of consumption, and not to any means of production such as machinery, tools or raw materials. However, it is an outstanding view in the light of the dual equation that aggregates multi-sectors including those of means of production, in which the net economic resource is expressed by Q_3 / E and its total in society is Q_3. Indeed, the wealth of nations lies exactly in "all the necessaries and conveniencies of life" produced and consumed by "the annual labour of every nation".

His next proposition that "According therefore, as this produce, or what is purchased with it, bears a greater or smaller proportion to the number of those who are to consume it, the nation will be better or worse supplied with all the necessaries and conveniencies for which it has occasion" can be expressed in the dual equation by $Gc_w + \frac{C}{E}$ in terms of one unit of labor and by $EGc_w + C$ in terms of the national total: i.e. the total amount of means of consumption for workers (i.e. Ec_w) and for the other unproductive classes (i.e. C), and the resource of growth for the next period (i.e. Egc_w). It should be noted that while wealth is measured in terms of the total output of means of consumption, it also includes resource of capital accumulation for the next production period. Smith asks for conditions that maximize them all.

The most important condition of wealth is "the skill, dexterity, and judgment with which its labour is generally applied", the total effect of which can be measured by Q_3 / E that embodies the level of productivity in all sectors. The second most important condition, i.e. "the proportion between the number of those who are employed in useful labour, and that of those who are not so employed", corresponds to $(C / c_w) / E$ that reveals the level of social burden of unproductive consumption in terms of labor units.

As we have seen so far, the framework of Smith's argument is firmly built on the basis of the quantity system, and the method he adopts is incomparably precise from the viewpoint of the dual equation. On top of this framework, the price system that deals with wages and profits will be fleshed out from Chapter I onwards.

IX. The effect of workers' propensity to consume on growth

Here we change our assumption adopted so far that workers' households are actively engaged in wage struggles into that workers' households are passively accepting wages and conducting consumption activities. In other words, it is a change from workers' "propensity to save" as a coefficient of demanding the real wage rate over the real consumption rate and as a variable endogenously determined within the system into workers' "propensity to save" as a certain percentage, exogenously determined by socio-psychological factors, of a given wage rate.

From equations c_w (29) and w (12), workers' propensity to consume β_w is given as follows:

$$\beta_w = \frac{c_w}{w} = (1 - C)\frac{\left[\frac{R^2 a_{12} a_{23} + a_{13} R(1 - R a_{22})}{1 - R^2 a_{12} a_{21} - R a_{22}}\right] R l_1 + \left[\frac{R^2 a_{13} a_{21} + R a_{23}}{1 - R^2 a_{12} a_{21} - R a_{22}}\right] R l_2 + R l_3}{\left[\frac{G^2 a_{12} a_{23} + a_{13} G(1 - G a_{22})}{1 - G^2 a_{12} a_{21} - G a_{22}}\right] G l_1 + \left[\frac{G^2 a_{13} a_{21} + G a_{23}}{1 - G^2 a_{12} a_{21} - G a_{22}}\right] G l_2 + G l_3} \tag{35}$$

It is rewritten as follows with the numerator and the denominator on the right side as a function of R and G, respectively:

$$\frac{c_w}{w} = (1 - C)\frac{f(R)}{f(G)}$$
$$0 < f'(R)$$
$$0 < f'(G) \tag{36}$$

It is differentiated with respect to time with constant capitalist consumption:

$$\dot{c}_w - \dot{w} = \dot{f}(R) - \dot{f}(G) \tag{37}$$

Since an increase in workers' propensity to save means $\dot{c}_w - \dot{w} < 0$,

$$\dot{c}_w - \dot{w} = \dot{f}(R) - \dot{f}(G) < 0$$
$$\dot{f}(R) < \dot{f}(G) \tag{38}$$

Therefore,

$$\dot{R} < \dot{G} \tag{39}$$

An increase in workers' propensity to save is caused either by rising wages with constant consumption, decreasing consumption with constant wages or a combination of both. In either case, it has an effect of relatively raising the growth rate, with the rate of change in the growth rate exceeding that of the profit rate.

We adopted the assumption, however, that workers' "propensity to save" is a certain percentage of the real wage, exogenously determined by socio-psychological factors.

Therefore, it would lead us to suppose that an increase in workers' propensity to save causes, on the one hand, a decrease in workers' consumption from constant wages and, on the other, an increase in the growth rate and the constant profit rate. By a sharp contrast, if we adopt the assumption of workers' households aggressively demanding higher wage rate over the consumption rate by their propensity to save, it would lead to a lower profit rate and the constant growth rate, keeping workers' consumption to be constant. It suggests that passive and prudent workers contribute to a higher growth rate at the price of their own consumption.

X. The Cambridge equation: the effect of consumption of non-working classes

In this section, we examine similarities and differences between our dual equation and the so-called Cambridge equation. The dual equation that does not separate consumption of capitalists and others can be rewritten as follows:

$$G = \frac{w}{c} R \tag{40}$$

in which if capitalists and other non-working classes consume a lot, $w < c$ holds and thus, $G < R$ holds. The equation is very similar to the following Cambridge equation[13]:

$$g = s_c r \tag{41}$$

in which $G = 1 + g$, $R = 1 + r$ and s_c ($0 \leq s_c \leq 1$) denotes the ratio of capitalists' savings to profits (i.e. capitalist propensity to save). It is assumed that workers do not save.

Let us separate consumption of capitalists and other non-working classes from the consumption rate c of the dual equation and examine similarities to and differences from the Cambridge equation. Given β_c ($0 \leq \beta_c \leq 1$) as the capitalist propensity to consume, the following quantity and price equations hold:

$$Gc_w = \frac{Q_3 - \beta_c rwE}{E}$$

$$Rw = \frac{Q_3}{E} \tag{42}$$

In the dual equation, a multi-sector system is aggregated as if it were a simple "food-gathering economy" in which only labor force is necessary, so rwE in the quantity equation represents the total profit in units of means of consumption. It is multiplied by β_c to get capitalist consumption, which is then subtracted from Q_3 to get the residual net economic resource. From both equations above, we acquire

$$Gc_w = Rw - \beta_c rw$$

$$G = R\frac{w}{c_w} - \frac{\beta_c rw}{c_w}$$

Given β_w as workers' propensity to consume,

$$\beta_w = \frac{c_w}{w}$$

Substituting it into equation G above yields the following equation:

$$G = \frac{R}{\beta_w} - \frac{\beta_c r}{\beta_w}$$

$$G = \frac{1}{\beta_w}(R - \beta_c r)$$

(43)

As with the Cambridge equation, if workers do not save at all, $1 = \beta_w$,

$$G = R - \beta_c r$$

$$1 + g = 1 + r - \beta_c r$$

$$g = (1 - \beta_c) r$$

$$g = s_c r$$

(44)

It is exactly the Cambridge equation, although being the “multi-sector Cambridge equation” derived from the dual equation. Given s_w as workers’ propensity to save, the general formula is obtained by rewriting equation (44) as follows[14]:

$$g = \left(\frac{s_c}{1 - s_w}\right) r$$

(45)

We have analyzed consumption of capitalists and other non-working classes, firstly assuming it to be fixed and independent of profits, and then assuming it to be proportional to profits as with the Cambridge equation. Kalecki, who was said to have heralded macroeconomic theory, independently and one step ahead of Keynes, distinguished between workers’ consumption and capitalists’ consumption according to the Marxian economics tradition. He constructed a model with workers’ savings being zero and capitalists’ consumption being divided into a fixed part and a part proportional to profits (Kalecki 1935, pp.327-328). Given C as its fixed part, the Kaleckian-type “multi-sector Cambridge equation” is as follows:

$$g = s_c r - \frac{C}{c_w E}$$

(46)

Incorporating workers’ savings, it is rewritten as follows:

$$g = \left(\frac{s_c}{1 - s_w}\right) r - \frac{C}{c_w E}$$

(47)

XI. The neo-classical growth model and growth accounting: the source of growth

We finally examine the growth accounting based on Solow’s growth model (Solow 1957) and clarify differences from our modified von Neumann model. This will make it clear that both models have completely different interpretations of the source of economic growth.

The Cobb-Douglas aggregate production function that is linear homogenous and thus, displays constant returns to scale is assumed:

$$Y = AK^{\alpha}L^{1-\alpha}$$
$$0 < \alpha < 1 \tag{48}$$

in which Y, K and L denote net output of a country, fully utilized capital input and fully employed labor input, respectively. A is a coefficient that represents a certain level of technology (i.e. total factor productivity)[15]; α is capital share of income; and $1 - \alpha$ is labor share of income.

The function is differentiated with respect to time:

$$\dot{Y} = \dot{A} + \alpha\dot{K} + (1-\alpha)\dot{L} \tag{49}$$

If the production function is expressed per unit of labor,

$$y = \frac{Y}{L} = AK^{\alpha}\frac{L^{1-\alpha}}{L}$$
$$y = AK^{\alpha}L^{-\alpha}$$
$$y = A\left(\frac{K}{L}\right)^{\alpha} \tag{50}$$

It is differentiated with respect to time:

$$\dot{y} = \dot{A} + \alpha\left(\dot{K} - \dot{L}\right) \tag{51}$$

Since $\dot{K} - \dot{L}$ is the rate of change in capital equipment per unit of labor, the growth rate of national income per unit of labor is explained by the rate of technological progress and the rate of increase in capital equipment per unit of labor. They are the sources of economic growth.

By contrast, according to the dual equation obtained from the modified von Neumann system, the growth rate is expressed as follows:

$$G = \frac{Q_3}{Ec} \tag{52}$$

Q_3 / c indicates how many units of labor force could be employed by the total output of means of consumption; in other words, it measures how much resource exist for renewal and growth of productive forces in terms of labor force. By dividing it by the actual employment E, the growth rate G is obtained. Alternatively, it can be interpreted as follows: G is obtained by dividing the net economic resource for growth and consumption per unit of labor in terms of means of consumption, i.e. Q_3 / E, by the actual consumption rate c. The output and allocation of means of production that enable this growth rate is determined by equations Q_1 and Q_2.

Comparison between our growth equation and Solow's reveals that the former has neither capital share of income nor an arbitrary rate of technological progress as “residual”.

The growth rate per unit of labor is simply determined by dividing social average productivity of labor Q_3 / E by the social consumption rate c. Technological progress is calculated as the rate of increase in social average productivity of labor. It should be noted that in the quantity system a certain absolute level of technology (i.e. social average productivity of labor) defines the growth rate, and that the rate of technological progress is *not* a determinant of the growth rate, but it is the determinant of the sum of the two rates of change in the growth rate and the consumption rate. Namely, $\dot{Q}_3 - \dot{E} = \dot{G} + \dot{c}$ holds; economic growth is caused neither by an *increase* in workers' capital equipment ratio nor technological *progress*, but is determined under a certain *level* of technology and industrial *structure*.

A few propositions come out of the examination above:

First, the growth rate decreases as a result of an increase in the social consumption rate, which may be an improvement of the standard of living for workers, an increase in unproductive consumption of capitalists or an enlargement of government organizations. In growth accounting, by contrast, only the supply side of growth is depicted, and such demand side analysis is lacking.

Second, an increase in the capital equipment ratio, expressed as an increase in the ratio of the means-of-production sector to the means-of-consumption sector, may reduce the growth rate, unless it contributes to reducing direct labor input of any sector. Solow's growth model says that an increase in the capital equipment ratio directly enhances the growth rate.

It is now apparent that the two models have a completely different understanding of the fundamental issues about what economic growth is and what its sources are.

Conclusions

The objective of the research note was, on the basis of the modified von Neumann model, to aggregate industrial sectors and reduce the complex input-output reproduction system into the dual equation, which now enables us to understand resource allocation between growth and consumption and income distribution between profits and wages as simply and clearly as possible. In addition, the net economic resource in the dual equation can measure the whole results of technological improvement as long as the social average productivity of labor is concerned. The simple dual equation sheds a clear analytical light on some basic claims of classical economists, such as Smith, Ricardo, Malthus and Marx, and verifies their validity. It also resurrects Ricardo's old corn-ratio theory in a modern style and encompasses Keynes's national income system. The note takes, however, a critical stance towards the growth accounting based on Solow's growth model.

Although we adopted the minimum 4-sector model for our analysis, the number of sectors, large or small, does not affect the analytical results presented here. They hinge, not on the number of sectors, but on the duality between the price and quantity systems, aggregate output of means of consumption and the price and quantity numeraire.

The next step that the present author would like to take is to incorporate foreign trade into the modified von Neumann model and examine the real effects of trade imbalance, which would finally eliminate the fallacy of mercantilism that is still haunting modern economics: i.e. Adam Smith's task yet unaccomplished.

(23 December 2019)

Notes

1) It might be appropriate to mention here in advance our relationship with the Keynesian national income theory, which is a macro system along with the von Neumann system. Keynes began Chapter two "The postulates of the classical economics", virtually the beginning of *The General Theory*, with the following words:

> "Most treatises on the theory of Value and Production are primarily concerned with the distribution of a *given* volume of employed resources between different uses and with conditions which, assuming the employment of this quantity of resources, determine their relative rewards and the relative values of their products." (Keynes 1936, p.4.)

Here he added a footnote "This is in the Ricardian tradition. For Ricardo expressly repudiated any interest in the *amount* of the national dividend, as distinct from its distribution" (*Ibid*.) and quoted Ricardo's letter to Malthus of October 9, 1820:

> "Political Economy you think is an enquiry into the nature and causes of wealth — I think it should be called an enquiry into the laws which determine the division of the produce of industry amongst the classes who concur in its formation. No law can be laid down respecting quantity, but a tolerably correct one can be laid down respecting proportions. Every day I am more satisfied that the former enquiry is vain and delusive, and the latter only the true objects of the science." (*Ibid*.) (Ricardo 1952b pp.278-279.)

Keynes constructed a macroeconomic system, which was an inseparable mixture of two systems of quantity and price. The clear distinction and reconnection between them would be fulfilled by von Neumann, who was born 20 years later than Keynes.

2) Morishima (1969) praised (a), (e) and (g) in particular, and called it the "von Neumann Revolution" in the history of economic theories. See Kurz and Salvadoli (2001) for exploring the relationship between Sraffa theory and von Neumann theory, using Sraffa's unpublished papers and letters.

3) The basic analytical results coming out later in this article will not be affected even if materials input into Sector I is allowed, or Sectors I and II produce means of consumption as well as means of production, although certain minor modifications are necessary. It should be noted, however, the classification of "sectors" and "industries" here is based on their "economic activities" as seen in the input-output analysis, in which output of means of production (i.e. materials, and parts and components) and that of means of consumption are separated into different "economic activities" even if they are produced by the same companies. Therefore, Sector III aggregates all the outputs of means of consumption in an economy.

The separation as such into these two types of economic activities has a good theoretical basis; as we see later, given constant productivity, the growth rate g and the consumption rate c are in an inverse relation, and so are output of means of production and that of means of consumption. Hence, their integration into the same sector with rigid output coefficients would certainly cause analytical problems.

4) Introduction of different compositions of the same consumption goods and services among different social groups may be one way of analytical development. And another promising direction may be to introduce a separate sector of luxury goods that are consumed only by capitalists

and other non-working members of the society.

5) See “assumption *d*” in von Neumann [1938] p.2.

6) See section 6 of Itaki (2018) for details.

7) See equations (3) and (4).

8) Sraffa (1960) showed the monotonous inverse relation between the profit rate and the real wage rate by constructing a composite commodity that is not affected by income distribution. On the other hand, in our article, the inverse relation was revealed by the duality between the price system and the quantity system.

9) However, even in the simplest four-sector model, actual calculations are difficult.

10) Keynes paid close attention to the issue of measurement units and adopted quantity of employment as his measurement units. That is, exogenously given the nominal monetary wage rate of “ordinary labor”, distinguished from special labor and skilled labor, it divides the commodity price or the output value, thereby is their real price or value in terms of employment (Keynes 1936, pp.40-41, 43-44.). Independent of Keynes, Kalecki, who is said to have been a precursor of the macroeconomic theory, was completely indifferent to the problem of measurement units and made no mention in Kalecki (1935).

11) Specifically, the industrial structure and employment shift in the direction of the 2nd sector < the 1st sector < the 3rd sector. The total employment is determined by the amount of investment.

12) Malthus wrote “unproductive consumption on the part of the landlords and capitalists” (the letter from Malthus to Ricardo on 16 July 1821 in Ricardo 1952c p.20.), suggesting capitalists’ consumption as well to be unproductive. Keynes also quoted the letter in *The General Theory* (Keynes 1936 p.363.).

13) As for the Cambridge equation, see Kaldor (1955-56), Robinson (1956) and Pasinetti (1962), (1974) and (1977).

14) In chapter 24 “Concluding notes on the social philosophy towards which the General Theory might lead”, *The General Theory*, Keynes criticized the belief that “for a large proportion of this growth we are dependent on the savings of the rich out of their superfluity” and asserted that “up to the point where full employment prevails, the growth of capital depends not at all on a low propensity to consume but is, on the contrary, held back by it; and only in conditions of full employment is a low propensity to consume conducive to the growth of capital.” (Keynes 1936, pp.372-373.) This is in contradiction to the result of the “Cambridge equation” derived from the modified von Neumann dual equation: full employment is not assumed in the dual equation. Keynes seems to confuse the long-term growth rate with the short-term multiplier effect.

15) According to Solow, it reflects not only the level of technology *per se*, but also includes various institutional factors such as the level of education among workers. It is exactly “Solow’s residual”.

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(ITAKI, Masahiko, Professor, College of International Relations, Ritsumeikan University)

The dual equation and the net economic resource（ITAKI）

双対方程式と純経済資源

本研究ノートでは、修正フォン・ノイマン型４部門モデルを構築して、消費と生産の再生産構造、生産力と人間集団と社会関係の再生産構造を、できるだけ単純化された形で描写している。そうすることで、成長と消費の間の資源配分関係と利潤と賃金の間の所得分配関係の密接なかかわりが観察されている。その際、「純経済資源」という新しい概念が分析の中核にすえられる。そして、これを媒介項として、資源配分と所得分配の「双対方程式」が導出される。この一組の方程式は、驚くほど単純な形をとりながらも、産業諸部門間の複雑な再生産構造の本質的な特徴を抽出することに成功している。さらに､労働１単位当たり純経済資源によって、技術発展と生産性上昇の成果を総合的に計測することが可能になり、ソローの成長会計の問題点が克服されている。

本稿ではまた、スミス、リカード、マルサス、マルクス以降、スラッファ、パシネッティに至る、広い意味での「古典派」の議論の中心的な論点についても検討を加えている。さらに、貯蓄性向に頼ることなくマクロ・モデルを構築し、ケインズ国民所得体系を多部門体系の中に包摂している。

（板木　雅彦，立命館大学国際関係学部教授）