Abstract of Doctoral Thesis

Title：Essays on stochastic calculus in relation to number theory and representation theory

Doctoral Program in Advanced Mathematics and Physics

Graduate School of Science and Engineering

Ritsumeikan University

ふりがな　かずひろ よしかわ

氏　名　Kazuhiro Yoshikawa

The theory of stochastic calculus established in the 20th century certainly contributed the great progresses of probability theory and mathematical statistics and many other fields. Stochastic processes having independent and stationary increments such as Wiener processes and Poisson processes are usually called Lévy processes. The marginal distributions of Lévy processes are always infinitely divisible in a certain sense and often studied analytically in terms of characteristic functions. In this thesis, we consider some prospects of stochastic calculus through characteristic functions in relation to number theory and representation theory.

Chapter 1 is an introduction to this thesis, where the backgrounds are also given.

In chapter 2, we discuss multivariable and multiple zeta functions and their definable multidimensional discrete distributions, especially multidimensional Shintani zeta functions and distributions which were introduced in Aoyama and Nakamura (2013). This class includes many kinds of multidimensional discrete distributions. In fact, multinomial or negative binomial distributions are of the multidimensional Shintani zeta class, which allows us to define some classes regarded as their generalizations in view of zeta distributions. We draw exact outlines of these classes by giving the necessary and sufficient conditions for some cases of multidimensional Shintani zeta functions to generate probability distributions.

In chapter 3, we show some results which are focused on Euler products. Aoyama and Nakamura (2013) introduced multidimensional polynomial Euler products which were generalized to be multivariable and multiple infinite products. Furthermore, they gave the necessary and sufficient conditions for those products to generate some infinitely divisible characteristic functions. In their cases, non-principal Dirichlet L-functions are not included. As a main result, we give the necessary and sufficient condition for a product of a real-valued Dirichlet L-function and the Riemann zeta function to generate an infinitely divisible characteristic function.

In chapter 4, we study a Fermion Fock space on Wiener functionals and consider its applications. First, we prove that all Wiener functionals in the Fermion Fock space can be expressed as a polynomial of first order integrals and second order anti-symmetric integrals. The second order anti-symmetric integrals are called stochastic areas, whose characteristic functions are known to be explicitly given by trigonometric functions. Secondly, we see explicit forms of the characteristic functions of some joint distributions with stochastic areas. As an application of the first and second results, we propose an approximation scheme based on the anti-symmetric calculus over Wiener space.