［様式－学 5］

主　論　文　要　旨

論文題名

Essays on the discretization of stochastic differential equations and their applications

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主論文要旨

The discretization of stochastic differential equations (SDEs) has been very important in many applications such as mathematical finance and nonlinear filtering. The aim of this thesis is to establish methods to construct a higher-order (or high accuracy) discretization scheme for general SDEs.

In the first chapter we give an overview of this research and briefly review the mathematical idea discussed throughout the thesis.

In the next three chapters, we propose several techniques for the construction of higher-order weak approximations of SDEs. Chapter 2 is devoted to an operator approach, often called the operator splitting method, which helps us to construct a higher-order scheme and to determine the rate of convergence. The discussion includes the analysis of approximations of Lévy-driven SDEs. Chapter 3 reviews the cubature formulas introduced by Lyons-Victoir (2004) and their relation with the operator splitting method. In Chapter 4, we introduce a space-time discretization scheme which can be applied to the computation of conditional expectations appeared in pricing American options and forward-backward SDEs.

In Chapter 5, several strong convergence results of an accelerated numerical scheme applied to perturbed SDEs are shown. The scheme introduced here was originally analyzed by Takahashi-Yoshida (2005) for weak approximations. We study the scheme from the viewpoint of strong convergence and the multi-level Monte Carlo method.

Finally in Chapter 6, we study a discrete-time approximation scheme for the nonlinear filtering problem. Picard (1984) showed that the scheme is a first-order approximation scheme under suitable conditions. We discuss a rigorous error analysis of the scheme using various techniques in infinite dimensional spaces, and in particular give a generalization of Picard's result.