Master's Thesis

A New Opportunity of Bitcoin for Improving Portfolio Efficiency in Japan

By

NAM Yonghyeon

52115003

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Certification of Originality

I, **NAM Yonghyeon (Student ID 52115003)**, hereby clarify that this thesis is my own original work and has not been submitted in any form for the award of another degree at any university or educational institute. Any information derived from the published or unpublished journal of others has been properly cited or acknowledged appropriately.

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Summary

Background

Against the backdrop of the global financial crisis, the digital currency has recently attracted substantial public attention. On the contrary to the existing centralized financial market infrastructure, bitcoin uses peer-to-peer technology to operate without central authotiry or banks (Nakamoto, 2008). The transparent and decentralized natures of bitcoin enable to become the most popular alternative currency.

Recent policital and economic events in Japan make plausible for the digital currency trading to be attractive. The Japanese government accepted bitcoin as a virtual currency to protect users. Moreover, Japan looks to end sales-tax collection on purchases of virtual currencies in spring, 2017 (Nikkei, 2016).

The global economic uncertainty also influenced on purchasing bitcoin. In 2016, the price of bitcoin surged more than 50% in July over January, which has benefited from recent economic events. For example, China devalued the yuan and the UK dramatically exited the European Union (Bovaird, 2016). Furthermore, a zero interest-rate policy has led the Japanese to find a better opportunity for investment as well as for protection from diminishing their assets (Dhaliwal, 2016).

In the academic world, bitcoin also has drawn significant attention from law and computer science scholars. However, few scientific studies have yet focused on examining bitcoin from an investment point of view. The absence of empirical works addressing bitcoin as an investment vehicle is the motive for this study. Therefore, in this paper, we extend the literature on bitcoin as an investment feature in view of a Japan investor.

Objective

This research aims to address the following questions: "Can bitcoin improve portfolio's efficiency" and "Which portfolio optimization strategy can make the best risk-return profile for portfolio including bitcoin".

Due to the lack of understanding about the influence of bitcoin on portfolio efficiency among Japan society and in general, this research examines the effect of bitcoin on a Japan investor's portfolio. After then, we figure out the best portfolio optimization strategy by comparing various models which have different objectives.

This study will contribute to the understanding of bitcoin in the portflio management for investors who are both individual investors and institutional investors. In addtion, this study will facilitate future researchers to use this ideas as reference in conducting other related literature and to consider bitcoin as an alternative investment to enhance portfolio value.

Methods

This paper adopted the view point of a Japan investor and constructed well-diversified portfolios including various indices such as bond, bond, equity, currency, real-estate and commodity. Then, we developed three portfolio optimization frameworks originated from the risk-based allocation and modern portfolio theory such as maximum efficient portfolio approach, minimum risk portfolio approach and risk budgeting portfolio approach. In addition, we used the robust risk measures (e.g. VaR, CVaR) to account for the bitcoin's non-normality and the investor's aversion toward downside risk.

In order to calculate the performance of a portfolio strategy, we applied the out-of-sample backtesting technique over the sample period, from July 30, 2010 to September 30, 2010, to assess the performance of models by comparing the risk-return ratios (e.g. sharpe ratio and sortino ratio).

Conclusions

First, this paper illustrated that bitcoin exhibits fat-tailed distribution with very high kurtosis but relatively low skewness. This symmetric heavy-tailed distribution was compensated not only by low correlations with other assets, but also high returns. Overall risk-return ratios were thus increased after adding bitcoin into the three different frameworks. Especially, the result showed that bitcoin increased sortino ratio in all frameworks, which means that bitcoin significantly decreased downside risk. As a result, the effect of adding bitcoin into the portfolio demonstrated the improvement in the portfolio's efficiency by increasing positive returns and decreasing negative returns.

Second, considering bitcoin invested into the portfolio, the framework 1, maximum efficient portfolio approach, achieved the highest sharpe ratio and sortino ratio. The weights of bitcoin illustrated that framework 1 which had the largest average bitcoin weights also gained the highest sortino ratio. The findings suggest that framework 1 was designed effectively for bitcoin to increase both sharpe ratio and sortino ratio. In other words, the objective to maximize reward to risk of framework 1 caused the higher average weights of bitcoin and resulted in the highest sortino ratio. As a result, framework 1, maximum efficient portfolio approach, could make the best risk-return profile for the portfolio including bitcoin.

Abstract

As an open source peer-to-peer electronic cash system which operates without any central authority, bitcoin has attracted users and investors who understand the risk of existing financial system such as negative interest rate policies and high levels of government debt, and concern the next global economic crisis (Nakamoto, 2008)

The incentive for the investment of bitcoin is high among Japan investors in terms of increasing bitcoin accepted shop, bitcoin-friendly regulation, unprecedented stability of bitcoin price. As Japanese are more interested in bitcoin as an investment and alternative currency, Japan is now the world's third-largest bitcoin market in the world after China and United Stated. However, few papers have emphasized on bitcoin as an investment aspect. Yermack (2013) argued that bitcoin behaves like an investment vehicle. Brière, Oosterlinck and Szafarz (2013) provided a tentative first look at how bitcoin might be of value in an investment portfolio optimization process.

Therefore, this research examines how bitcoin can increase the efficiency of a Japan investor's portfolio and finds which portfolio optimization strategy can make the best risk-return profile for a well-diversified portfolio including bitcoin.

By using out-of-sampling backtesting over the period from July 30, 2010 to September 30, 2016, we analyze a bitcoin investment from a Japan investor's standpoint with a well-diversified portfolio including both broad range of asset classes (equity, bond, commodity, real-estate and currencies) and Japan's investment market trends (domestic equities, high-yield bond and REITs). This study develops three different frameworks based on modern portfolio theory and risk-based allocation which have

genuinely different objectives (framework 1: maximum efficient portfolio approach, framework 2: minimum risk portfolio approach, framework 3: risk budgeting portfolio approach).

Distribution of the returns for bitcoin shows highly distinctive features, including exceptionally high average return and volatility. Its correlation with other assets is remarkably low, which makes bitcoin valuable as an investment.

The backtesting results confirm that overall risk-return ratios were increased after adding bitcoin into the three different frameworks. Especially, bitcoin offered significant improvement in the portfolio efficiency by reducing downside risk and increasing returns.

Results also illustrate that that bitcoin significantly contributed to the increase of risk-return profile efficiently in the framework 1, maximum efficient portfolio approach. In other words, the objective to maximize reward to risk of framework 1 caused the higher average weights of bitcoin and resulted in the highest sortino ratio.

Consequently, considering the features of bitcoin (e.g. low correlation with other assets, high return and heavy-tailed distribution), the approach concerning not only more robust risk measures, but also return is suitable for building the optimal portfolio model.

Keywords Bitcoin, Portfolio Theory, Investment, Efficiency, Optimization, Japan, VaR, CVaR, Risk Parity, Mean-Variance

CHAPTER1 Introduction

1.1 Background

As the recent financial system crumbles, an alternative concept of the financial system has become more relevant and credible. On the contrary to the discretionary decision-making of a central bank, bitcoin system is transparently operated by a peer-to-peer network without a central authority. These tasks are managed by an open-source computer algorithm, which facilitates the reliability of expectations about the future supply of bitcoin and maintains its integrity (Nakamoto, 2008).

Japan was once one of the largest bitcoin markets before Mt.Gox filed for bankruptcy due to unprofessional conduct, deception, and theft. This experience has put most people in fear about trading in any crypto-currency in Japan. However, recent political and economic events make it plausible for this digital currency trading to be attractive and rebound back to the true value (Coincheck, 2016).

One of the reasons is that regulations have been more bitcoin-friendly. For example, the government of Japan had passed a bill about virtual currency exchanges to protect users. Moreover, Japan looks to end sales-tax collection on purchases of virtual currencies in spring, 2017 (Nikkei, 2016).

The global economic uncertainty also influences on purchasing bitcoin. In 2016, the price of bitcoin surged more than 50% in July over January, which has benefited from recent economic events. For example, China devalued the yuan and the UK dramatically exited the European Union (Bovaird, 2016). In addition, the Japanese government, led by prime minister Shinzo Abe, has struggled with deflation and

stimulating domestic demand. The economic stimulus package, known as "Abenomics", has recently regarded by economic financial experts as underwhelming. This economic policy uncertainty causes investors and users to find alternative financial system. Furthermore, a zero interest-rate policy has led Japanese to find a better opportunity for investment as well as for protection from diminishing their assets (Dhaliwal, 2016). Actually, according to Brian (2016), 80% of users are using bitcoin as an investment, and 20% of users are using it as a wallet for daily spending.

As Japanese are more interested in bitcoin for investment and alternative currency, Japan is now the world's third-largest bitcoin market in the world after China and United States of America. At the beginning of 2016, one bitcoin was trading at around 38,000 JPY, where now one bitcoin is worth almost 60,000 JPY. Furthermore, the number of shops in Japan that has accepted bitcoin reached around 2,500 stores (Nikkei, 2016).

In the academic world, bitcoin also has drawn significant attention from law and computer science scholars. Many papers have been published focusing on descriptive analysis of the bitcoin network (Ron and Shamir, 2013), the potential risk of double-spending (Karame, Androulaki & Capkun, 2012), as well as the implications of the availability of a public ledger containing all bitcoin transaction ever made (Meiklejohn, Pomarole, Jordan, Levchenko, McCoy, Voelker & Savage, 2013). However, few scientific studies have yet focused on examining bitcoin from an investment point of view. Yermack (2013) argued that bitcoin behaves like an investment vehicle. Brière *et al*. (2013) provided a tentative first look at how bitcoin might be of value in an investment portfolio optimization process. Wu and Pandey (2014) found out that bitcoin

could play an important role in enhancing the efficiency of an investor's portfolio. Eisl, Gasser and Weinmayer (2015) indicated that bitcoin could contribute to the risk-return ratios of optimal portfolios by adopting a Conditional Value-at-Risk framework.

The absence of empirical works addressing bitcoins as an investment vehicle is the motive for this study. Therefore, in this paper, we extend the literature on bitcoin as an investment feature in a Japan investor's standpoint. We, therefore, look at the impact of bitcoin on the portfolio by comparing the results of different strategy and find the best strategy to maximize portfolio performance.

1.2 Research Questions and Objectives

This research aims to address the following questions: "Can bitcoin improve portfolio's efficiency" and "Which portfolio optimization strategy can make the best risk-return profile for portfolio including bitcoin".

Due to the lack of understanding about the influence of bitcoin on portfolio efficiency among Japan society and in general, therefore, this study considers on this aspect. Therefore, this research examines the effect of bitcoin on an investor's portfolio in Japan where the demand and opportunities of bitcoin as an investment are rising. After then, we figure out the best portfolio optimization strategy by comparing various models which have different objectives. These results can be used as practical information for Japan investors who consider bitcoin as an investment opportunity.

This study will contribute to the understanding of bitcoin in the portfolio management for investors who are both individual investors and institutional investors. I hope that this research will encourage them to be drawn to the frameworks of stretegy and to adapt it as an effective investment strategy that will benefit the Japan investors.

In addition, this study will facilitate future researchers to use theses ideas as a reference in conducting other related literature and to consider bitcoin as an alternative investment to enhance portfolio value.

The outcomes to be considered consist of the following: understanding statistical properties of bitcoin; the improvement of portfolio efficiency by adding bitocin in three frameworks; enhancing portfolio's returns; the level of weights of bitcoin to make the best risk-return profile, and eventually the development of a positive attitude towards bitcoin as an investment.

1.3 Structure of the Paper

The structure of the paper is following. Introduction chapter explains the background of the subject in hand. Chapter 2 and Chapter 3 review the theoretical background for an understanding of bitcoin and portfolio theory respectively. Chapter 4 explains the methodology. In Chapter 5, data is examined and empirical results are presented and interpreted. Chapter 6 presents the conclusion, limitation and managerial implication.

CHAPTER2 Introduction to Bitcoin

2.1 History of Bitcoin

Bitcoin is a form of digital currency and system released as open source software in January 2009. Satoshi Nakamoto is known as the pseudonymous creator of bitcoin, who continues to this day to remain unknown. bitcoin was first introduced in a paper entitled "Bitcoin: A Peer-to-Peer Electronic Cash System" in 2008 (Nakamoto, 2008)

The main advantage of this digital currency is the lack of centralized third-parties or authorities such as a bank or credit card company who is keeping track of users' deposits and withdrawals. Bitcoin system allows users to share and process transactions, which greatly reduce transaction costs and time lag compared to traditional currency systems where banks require more time and charge fee for transactions. This peer-to-peer system is also designed to verify and record all transactions in a public ledger known as the blockchain (Velde 2013). However, it is described as an anonymous currency because users are identified by "bitcoin address" only (Doguet, 2012).

In many ways, bitcoin, commonly referred to as a "virtual currency" or "cryptocurrency", has been used as a currency which can buy goods and service. Users can purchase bitcoin from either an online exchange or directly from other users. The exchange is acting similar to a foreign currency exchange where individual bitcoin buyers and sellers can find each other.

In February 2010, the very first of bitcoin exchange was established by Dwdollar who was a member of bitcoin online forums. After the first bitcoin exchange, a Tokyo-based online exchange, Mt.Gox, was officially launched in July 2010. Growing trading volume of bitcoin on the Mt.Gox led to $1 million market capitalization of bitcoin by November 2010. By 2013, it handled approximately 70% of the world's bitcoin trades (Jossep, 2015).

In 2013, some mainstream websites began to accept bitcoin as a currency such as WordPress, Expedia, Dell, Microsoft and so on. In October 2013, Baidu, China-based search engine, accepted bitcoin as a payment method. By November 2013, China-based bitcoin exchange reached the world's largest bitcoin trading exchange. On the other hand, bitcoin's anonymity made it powerful currency for online black markets. In October 2013, the U.S Federal Bureau of Investigation (FBI) shut down the Silk Road website which was an online market for selling illicit drugs by bitcoin (Chwierut, 2016).

In February 2014, another crisis of bitcoin occurred. Mt.Gox collapsed and lost a reported 850 thousand bitcoin, which led the value of bitcoin to fell close to 23% (Böhme, Christin, Edelman & Moore, 2015; Yermack, 2013). However, the price recovered after six months following the bitcoin crash (Ngo, 2015). After its biggest exchange's collapse, governments began to pass regulation to control bitcoin. The Internal Revenue Service (IRS) declared to consider bitcoin as property to be taxed and the People's Bank of China (PBOC) required Chinese banks to close the accounts of bitcoin exchanges (Chwierut, 2016).

Since 2015, interest in the bitcoin technology, "blockchain", surged in banks and financial industry. For example, Microsoft launched blockchain-as-a-service (BaaS)

within its Azure service portfolio, and Barclays announced that it would become the first UK bank to start accepting bitcoin for users to make charitable donations (Macfarlan, 2015). On the other hand, there was also a debate on the expansion in block size. The limited block size of bitcoin is not enough to supply block space according to the increasing demands of users.

Many of trends in 2015 are continuing in 2016. More companies are interested in blockchain technology, uncertainty over the block size is still in debate and the development of alternative cryptocurrencies is emerging such as Ethereum, Ripple, Litecoin, and so on. However, bitcoin maintains its reputation as the pioneer of cryptocurrency with the largest market capitalization and monthly trade volume among cryptocurrencies, which is exhibited in Table 2.1 and Table 2.2.

Table 2.1. Top10 Cryptocurrency Market Capitalizations (November 2, 2016)

Rank	Name	Market Cap (USD)
1	**Bitcoin**	**$11,683,126,523**
2	Ethereum	$918,655,230
3	Ripple	$291,158,743
4	Litecoin	$197,222,743
5	Ethereum Classic	$75,731,299
6	Monero	$61,359,241
7	Dash	$61,076,190
8	Augur	$48,823,830
9	NEM	$34,568,460
10	Waves	$34,370,000

Note: Adapted from "Crypto-Currency Market Capitalizations," by CoinMarketCap, 2016, Retrieved from http://coinmarketcap.com/

Table 2.2. Top 10 Monthly Trade Volume Rankings (November 2, 2016)

Rank	Name	Volume (30days)
1	**Bitcoin**	**$2,098,122,432**
2	Ethereum	$319,789,879
3	Monero	$94,020,646
4	Litecoin	$86,461,275
5	Ripple	$83,670,810
6	Augur	$50,169,162
7	Ethereum Classic	$39,903,922
8	Dash	$24,051,127
9	PotCoin	$22,056,849
10	Factom	$21,457,598

Note: Adapted from "Monthly Volume Rankings (Currency)," by CoinMarketCap, 2016, Retrieved from http://coinmarketcap.com/

2.2 Bitcoin in Japan

Japan now has 11 bitcoin exchanges and the world's third largest market. The total market exchanging volume in Japan is 17.8 million BTC over the period from February 2011 to July 2016, which is the third largest amount after China and United States of America, which is shown in Figure 2.1 (Bitcoin 日本情報サイト, 2016).
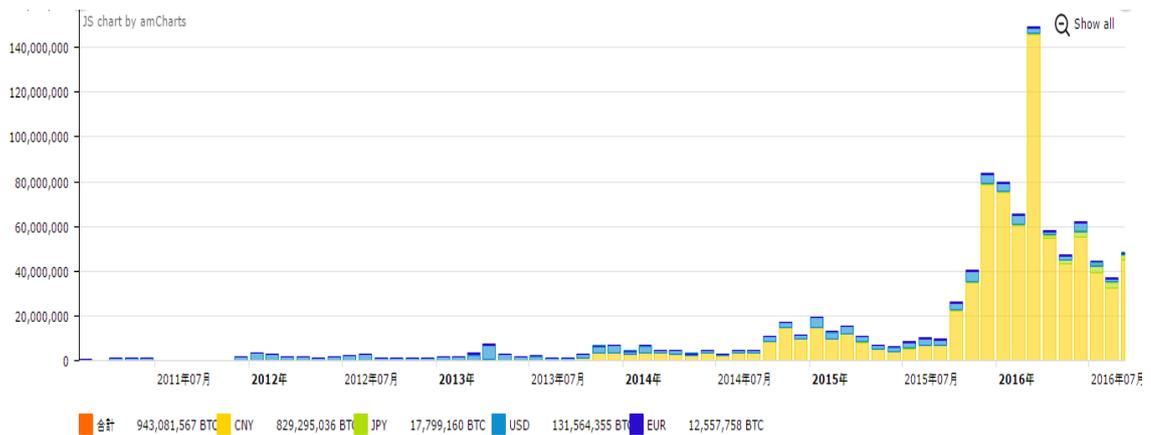

Figure 2.1. Monthly Exchanging Volume (Bitcoin 日本情報サイト, 2016)

Bitcoin trading volume and price are sharply increased recently. As showed in Figure 2.2, 430 million yen ($4.25 billion) in bitcoin were traded in Japan from January to Jun 2016 which is 50 times more than the trading volume in the same

period of the previous year. As the trading volume of JPY is increasing, bitcoin price is also steadily increasing from 38,316 yen on January 1, 2015, to 61,876 yen on September 15, 2016 (BitcoinCharts, 2016).

Figure 2.2. Bitcoin Trading Volume and Price (JPY) (BitcoinCharts, 2016)

One of the factors for investors and users to exchange bitcoin actively is that the government made it acceptable as a currency. The introduction of the rules and regulatory have been instrumental in the recovery process from the Mt.Gox scandal which was hacked and eventually filed for bankruptcy in February 2015. In February 2016, Japanese regulators proposed a draft which defines cryptocurrencies as digital currencies rather than commodities. In May 2016, Japan has passed a bill that mandated the rules and regulations of the bitcoin and the virtual currency exchanges by the Financial Services Agency. Moreover, Japan looks to end sales-tax collection on purchases of virtual currencies in spring 2017. This change would not only reduce burden of costs for buyers and operators, but also encourage bitcoin adoption among

investors and adopters who will see the cryptocurrency as a store of value or a transactional currency used as an alternative to the fiat money (Nikkei, 2016).

Another factor to boost bitcoin price is macroeconomic uncertainty. The incumbent government, ruled by Prime Minister Shinzo Abe, has struggled with deflation and stimulating domestic demand by the radical action to end economic stagnation, called "Abenomics". The uncertainty in economic policy causes instability in economic. These factors attract users and investors who are wary of traditional financial systems and who are like the volatility as an investment opportunity. Actually, Yuzo Kano, chief executive of bitFlyer which is one of the bitcoin exchanges in Japan, mentioned that the number of customers has already surpassed 200,000 by August 2016 (Solana, 2016).

As the prevalence of bitcoin in Japan has increased, the number of shops in Japan that accept bitcoin payments has been increased. Currently, around 2,500 stores in Japan have accepted bitcoin as a payment currency. One example is DMM.com which is one of the most well-known online content platforms in Japan. In March 2016, it started accepting bitcoin payment through coinceck's processing service. The bitcoin start-up ResuPress also plans to accept the cryptocurrency as payment for electricity charges, which is expected to reduce the payment owed by from 4% to 6%. Similarly, bitcoin exchange and service firm, Coincheck, will enable Japanese citizens to pay utility bills by bitcoin in November 2016. Bills will be cheaper, compared to payments made through traditional means. There are now more than 2,500 merchants and online businesses that accept bitcoin in Japan and merchant adoption is accelerating by the day (Southurst, 2016).

In business aspect, many corporations are also interested in bitcoin as an opportunity to growth. For example, Japan's biggest financial group, Mitsubishi UFJ Group, announced to invest Coinbase which is the bitcoin venture company offering a service for users to store the virtual currency and make payment with it (Fukase, 2016).

2.3 Bitcoin as investment

Users can use bitcoin either as payment for goods and services or alternative currency converted to fiat currencies in various exchanges. Because the value of bitcoin is not controlled by any central bank, nor is it backed by any government, the price of bitcoin fluctuates freely based on supply and demand and the public's perception of bitcoin as a store of wealth. The Bank of America Merrill Lynch (BAML) also predicted that, as both "a medium of exchange as well as a store of value," bitcoin can become "a major means of payment for e-commerce and may emerge as a serious competitor to traditional money transfer providers". In the report, Woo, Gordon and Iaralov (2013) analyzed Cost-Benefit of Bitcoin, which is showed in Table 2.3.

Table 2.3. Cost-Benefit Analysis of Bitcoin

		Medium of Exchange	Store of Value
Advantages		Low transaction costs	Value protected by finite supply
		More secure, transparent, and portable than cash	Evasion of capital controls
		Disincentives experimentation with alternate digital currencies	Like gold, large benefits given negative correlation with risk sensitive assets
Disadvantages		Further regulation would increase transaction costs	Price volatility
		Bitcoin exchanges vulnerable to hacking	Seigniorage accrues to bitcoin miners, incentivizing government crackdown
		Payment confirmation delays	Status as non-fiat currency

Note: Adapted from "Bitcoin: a first assessment," by Woo, Gordon and Iaralov, 2013, Retrieved from http://www.pequenocerdocapitalista.com/wp-content/uploads/2015/07/banks-research-report-on-Bitcoin.pdf

Glaser, Zimmermann, Haferkorn, Weber and Siering (2014) studied bitcoin in the view point of user perspective. In this paper, they revealed bitcoin users use it as an asset than as a currency. In addition, the new bitcoin users tend to trade bitcoin for speculation purposes. Actually, Brian (2016) indicated that 80% of users are using bitcoin as an investment, and 20% of users are using it as a wallet for daily spending. Notably, a proposed bitcoin Exchange-traded Fund (ETF) of Winklevoss bitcoin trust shows that Bitcoin is now a credible investment vehicle (Higgins, 2014).

One of the reasons to attract investors to buy bitcoin is that the price of Bitocin has been more stable than ever before. In the recent 5 years, bitcoin has shown the unprecedented stability as a financial instrument. The bitcoin volatility and a downward linear trend are exhibited in Figure 2.3.
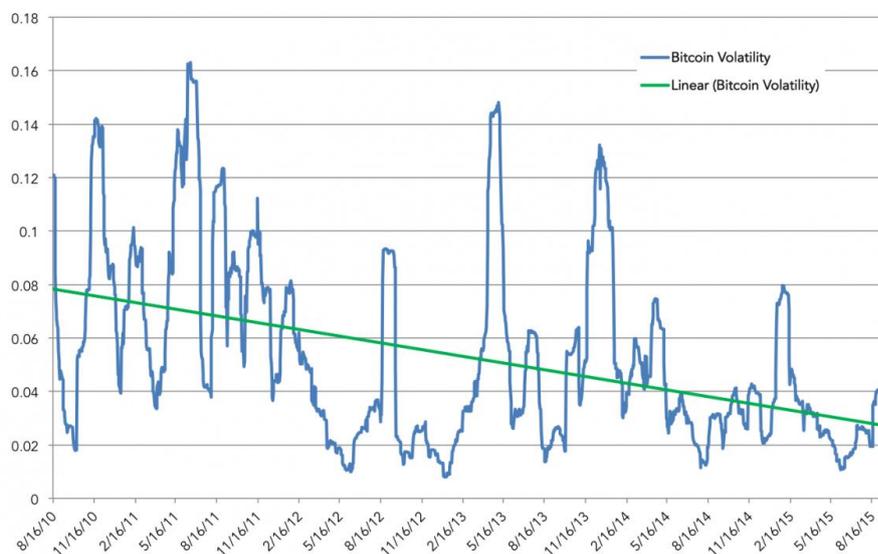

Figure 2.3. Bitcoin Volatility (Brade, 2016)

With the increase of bitcoin usage, a single trade has a lesser effect on the price, which results in decreasing volatility and attracting investors to the market. Another reason is the value of bitocin is not closely correlated with the equity and currencies.

(Brade, 2016). Therefore, bitcoin can offer significant diversification benefits for investors (Brière *et al*., 2015). Weak national currencies are also a great basis of demand for bitcoin. The growth of bitcoin trading volume is high in countries such as China, Latin America, South Africa and India. People in these countries are increasingly exchanging their traditional currency into bitcoin because of the weak valuation of local currencies (Bitcoinist, 2016; Durben, 2016; Redman, 2016; Singh and Vega, 2016).

In particular, these factors also seem to attract Japanese investors who are considering bitcoin as an investment. A zero interest-rate policy has led the Japanese to find a better opportunity for investment as well as for protection from diminishing their assets (Dhaliwal, 2016). Increasing trade volume of bitcoin in Japan (exhibited in Figure 2.2) does not mean that potential users are suddenly rushing into bitcoin trading for protecting a fall in the value of Japanese yen. However, it seems that traders and investors are seeking alternative investments to find yield against volatility.

CHAPTER3 Portfolio Optimization

3.1 Asset allocation

This chapter reviews the theoretical background for an understanding of the subject. In order to create frameworks for asset allocation, we need to know relevant portfolio optimization theory and understand the theoretical foundation that this paper builds upon. Frameworks adopt different approaches from the traditional strategies of asset allocation to recently highlighted strategies.

Asset allocation refers to the set of weights of broad asset classes within a portfolio so as to achieve an investment objective and goal. Once an investor has defined an investment goal and objective, the investor selects universe of investable assets in developing an investment program. Allocating the weight of each asset will define the overall behavior of the portfolio, which should be matched with the risk and return targets for the investor. Once the model portfolio has been chosen, the portfolio should be evaluated in order to examine whether the model meets the investor's criteria for performance and volatility.

Here we have a simple portfolio with multiply assets. We have n risky assets. Let R_n represent the return on asset i. We will allocate w_i to asset i. The total expected return of portfolio P is the weighted average of the returns on individual assets in portfolio, which is defined as:

$$E[R_P] = w_1\,R_1 + w_2\,R_2\ldots + w_n\,R_n = \sum_{i=1}^{n} w_i\,R_i \tag{1}$$

By definition, the sum of w_i, called "weight" in the portfolio allocation problem, which must be equal to 1

$$w_1 + w_2 \dots w_n = \sum_{i=1}^{n} w_i = 1 \tag{2}$$

On the other hand, one of the most commonly used risk measures is variability. Variance is a deviation of a set of expected returns, which is defended as:

$$\sigma_P^2 = var[R_P] = var\left(\sum_{i=1}^{n} w_i \, R_i\right) = \sum_{i,j=1}^{n} w_i w_j Cov(R_i, R_i) \tag{3}$$

Covariance is then:

$$cov[R_i, R_j] = E[(R_i - E[R_i])(R_j - E[R_j])] \tag{4}$$

We can also express variance using matrix notation as:

$$\sigma_P^2 = var[R_P] = \sum_{i,j=1}^{n} w_i w_j Cov(R_i, R_i) = \omega^t \Sigma \omega \tag{5}$$

where ω is a column vector whose components are the ω_i, ω^t is the row vector that is the transpose of ω, and Σ is the covariance matrix whose entries are the variance (3) and covariance (4).

3.2 Mean-Variance Framework

Harry Markowitz is regarded as the cornerstone of modern portfolio theory. He was awarded the 1990 Noble Prize for his contributions to financial economics and corporate finance field, supported in his "Portfolio Selection" (1952) essay and more extensively in his book "Portfolio Selection: Efficient Diversification" (1959). Starting with the historic work of Markowitz, past historical research includes many attempts to optimize portfolio using the risk-return analysis.

One of the most common approaches to practical asset allocation decisions is the mean-variance approach, developed by Markowitz (1952), for analyzing the trade-off between risk and return for portfolios comprising several assets. In the context of mean-variance analysis, a portfolio is called "efficient" if the portfolio offers the highest expected return for each level of risk. The efficient frontier consists of the set of efficient portfolios, which is showed in Figure 3.1 (Bodie and Marcus, 2008).
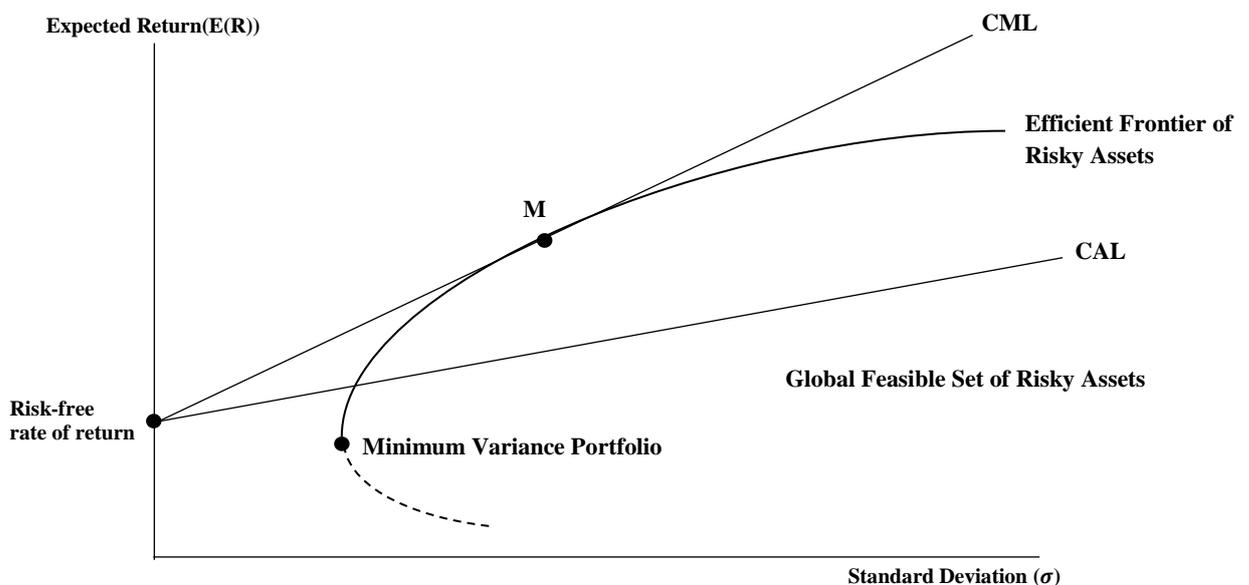

Figure 3.1. Markowitz Efficient Frontier of Risky Asset (Bodie and Marcus, 2008)

The efficient frontier considers a portfolio which is comprised of risky assets. However, investors also can choose to invest a risk-free asset whose standard deviation and correlation with risky assets are zero. Any combination of risky portfolio and the risk-free asset in an investor's portfolio can change the efficient frontier into a straight line, which is called Capital Allocation Line (CAL) drawn from the risk-free rate of return through a risky portfolio, which is shown in Figure 3.1

The ultimate CAL tangent to the optimal risk portfolio is called Capital Market Line (CML), which offers the highest expected return for all level of risk, and the lowest risk for all level of expected return. The equation describing the CML can be written as:

$$E[R_i] = R_f + \sigma_i \times \left(\frac{E[R_m] - R_f}{\sigma_m}\right) \qquad (6)$$

where $E(R_i)$ is the expected return on asset i. $E(R_m)$ is the expected return of the market portfolio, R_f is the risk-free return, σ_i is the standard deviation on asset i, and σ_m is the standard deviation of the market portfolio.

The graph of CML (shown in Figure 3.1) states that the intercept is the risk-free rate (R_f) and the slope represents the market premium ($E[R_m] - R_f$).

3.3 Sharpe ratio

The portfolio optimization is based on the Tobin's Separation Theorem, which explains that finding an optimal portfolio can be separated into two problems. The first is to find an optimal combination of risky assets and risk-free asset, which does not vary with the risk tolerance of investors, and second is to decide whether to lend or borrow

based on an investor' attitude toward risk. Within this framework, the tangent portfolio, CML, is regarded as the optimal risk portfolio on the Markowitz efficient frontier, which dominates all other risky portfolios regardless of risk preferences (Tobin, 1958).

The optimal portfolio chooses the highest expected return-to-risk ratio which is called sharpe ratio:

$$Sharpe\ ratio = \frac{E[R_P] - R_f}{\sigma_P} \tag{7}$$

where $E[R_P]$ is expected return of portfolio P, R_f is risk-free return, and σ_P is standard deviation of portfolio P.

Sharpe ratio indicates how well the portfolio performs in comparison to the risk-free rate by measuring the reward per unit of risk, which is also called as "risk-efficient". Thus, it is often used to evaluate the performance of a portfolio (Sharpe, 1966, 1994).

3.4 Risk-based allocations

The mean-variance framework from Markowitz's (1952) modern portfolio theory indicates the method to construct a portfolio having the balance between risk and return. However, the interest has shifted towards risk-based allocation schemes due to unrealistic assumptions of the mean-variance framework, (i.e. returns from normally distributed assets). Risk-based allocation schemes do not require an explicit estimation for returns on assets. This approach relies mostly on the estimation of asset variances and covariance for managing portfolio risk and increasing diversification. The followings are the major weaknesses of the mean-variance framework.

1) Returns from normally distributed assets

The assumption of Markowitz's mean-variance model is a normal distribution of returns, which is an unrealistic estimate of the actual performance of financial assets. Selecting variance as a risk measure is also reasonable if asset returns follow a normal distribution as the loss is distributed entirely by the expectation and variance of the returns in the man-variance framework. However, practically, two distributions having the same variance do not demonstrate the same loss profiles as variance does not incorporate the skewness of the returns.

2) Portfolio concentration

In the mean-variance framework, allocation of assets is often biased to the few assets in the investment universe in order to make the highest sharpe ratio. The high concentration of assets results in the high sensitivity of limited assets and the potential for a loss.

3) Lack of robustness and stability

As shown by Black and Litterman (1992), the optimal portfolio selected with mean-variance optimization is not robust with respect to the significant change of results which is caused by a minor change in inputs such as the expected returns, variance of the assets, and their correlation matrix. Chopra and Ziemba (1993) explained that the impact of an error in returns is greater than that of an error in variances and correlations. Therefore, the emphasis on the improvement of mean-variance framework has been placed on a more accurate estimation of variance and correlations while shrinking the estimation of returns.

3.4.1 Risk-Budgeting approach

The period selected to define the expected return and its underlying risk does not necessarily present a profile that will be consistent with future events. In addition, the assumption, returns are normally distributed, is not always the case. Risk-budgeting approach based on risk-based allocations allocates a risk budget to assets and requires only the estimation of volatilities. This style puts diversification at the heart of the investment process.

3.4.1.1 Diversification Ratio

Choueifaty and Coignard (2008) introduced how to achieve "maximum diversification portfolio" with different finance assets. They introduced a ratio of weighted average asset volatilities to portfolio volatility, called diversification ratio:

$$Diversification\ ratio = \frac{\sum_{i=1}^{n} w_i \sigma_i}{\sigma_P} \qquad (8)$$

where w_i is weight of asset i, σ_P is standard deviation of portfolio P and σ_i is standard deviation of asset i.

It represents that the higher the ratio is, the more the portfolio is diversified. The Maximum Diversified Portfolio (MDP) is the portfolio with weights of assets that maximize the diversification ratio.

3.4.1.2 Risk Parity

The concept of risk parity was introduced from Bridgewater embedded in research in the 1990s. Risk parity aims to equate the weighted marginal contribution to risk within across all portfolio constituents for a certain level of portfolio volatility.

Both of risk parity and MDP seeks to reduce risks based on the maximizing diversification. However, risk parity takes into consideration of the covariance which is the interactions that exist between different assets of portfolio, while MDP emphasis on the variance of assets. In other words, this approach considers not only the volatility of each asset, but also their correlation. It aims to balance risk exposures in a given portfolio, so as to avoid risk concentration.

The contribution of each asset class to the total risk of the portfolio is defined as Marginal Risk (MR). The general definition of MR of asset i to the total risk of portfolio P is given by the following expression:

$$Marginal\ Risk\ (MR_i) = \frac{\partial \sigma_P}{\partial w_i} = \frac{(\Sigma\ \omega)_i}{\sqrt{\omega^t \Sigma\ \omega}} \tag{9}$$

where w_i represents the weight of asset i in the portfolio, σ_P is the volatility of portfolio P, ω is the vector of weights, ω^t is the row vector that is the transpose of ω, and Σ is the covariance matrix.

The risk contribution is as follows:

$$Risk\ contribution\ (RC_i) = w_i \frac{\partial \sigma_P}{\partial w_i} = w_i \frac{(\Sigma\ \omega)_i}{\sqrt{\omega^t \Sigma\ \omega}} \tag{10}$$

Then, the total risk of portfolio P will be:

$$Total\ Risk(TR) = \sigma_P = \sum_{i=1}^{n} RC_i = \sum_{i=1}^{n} \omega_i \frac{\partial \sigma_P}{\partial \omega_i} = \sum_{i=1}^{n} \omega_i \frac{(\Sigma\ \omega)_i}{\sqrt{\omega^t \Sigma\ \omega}} \tag{11}$$

The most formal and widely recognized approach to robust risk parity approach is Equal Risk Contribution (ERC) model, which aims to equalize the RC from each asset class (Maillard, Thierry & Jérôme, 2010). The equation describing ERC allocation method can be written:

$$RC_i = RC_j = w_i \frac{\partial \sigma_P}{\partial w_i} = w_j \frac{\partial \sigma_P}{\partial w_j} \tag{12}$$

3.4.2 Downside risk approach

The main concern for investors may not be the variance but the downside risk as the general assumption is that if a return is below their expected value, investors will become more unsatisfied than if the return is above their expected value. Because of this aversion to downside risk, Value at Risk (VaR) and Conditional Value at Risk (CVaR) were introduced as an alternative method of variance which does not capture extreme risks adequately. These two concepts are also used to build the portfolio optimization models under the framework of risk-based allocation.
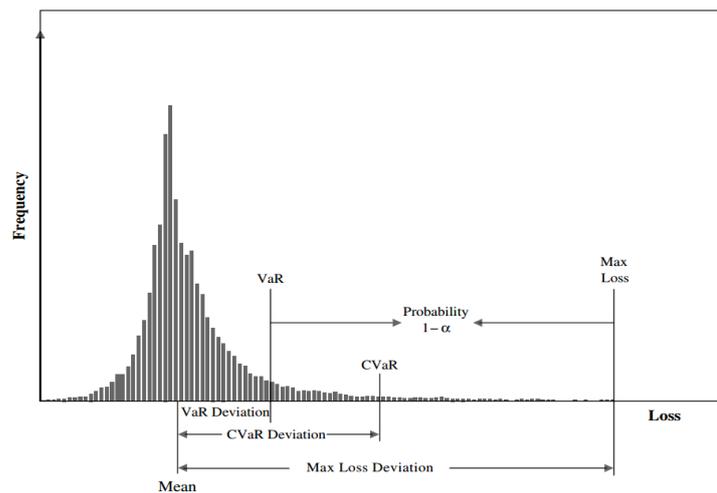

Figure 3.2. VaR, CVaR, Deviations (Sarykalin, Serraino & Uryasev, 2008)

3.4.2.1 VaR

Value at Risk (VaR) is a general risk measure that can be used for a portfolio regardless of its return distribution, which is shown in Figure 3.2. It represents an amount of loss to be exceeded with a probability $1 - p$ in a given time horizon t and confidence level α (Choudhry, 2013). For example, the VaR at level 95% is defined as the minimal amount of capital which is required to cover the losses in 95% of cases.

The VaR of X given parameter $0 < α < 1$ is:

$$VaR_\alpha(X) = \inf\{x : \Pr(L \geq x) > 1 - \alpha\}$$ (13)

where L is a random variable representing loss and α is the confidence level.

The advantage of using VaR is that it can incorporate skewness and kurtosis in the measure of total risk. In addition, VaR is easy to interpret and use in analysis as it is measured in price units or as percentage of portfolio value. This is why many financial institutions adopted it as a risk measurement. For example, the Basle Committee on banking supervision announced in 1995 that capital adequacy requirements for commercial banks were to be based on VaR (Jorion, 1996).

Despite of popularity of VaR, it VaR shows various limits. For example, VaR assumes that returns are normally distributed and VaR does not explain any information about the shape of the left tail of the distribution (Rockafellar and Uryasev, 2002). Additionally, Artzner, Delbaen, Eber and Heath (1999) show that VaR does not fully consider the benefit of diversification, which is not a coherent risk measure.

3.4.2.2 CVaR

The criticisms of VaR resulted in the emerging CVaR which is a coherent risk measure for any type of loss distribution (Rockafellar and Uryasev, 2002). Conditional value-at-risk, CVaR, also known as "expected shortfall" or "average value-at-risk" or "tail value-at-risk", is defined as the expected loss exceeding VaR. VaR is concerned with the (1 – α) percentiles of the distribution, while CVaR focuses on the tail of the loss distribution, which is shown in Figure 3.2 (Sarykalin *et al.,* 2008).

Mathematically, CVaR is defined as:

$$CVaR_\alpha(X) = E\big(L|L >> VaR_\alpha(X)\big) \tag{14}$$

where L is a random variable representing loss and α is the confidence level.

CHAPTER4 Methodology

4.1 Out-of-sample Backtesting

This paper adopts the view of a Japan investor and constructs a well-diversified portfolio including various indices such as bond, equity, currency, real-estate, and commodity. Then, in order to calculate the performance of portfolio strategies based on the optimal weights w_i of each asset i given objectives of frameworks, this research uses out-of-sample backtesting which is a process of assessing a trading strategy using historical data.

This process applies a two-month rolling horizon to estimate portfolio weights throughout the sample period. For example, the first two months, from July 30, 2010 to September 30, 2010, estimates for the initial weights estimation. Thus, the weight optimization process for each optimal portfolio is thereby subject to various parameters defined in three different portfolio optimization frameworks described below.

4.2 Frameworks

In order to answer the two questions of this study, this paper develops three different frameworks based on modern portfolio theory and various portfolio optimization theories which are discussed in Chapter 3. The objectives of three weighting schemes are genuinely different. Framework 1, "maximum efficient portfolio", optimizes the balance between risk and return, whereas framework 2, "minimum risk portfolio" only focuses on the risk side of the portfolio. On the other hand, framework 3, "risk budgeting portfolio" puts an emphasis on diversification effects of assets (described in Figure 4.1).

To answer the first question, "Can bitcoin improve portfolio's efficiency", this research compares the result of two different portfolios which are a portfolio with bitcoin (with BTC) and a portfolio without Bitcoin (without BTC), given each framework. In addition, comparing the performance of the with BTC with three different frameworks explains the second question, "Which portfolio optimization strategy can make the best risk-return profile for portfolio including bitcoin".
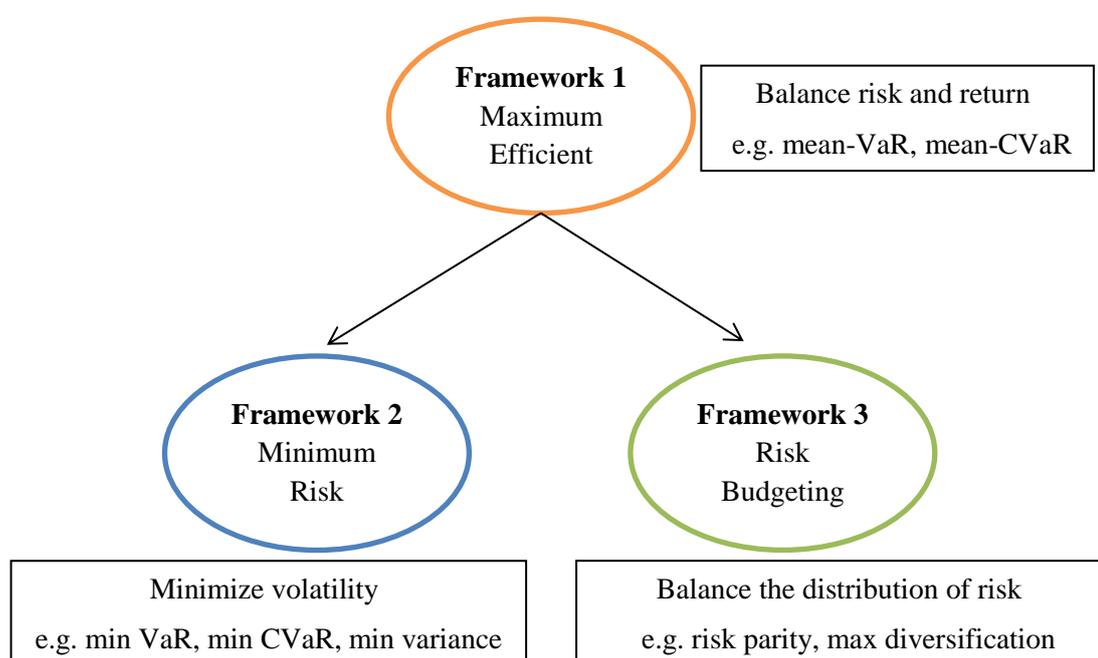

Figure 4.1. Frameworks

To evaluate the performance of portfolios, sharpe ratio is used with variance as a risk measure (Sharpe, 1966). However, the variance puts equal weights on positive and negative returns even though investors' attitudes towards risk are different. Investors generally are more concerned about the downside variability of their investments than the upside gains (Kahneman & Tversky, 1979). Moreover, since the global financial crisis in 2008 to Brexit in 2016, the desire to protect assets against "left tail" events, or significant portfolio losses, has increased considerably (Harrison, 2016; Schroder, 2016).

This study thus adopts the sortino ratio as a performance measure, which considers downside risk. In the early 1980s, Dr. Frank Sortino had undertaken research to come up with an improved measure for risk-adjusted returns, which is called sortino ratio. The sortino ratio is a modification of the sharpe ratio but uses downside deviation rather than standard deviation as the measure of risk (Sortino & Van Der Meer, 1991).

The sortino ratio is defined as:

$$Sortino\ Ratio = \frac{E[R] - MAR}{Downside\ deviation} = \frac{E[R] - MAR}{\sqrt{\frac{1}{N}\sum_{i=1}^{n}(R_t - MAR)^2}} \tag{15}$$

where $R_t < MAR$, MAR is minimum acceptable return, R_t is the return on the portfolio for sub-period t.

4.2.1 Framework 1: Maximum Efficient Portfolio Approach

Maximum efficient portfolio approach is based on return and risk management framework, which is called mean-variance model from Markowitz (1952). In the return and risk management framework, a measure of financial performance is sharpe ratio, equation (7), which represents the expected return per unit of risk. Therefore, the portfolio with maximum sharpe ratio gives the highest expected return per unit of risk, and is the most "risk-efficient" portfolio. However, sharpe ratio based on mean-variance model is a meaningful measure of risk when risk can be sufficiently measured by the standard deviation and return can be distributed normally, which is an unrealistic assumption. Thus, Campbell, Huisman and Koedijk (2011) developed the concept of mean-VaR which maximizes expected return subject to a downside risk constraint rather than standard deviation. In addition, to build comparable sharpe ratio of mean-

variance strategy, Dowd (1998), Alexander and Baptista (2003) suggested Reward to VaR (RTV) which uses VaR as the risk measurement:

$$Reward\ to\ VaR(\text{RTV}) = \frac{E[R] - R_f}{VaR_\alpha} \tag{16}$$

where VaR_α is the Value at Risk with α confidence.

Likewise, Martin, Rachev and Siboulet (2003) introduced the STARR (Stable Tail Adjust Return Ratio) and Sigmundsdóttir and Ren (2012) developed the concept of downside risk ratio which uses the same approach of sharpe ratio except using expected shortfall (CVaR) as the risk measure.

The downside risk ratio is:

$$Downside\ risk\ ratio = \frac{E[R] - R_f}{ES} \tag{17}$$

where ES is expected shortfall of the portfolio

Similar to mean-variance model, maximization of RTV and the downside risk ratio can be interpreted as the most efficient risk-return portfolio under the risk measure of VaR and CVaR. Therefore, this framework adopts two models with different risk measures, which are mean-VaR and mean-CVaR.

4.2.1.1 Mean-VaR model

Based on the research of Stoyanov, Rachev and Fabozzi (2007), Parrák and Seidler (2010), mean-variance can be transferred into the optimal portfolio under mean-VaR model to make the maximize returns with the least amount of risk, which makes

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the maximum RTV. However, a modification is needed into RTV parameter in order to take negative excess returns into consideration. One possibility that has been suggested is to change the formula to the following:

$$Modified\ Reward\ to\ VaR (MRTV) = \frac{E[R]}{VaR_\alpha} \qquad (18)$$

Then, we form the optimal portfolio model using MRTV under mean-VaR model:

$$Maximize \quad MRTV = \frac{E[R]}{VaR_\alpha}$$

$$Subject\ to \qquad (19)$$

$$w_i \geq 0, \quad \sum_{i=1}^{n} w_i = 1$$

The objective of mean-VaR model is to maximize MRTV under the constraints. The maximum of MRTV represents the highest expected return per unit of risk which means the most "risk-efficient" portfolio. However, this model uses VaR as a risk measure for the advantages, discussed in 3.4.2.1. The portfolio model should therefore, yield optimal portfolio with the highest risk-return ratios of all portfolio frameworks.

All frameworks have the same constraints. One of the constraints is a short-selling constraint, described as the equation; $w_i \geq 0$. It reflects possible restrictions involved with short-selling certain assets that are included in the portfolio. As of now, it is also not clear whether a short position in bitcoin is feasible. Thus, the sum of all asset weights is 100%, described as the equation, $\sum_{i=1}^{n} w_i = 1$.

4.2.1.2 Mean-CVaR model

Yu, Sun and Chen (2011) also developed the optimal portfolio under mean-CVaR model, which the maximize returns while controlling expected shortfall (CVaR). However, a modification is also needed into downside risk ratio in order to take negative excess returns into consideration. One possibility that has been suggested is to change the formula to the following:

$$Modified\ Downside\ risk\ ratio(MDRR) = \frac{E[R]}{CVaR_\alpha} \tag{20}$$

Then, we form the optimal portfolio model under mean-CVaR model:

$$Maximize\ \ MDRR = \frac{E[R]}{CVaR_\alpha}$$

$$\tag{21}$$

$$Subject\ to$$

$$w_i \geq 0,\ \sum_{i=1}^{n} w_i = 1$$

Likewise, the objective of mean-CVaR model is to maximize risk-efficiency under the constraints. However, this model uses CVaR as a risk measure for some advantages, discussed in 3.4.2.2.

Framework2: Minimum Risk Portfolio approach

One of the critiques of mean-variance is that it is very sensitive to the input parameters such as the expected returns and covariance matrix of the assets (Chopra and Ziemba, 1993). Marton (1980) pointed out that covariance of the assets can be estimated more accurately than expected returns from historical data. In addition, most asset

returns are non-normally distributed and this can be proved as an extreme tail risk in the current crisis. Consequently, the last few decades, the interest on portfolio technique has shifted towards risk-based allocation schemes, which ignore estimation of the returns (Lee, 2011). Thus, the framework of minimum risk approach also focuses only on the risk measure. The framework adopts three models with different risk measures, which are minimum-variance model, minimum-VaR model and minimum-CVaR model.

4.2.1.3 Minimum-variance (MV) model

Minimum-variance (MV) portfolio is an optimal portfolio to make the lowest risk level. The model is expected to have the lowest possible volatility and that can be uniquely determined by a covariance matrix. As reviewed in Chow, Hsu, Kalesnik, and Little (2011), MV portfolios have been defined and analyzed from the start of modern portfolio theory (i.e., 1960s) as a special case of mean-variance efficient portfolios. In Figure 3.1, MV portfolio sits on the efficient frontier with a minimal risk. Although MV portfolio generally has the disadvantage of a high concentration ratio, it can be limited through diversification (Qian, 2005). Clark, de Silva, and Thorley (2011) showed that with the 1,000 largest-capitalization stocks in the U.S. from 1968 to 2005, various versions of the MV portfolio are found to have higher returns and lower volatilities. In another study, Behr, Güttler, and Miebs (2008) reported that with the entire Center for Research in Security Prices (CSRP) dataset from April 1964 to December 2007, many different MV portfolios with different constraints on weights outperformed the market capitalization.

Then, we form the optimal portfolio under MV model:

$$Minimize \quad \sigma^2{}_p$$

(22)

$$Subject \ to$$

$$w_i \geq 0, \quad \sum_{i}^{n} w_i = 1$$

where $\sigma^2{}_p$ is variance of portfolio P.

The objective of MV model is to minimize variance which is considered as a risk. In short, the model only considers risk rather than return as return is hard to be estimated. Then, the portfolio model is expected to perform better result with a low risk and a high return.

4.2.1.4 Min VaR (Minimum VaR Portfolio) model

Value at Risk (VaR) was popularly embraced for measuring downside risk in a portfolio. VaR is defined as the pth percentile of portfolio returns at the end of the planning horizon. It can be thought of as identifying the "worst case" outcome of portfolio performance. Stambaugh (1996) outlined the uses of VaR as 1) providing a common language for risk, 2) allowing for more effective and consistent internal risk management, risk limit setting and evaluation, 3) providing an enterprise-wide mechanism for external regulation, and 4) providing investors with an understandable tool for risk assessment. Moreover, VaR has been accepted by managers of firms as an integrated and functional internal risk measure and by investors as an intuitive presentation of overall risk using a single currency valued number allowing for easy comparison among investment alternatives.

Then, we form the optimal portfolio under min VaR model:

$$Minimize \ \ VaR_\alpha$$

(23)

$$Subject \ to$$

$$w_i \geq 0, \ \sum_{i=1}^{n} w_i = 1$$

Similar to MV model, the objective of VaR model is to minimize risk which is VaR. As we discussed the advantages of VaR from Stambaugh (1996), the model is expected to bring better outcome than MV model.

Min CVaR(Minimum CVaR Portfolio)

Rockafellar and Uryasev (2000) proposed a scenario-based model for portfolio optimization using Conditional Value at Risk (CVaR) which is defined as expected value of losses exceeding VaR. Uryasev (2000) showed a simple description of the approach for minimization of CVaR and optimization with CVaR constraints.

Then, we form the optimal portfolio under min CVaR model:

$$Minimize \ \ CVaR_\alpha = E\big(L\big|L >> VaR_\alpha(X)\big)$$

(24)

$$Subject \ to$$

$$w_i \geq 0, \ \sum_{i=1}^{n} w_i = 1$$

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Likewise, the objective of min CVaR model is to minimize risk under the constraints. However, this model uses CVaR as a risk measure for some advantages, discussed in 3.4.2.2.

4.2.2 Framework3: Risk Budgeting Portfolio approach

Mean-variance approach failed to stand up the reality of the market as real assets are not normally distributed (Marton, 1980). The MV portfolios also have the drawback of a high concentration (Qian, 2005). As a result, researchers (Booth and Fama, 1992; Qian, 2005) suggested the concept of risk budgeting portfolio approach, which puts diversification at the heart of the investment process without any consideration of returns. Maximum Diversification Portfolio (MDP) and risk parity portfolio are examples of adopting the risk budgeting approach on the idea to balance risks.

4.2.2.1 Risk Parity(RP)

Qian (2005) proposed risk parity portfolio which is a portfolio allocation strategy by risk contribution to the portfolio. In addition, Maillard *et al*., (2010) proposed an approach to compute an Equal Risk Contribution (ERC) portfolio, which achieves a truly diversified portfolio for each asset to contribute the same extent to the overall risk.

Then, we form the optimal portfolio under risk parity model:

$$Minimize \quad \sigma_P = \sqrt{\omega^T \Omega \omega}$$

(25)

$$Subject\ to$$

$$CR_i = CR_j = \frac{\sigma_P}{n}$$

$$w_i \geq 0, \quad \sum_{i=1}^{n} w_i = 1$$

where   MR=marginal risk

CR= risk contribution

$$Marginal\ Risk\ (MR_i) = \frac{\partial \sigma_P}{\partial w_i} = \frac{(\Sigma\ \omega)_i}{\sqrt{\omega^t \Sigma\ \omega}}$$

(26)

$$Risk\ contribution\ (RC_i) = w_i MR_i = w_i \frac{\partial \sigma_P}{\partial w_i} = w_i \frac{(\Sigma\omega)_i}{\sqrt{\omega^t \Sigma\ \omega}}$$

(27)

The objective of this model is to minimize standard deviation while each asset has the same risk contribution to maximize diversification effect.

4.2.2.2 Maximum Diversification Portfolio (MDP)

Maximum Diversification Portfolio (MDP) is an optimal portfolio model to maximize the ratio of weighted average asset volatilities to portfolio volatility which is Diversification ratio, introduced by Choueifaty and Coignard (2008).

We form the optimal portfolio under MDP model for long-only, constrained maximum diversification portfolios, similar to Clarke *et al.,* (2011).

$$Maximize\ DR = \frac{\sum_{i=1}^{n} w_i \sigma_i}{\sigma_P}$$

(28)

$$Subject\ to$$

$$w_i \geq 0, \quad \sum_{i=1}^{n} w_i = 1$$

Compared to risk parity model, the objective of this model is to maximize diversification effects. However, risk parity model makes it by making equal risk weight of each asset while MDP model achieves diversification effects by maximizing the diversification ratio.

CHAPTER5 Data and Analysis

5.1 Presentation of data

Bitcoin data was obtained from CoinDesk's bitcoin price index, a close price of global Bitcoin-USD exchange prices. Since historical data on bitcoin is available starting from July 18, 2010 on CoinDesk.com, the sample period in this study covers under 74-months from July 30, 2010 to September 30, 2016.

In the process of portfolio optimization, this study adopted the viewpoint of a Japan investor. In order to allow a well-diversified portfolio, this paper considered not only a broad range of assets classes from global market indices, but also Japan's investment market trends. Figure 5.1 shows the product market trends of Japanese investors. In the map, domestic equities place high ranked on the demand scale, together with domestic and foreign REITs. According to the Nomura research Institute (NRI) (2015), high-yield bond products have also been high ranked since 2015. Therefore, we added domestic equities, high-yield bond and domestic and foreign REITs to globally well-diversified portfolio.
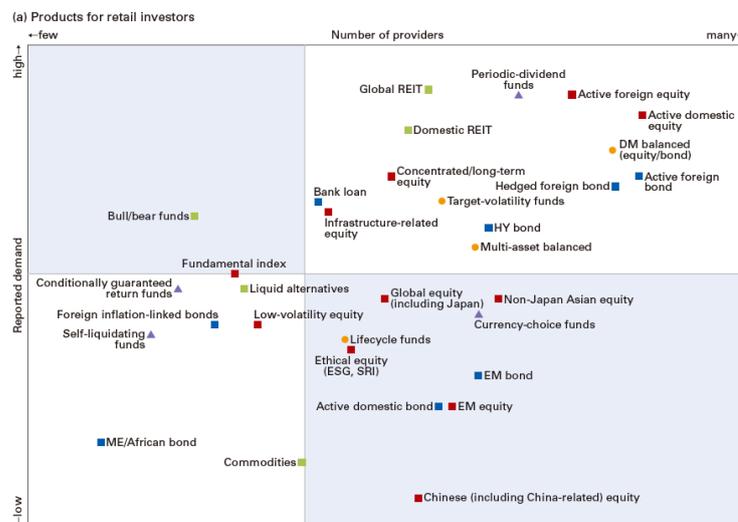

Figure 5.1. Product Supply and Demand Maps by Client Segment (NRI, 2015)

The line-up of asset classes in the portfolio therefore comprised equity, bond, high-yield bond, commodity, real-estate and currencies, represented by at least one or some broad and liquid financial indices. The data is gathered using Federal Reserve Economic Data (FRED) and Bloomberg. A detailed overview of all assets is shown in Table 5.1.

Table 5.1. Asset Class in the Portfolio

Asset class	Region	Mnemonic	Index
Bond	Japan	JBI	S&P Japan Bond Index
Bond	Global	GBI	S&P Global Developed Sovereign Bond Index
High yield bond	Global	HYBI	S&P Municipal Bond High Yield Index
Equity	Japan	NK225	Nikkei 225
Equity	Japan	J500	S&P Japan 500
Equity	Global	MSCI	MSCI world Index
Currency	Japan	BTP	Bitcoin (JPY/BTC)
Currency	Euro	EUR	Euro (JPY/EUR)
Currency	USA	USD	US Dollar (JPY/USD)
Currency	UK	GBP	British Pound (JPY/GBP)
Real estate	Japan	JREIT	TSE REIT Index
Real estate	Global	GREIT	S&P Global REIT
Commodity	Global	COMD	S&P GSCI Commodity Index

5.2 Data description

To make a better understanding of the final results, data statistics will be presented in this section. The summary of the data statistics for the observed time zone, 30th of July 2010 to 30th of September 2016, is shown in Table 5.2.

Table 5.2. Data Statistics (from 30th of July 2010 to 30th of September 2016)

	BTP	JBI	JREIT	MSCI	NK225	USD	EUR	GBP	GREIT	GBI	COMD	HYB
Mean (%)	**0.63**	0.01	0.05	0.07	0.04	0.01	0.00	0.00	0.05	0.01	-0.04	0.03
Standard deviation (%)	**7.07**	0.11	1.18	1.80	1.46	0.62	0.74	0.80	0.94	0.37	1.30	0.19
Kurtosis	**7.83**	9.18	7.62	64.51	5.01	4.03	4.24	31.58	6.03	2.26	5.29	31.58
Skewness	**0.25**	-0.49	-0.03	-3.54	-0.56	0.23	-0.13	-1.97	-0.56	-0.17	-0.61	-2.81
Max (%)	**49.8**	0.7	7.5	10.4	7.4	3.3	3.9	3.9	6.0	1.8	5.5	1.4
Min (%)	**-45.1**	-1.0	-8.2	-31.2	-11.2	-3.5	-5.7	-11.7	-7.3	-2.2	-11.3	-2.2
Sharpe ratio (%)	**8.9**	7.4	3.9	3.8	2.5	1.7	0.0	-0.2	5.2	1.9	-3.2	16.6
VaR (%)	**-11.0**	-0.2	-1.9	-2.9	-2.4	-1.0	-1.2	-1.3	-1.5	-0.6	-2.2	-0.3
CVaR (%)	**-9.4**	-0.2	-1.6	-2.4	-2.2	-1.0	-1.2	-1.2	-1.5	-0.6	-2.1	-0.2

Note: VaR: value at risk at level 95%, CVaR: conditional value at risk at level 95%

The results show that most of the assets described high kurtosis ranged from 4.03 to 64.51 and negative and positive skewness of returns ranged from -3.51 to +0.25. Since the basic assumption of mean-variance framework is the normally distributed returns, we conducted normality test, which are jarque-bera tests and shapiro-wilk test, to determine whether data has been drawn from a normally distributed population.

Jarque-bera tests and shapiro-wilk test were performed to test for normality of the results for all the indices. The results (described in appendix 3,4) show that p-value is less than 0.001 for all the indices. The null hypothesis, which is returns follow the normal distribution, is rejected.

In the case of bitcoin, many researchers demonstrated that bitcoin returns have substantially high negative skewness and very high kurtosis (Baek & Elbeck, 2015; Baur, Hong & Lee, 2015). However, the skewness of bitcoin is 0.25 which is relatively small while kurtosis is 7.83 which is relatively high compared to other indices such as MSCI index (64.51), high-yield bond (31.58).

To present the results in detail, we adopted the Q-Q plots, which are useful in highlighting distributional asymmetry, heavy tails, outliers, multi-modality, or other data anomalies, which is shown in Figure 5.2. The Q-Q plots of bitcoin returns illustrate that both tails of the distribution lie above the reference line, which produced frequent outliers than those of a normal distribution, called "heavy tail".

BTP Index
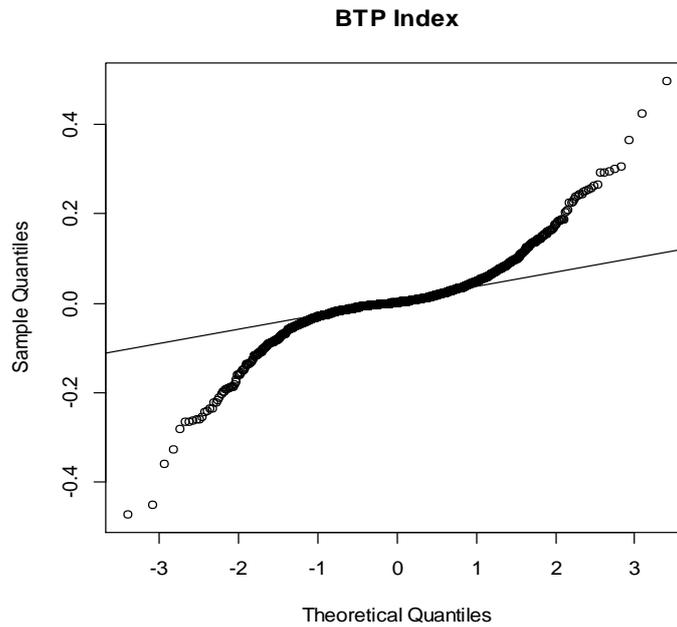
Figure 5.2. Normal Q-Q plot of bitcoin

One of the attractive factors on bitcoin as an investment is that it delivers high diversification benefits from low correlation with the other assets (Brière *et al,* 2013, Gasser, 2014). This paper also figured out that the correlation of bitcoin is substantially low compared to other assets, which is shown in Table 5.3.

Table 5.3. Correlation Matrix

	BITCO	JPBOND	JPREIT	JPSHARE	NIK225	USD	EUR	GBP	GRI	GBI	COM
BITCO											
JPBOND	-0.03										
JPREIT	0.04	0.07									
JPSHARE	0.04	-0.01	0.09								
NIK225	0.04	-0.19	0.45	0.10							
USD	0.11	-0.10	0.21	0.16	0.39						
EUR	0.08	-0.07	0.17	0.23	0.33	0.61					
GBP	0.08	-0.13	0.22	0.22	0.37	0.70	0.75				
GRI	0.07	0.02	0.25	0.49	0.24	0.04	0.30	0.26			
GBI	-0.09	0.23	-0.07	-0.08	-0.23	-0.79	-0.14	-0.33	0.18		
COM	0.03	-0.06	0.08	0.20	0.13	0.04	0.26	0.24	0.35	0.07	
HYB	-0.02	0.16	0.02	-0.02	-0.04	-0.10	-0.12	-0.12	0.05	0.17	-0.07

5.3 Empirical Results

Previously, two portfolios (the with BTC and the without BTC) with three different frameworks were constructed. Table 5.4 displays an overview of the main results for the portfolio without bitcoin.

Table 5.4. Optimal Portfolio without Bitcoin

without BTC	Framework 1 Maximum Efficient		Framework2 Minimum Risk			Framework3 Risk Budgeting	
	Mean-VaR	Mean-CVaR	MV	Min VaR	Min CvaR	RP	MDP
Return	10.2%	9.1%	3.5%	4.7%	4.8%	4.9%	3.8%
Variance	0.6%	0.6%	0.3%	0.3%	0.3%	0.1%	0.1%
Standard Deviation	7.9%	7.6%	5.0%	5.7%	5.4%	3.7%	3.4%
Downside Deviation	7.0%	6.8%	6.8%	7.4%	8.3%	2.8%	2.0%
Max Drawdown	-6.6%	-6.4%	-6.2%	-6.8%	-6.3%	-2.5%	-1.7%
Sharpe ratio	1.28	1.19	0.69	0.84	0.88	1.30	1.12
Sortino ratio	1.44	1.33	0.51	0.64	0.58	1.74	1.88

The results show that framework 1 has the highest returns, with 10.2% in mean-VaR model and 9.1% in mean-CVaR model, and the highest variances with 0.6% in both models. As we designed the models of framework 1 to maximize the efficiency of reward to risks (see Section 4), the objective of these models contributed to increasing not only return but also variance. Therefore, sharpe ratio which is the measurement of risk-efficiency also shows relatively high rate in framework 1. On the other hands, framework 2 and 3 indicate similarly low returns from 3.5% to 4.9%, while variances of framework 3 (0.1%) is relatively lower compared to framework 2 (0.3%), which means that the diversification effects of framework 3 on the without BTC contributed significantly to reducing risk then framework 2 which concentrated on minimum risk itself. This low risk of framework 3 caused the highest sharpe ratio with 1.30 in RP

model. Moreover, in terms of downside risk, framework 3 was also proved to have the lowest risks which are 2.8% in RP model and 2.0% in MDP model. As a result, framework 3, the risk budgeting portfolio approach, demonstrated to be benefited not only from the efficiency with the highest sharpe ratio, but also from the downside risk management with the highest sortino ratio.

Table 5.5 describes the results of the with BTC for all three portfolio optimization frameworks.

Table 5.5. Optimal Portfolio with Bitcoin

with BTC	Framework 1 Maximum Efficient		Framework2 Minimum Risk			Framework3 Risk Budgeting	
	Mean-VaR	Mean-CVaR	MV	Min VaR	Min CvaR	RP	MDP
Return	29.8%	28.2%	4.7%	7.1%	7.0%	10.5%	8.6%
Variance	5.5%	5.4%	0.3%	0.5%	0.3%	1.1%	1.0%
Standard Deviation	23.4%	23.2%	5.6%	6.7%	5.4%	10.6%	9.9%
Downside Deviation	7.2%	7.7%	7.2%	6.9%	4.6%	2.9%	2.2%
Max Drawdown	-6.6%	-6.5%	-6.1%	-6.1%	-3.9%	-2.6%	-1.9%
Sharpe ratio	1.27	1.22	0.84	1.05	1.28	0.99	0.87
Sortino ratio	4.16	3.68	0.66	1.02	1.53	3.62	3.89

The results show that framework 1 has the highest returns, with 29.8% in mean-VaR model and 28.2% in mean-CVaR model, and the highest variances with 5.5% in mean-VaR model and 5.4% in mean-CVaR model, which resulted in relatively high sharpe ratio with 1.27 in mean-VaR and 1.22 in mean-CVaR model as the increase rate of returns is higher than those of variance. Compared to the results of without BTC, a large increase in returns indicates that bitcoin in the portfolio contributed significantly to increasing returns than variance, which resulted in the more efficient portfolio. On

the other hands, framework 2 and 3 indicated relatively small returns from 4.7% to 10.5% compared to framework 1, while variances of framework 2, 0.3% in MV and min-CVaR models and 0.5% in min-VaR model, are slightly lower than those of framework 3, 1.0% in MDP model and 1.1% in RP model. Whereas, the framework 3 has considerably lower downside deviation, 2.2% in RP model and 2.9% in MDP model, which is the same result of the without BTC. The results of the low downside deviation in the framework 3 of without BTC and with BTC show that risk budgeting approach is superior to reduce downside risk regardless of including bitcoin. Thus, considering the risk of including bitcoin into the portfolio, the framework 3 is effective to reduce downside risk than the framework 2. However, when it comes to returns, the results of the with BTC and without BTC are different. Unlike the results of the without BTC, framework 1 demonstrated that the maximum efficient approach benefited not only from the efficiency with the highest sharpe ratio, but also from the downside risk management with the highest sortino ratio. This result shows that framework 1 is designed effectively for bitcoin to increase return and decrease risk.

5.4 Analysis of results

The first question, "**Can bitcoin improve portfolio's efficiency**", was studied comparing sharpe ratio and sortino ratio of the without BTC and with BTC in all three frameworks, which is shown in Table 5.6.

Table 5.6 Sharp Ratio and Sortino ratio

	Framework 1 Maximum Efficient		Framework2 Maximum Risk			Framework3 Risk Budgeting	
	Mean-VaR	Mean-CVaR	MV	Min VaR	Min CvaR	RP	MDP
(a) Sharp Ratio (without BTC)	1.28	1.19	0.69	0.84	0.88	1.30	1.12
(b) Sharp Ratio (with BTC)	1.27	1.22	0.84	1.05	1.28	0.99	0.87
(a') Sortino ratio	1.44	1.33	0.51	0.64	0.58	1.74	1.88
(b') Sortino ratio	4.16	3.68	0.66	1.02	1.53	3.62	3.89
Sharp Ratio (b-a)	-0.003	0.03	0.15	0.21	0.40	-0.32	-0.24
Sortino ratio (b'-a')	2.72	2.35	0.14	0.38	0.95	1.88	2.01

The difference of sharpe ratio between two portfolios shows the effect of bitcoin is most prominent in framework 2, where the sharpe ratio increased by 0.40 from 0.58 to 0.88 in Min-CVaR model, while decreasing in framework 3 by 0.32 from 1.74 to 1.3.

Comparing the cumulative returns of the with BTC and the without BTC across all model illustrates that adding bitcoin increased the overall returns throughout all investment period (described in appendix 5).

While sharpe ratio is increased or decreased depend on the framework, sortino ratio increased in all frameworks from 0.14 to 2.72. It means that bitcoin increased positive returns which affect sharpe ratio to increase risk and return, but decreased negative return which affect sortino ratio to decrease downside risk. As a result, the effect of adding bitcoin into the portfolio shows significant improvement in the portfolio's efficiency.

The second question, **"Which portfolio optimization strategy can make the best risk-return profile for portfolio including bitcoin"**, was studied comparing all three frameworks of the with BTC, which is shown in Table 5.5.

As we discussed the results of the with BTC (see chapter 5.3), framework 1, maximum efficient portfolio approach, achieved the highest sharpe ratio and sortino ratio, which is the different result of the without BTC. In order to examine what made the difference, Table 5.7 provides maximum, minimum and average bitcoin weights across all frameworks. The results show that framework 1 which had the largest average bitcoin weights with 1.0% in mean-VaR model and 1.2% in mean-CVaR model also gained the highest sortino ratio with 4.16 in mean-VaR model and 3.68 in mean-CVaR model. The findings suggest that framework 1 was designed effectively for bitcoin to increase both sharpe ratio and sortino ratio. In other words, the objective to maximize reward to risk of framework 1 caused the higher average weights of bitcoin and resulted in the highest sortino ratio. As a result, framework 1, maximum efficient approach, can make the best risk-return profile for the with BTC.

Table 5.7. Bitcoin Weights

	Framework 1 Maximum Efficient		Framework2 Maximum Risk			Framework3 Risk Budgeting	
	Mean-VaR	Mean-CVaR	MV	Min VaR	Min CvaR	RP	MDP
Max	9.6%	12.8%	0.4%	1.2%	1.1%	2.0%	2.5%
Min	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.0%
Average	1.0%	1.2%	0.1%	0.1%	0.1%	0.7%	0.6%

Additionally, Figure 5.3 depicts that the weights of bitcoin under all frameworks which we applied in this paper. Overall weights of bitcoin are more or less stable in the three different frameworks, hovering in the low range from 0.1% to 1.2% throughout

the entire investment period. The exceptions of these results are the mean-VaR and mean-CVaR model under the framework 1.

The relatively low and stable weights of bitcoin might be beneficial for investors from a liquidity point of view and more feasible to invest for investors who are concerned about risks of bitcoin.
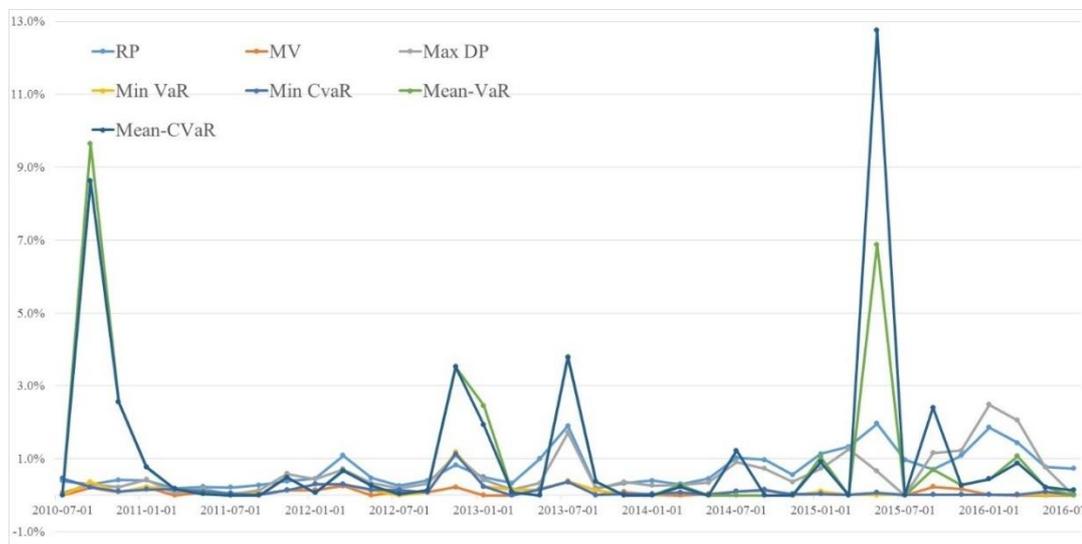
Figure 5.3. Bitcoin Weights Overview

CHAPTER6 Conclusion and limitation

6.1 Conclusions

As the open source peer-to-peer electronic cash system which operates without any central authority, bitcoin has attracted users and investors who understand the risk of existing financial system with extremely low interest rate policies and high levels of government debt, and concerns on the next global economic crisis (Nakamoto, 2008).

Especially, from the Japan investors' standpoint, the features of the bitcoin and a favorable environment for bitcoin encourage them to consider bitcoin as an investment asset and alternative currency. However, given the lack of economically-motivated literature on bitcoin, we aimed to answer two research questions on bitcoin: "Can bitcoin improve portfolio's efficiency" (Q1), and "Which portfolio optimization strategy can make the best risk-return profile for portfolio including bitcoin" (Q2).

In order to answer to our research questions, we adopted the three portfolio optimization frameworks originated from the risk-based allocation and modern portfolio theory such as maximum efficient portfolio approach, minimum risk portfolio approach and risk budgeting portfolio approach. In addition, we used the more robust risk measures (e.g. VaR, CVaR) to account for the bitcoin's non-normality and the investor's aversion toward downside risk. We applied the out-of-sample backtesting technique over the sample period, from July 30, 2010 to September 30, 2010, to assess the performance of models by comparing the risk-return ratios (e.g. sharpe ratio and sortino ratio).

First, this paper illustrated that bitcoin exhibits fat-tailed distribution with very high kurtosis but relatively low skewness. This symmetric heavy-tailed distribution was

compensated not only by low correlations with other assets, but also high returns. Overall risk-return ratios were thus increased after adding bitcoin into the three different frameworks. The exceptions of lower sharpe ratio are the RP and MDP models under the framework 3 which is substantially influenced by positive volatility of bitcoin due to high diversification effect. Whereas, the result showed that bitcoin increased sortino ratio in all frameworks from 0.14 to 2.72, which means that bitcoin significantly decreased downside risk. As a result, the effect of adding bitcoin into the portfolio demonstrated the improvement in the portfolio's efficiency by increasing positive returns and decreasing negative returns.

Second, considering investing bitcoin into the portfolio, framework 1, maximum efficient portfolio approach, achieved the highest sharpe ratio and sortino ratio. The weights of bitcoin illustrated that framework 1 had the largest average bitcoin weights with 1.0% in mean-VaR model and 1.2% in mean-CVaR model and also gained the highest sortino ratio with 4.16 in mean-VaR model and 3.68 in mean-CVaR model. The findings suggested that framework 1 was designed effectively for bitcoin to increase both sharpe ratio and sortino ratio. In other words, the objective to maximize reward to risk caused the higher average weights of bitcoin and resulted in the highest sortino ratio. As a result, framework 1, maximum efficient approach, could make the best risk-return profile for the portfolio with bitcoin.

6.2 Managerial Implication

In this paper, the relationship between bitcoin and the portfolio efficiency of a Japan investor was investigated. The results show that bitcoin is able to improve the efficiency of well-diversified portfolio by reducing risks and increasing returns, which is mainly caused by bitcoin's features, low correlations with other assets, fat-tailed distribution. These features also make bitcoin more attractive for corporations in Japan.

Many Japanese multinational companies have struggled with volatility in Japanese yen and the failure of the traditional financial system. For these reasons, bitcoin as an alternative concept of the financial system has become more relevant and credible. Moreover, bitcoin has returned to Japan with reliability and stability after the Mt.Gox scandal. First, Japan has passed a law regulating virtual currency and accepted bitcoin under the regulatory system. In addition, according to the increase of users and the total number of bitcoins, the high volatility which was the biggest obstacle to use bitcoin as an investment and currency was solved. Currently, the price of bitocin has been more stable than ever before (shown in Figure 2.1). Therefore, bticoin has a potential to be an alternative investment to protect corporations' assets against high correlations of other assets and uncertainty on monetary policies.

The relatively low and stable average weights of bitcoin in optimal portflios (exhibited in Table 2.1) is also practicable and beneficial for managers to consider bitcoin as an investment.

Moreover, for the investment managers or institutional investors, the results of the best framwork achieving the best risk-return profile on the bitcoin investment could be a helpful information. A growing amount of literature on portflio optimization

approaches focused on risks and diversification effects rather than on estimating expected returns. They have achievd many improvement in the field of the risk measurement (e.g. VaR and CVaR). However, the porftlio performance is evaluated by both returns and risks. Maximum efficient portflio approach which achieved the best risk-return profile of bticoin investment indicates that we should take into account returns in modeling the portflio including bitcoin. This finding can also support them to build or design a more developed model for bitcoin investment.

6.3 Limitation of the study

The major limitation of the study was based on the modeling and empirical study. In the optimization modeling, the asset classes might not be adequate to bring out the true and correct picture of the Japan investors. In addition, the results may change when considering the different assumptions such as allowing short selling, different rebalancing periods and limited range of asset weights. Even though this paper ignored transaction costs, turnover constrains and other legal frameworks to make it simple, these factors must be considered in the real world.

Empirical study has an inherent limitation which is an estimation error. The estimation error is the difference between actual results and estimated results. While theoretical models focus on the estimation of parameters such as expected returns standard deviation and correlation, practical implementation of models aim to predict future. The real world may show different results compared to expected results based on the historical data. In other words, if we know exactly the parameters of the distributions, we can form a portfolio that provides the highest level of returns for a given level of risk. Unfortunately, in the world we never know this information. We

only have estimates of this information for the uncertain future. Thus, improving the accuracy on the estimation of parameters need to be continued.

Lastly, the performance measurements, sharpe ratio and the sortino ratio, are quite sensitive to sample data as they vary from period to period, implying that the forecasting ability of these optimal weights might be limited.

Therefore, further studies may consider these factors to build the portfolio optimization models, and to select data. Besides, further researches may also adopt the different assumptions to reflect the real-world environment.

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Appendix

Appendix 1: Description of the returns for the indexes

**BTP Index**

**Probability Distribution - BTP Index**
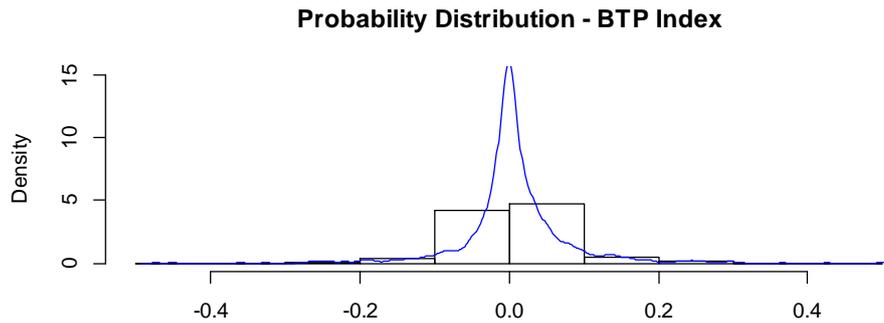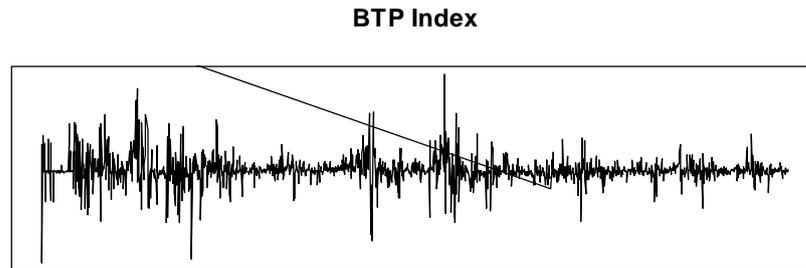
The graph above shows the daily log returns for the BTP index for the observed time zone (horizontal line: the daily log returns, vertical line: time period, 30th of July 2010 to 30th of September 2016). The graph beneath shows the density distribution of the returns for the BTP index.

JREIT Index
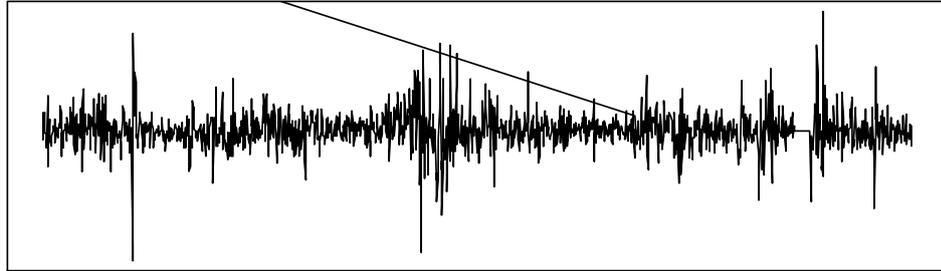
Probability Distribution - JREIT Index
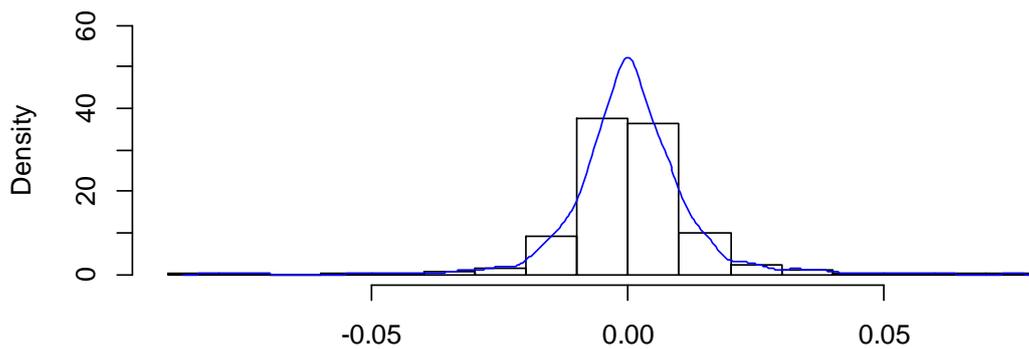
The graph above shows the daily log returns for the JREIT index for the observed time zone (horizontal line: the daily log returns, vertical line: time period, 30th of July 2010 to 30th of September 2016). The graph beneath shows the density distribution of the returns for the JREIT index.

## MSCI Index
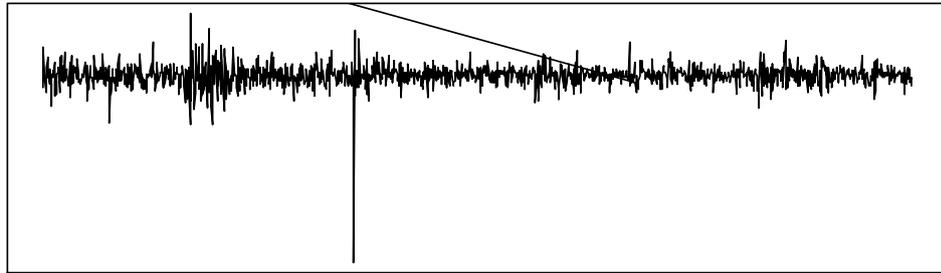
Probability Distribution - MSCI Index
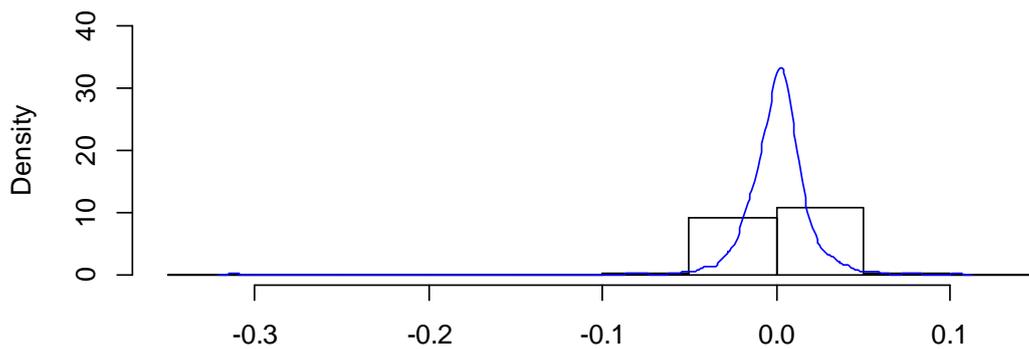
The graph above shows the daily log returns for the MSCI index for the observed time

zone (horizontal line: the daily log returns, vertical line: time period, 30th of July 2010

to 30th of September 2016). The graph beneath shows the density distribution of the

returns for the MSCI index.

NK225 Index
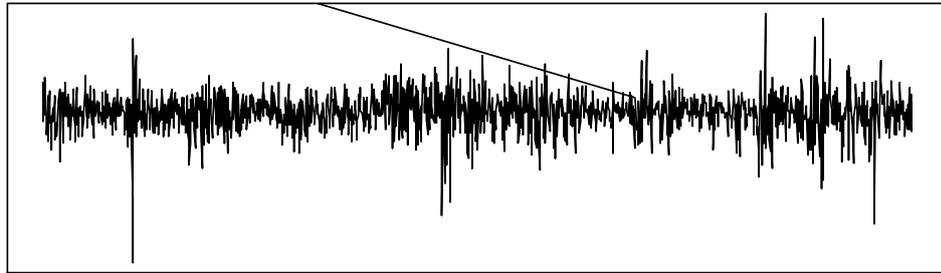
Probability Distribution - NK225 Index
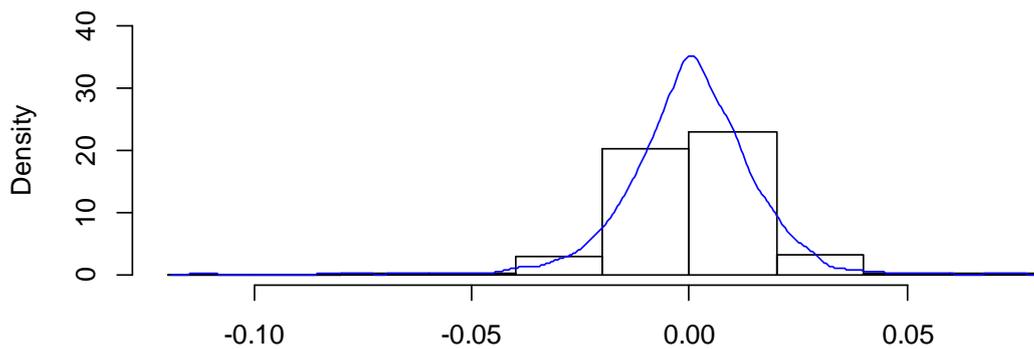
The graph above shows the daily log returns for the NK225 index for the observed time zone (horizontal line: the daily log returns, vertical line: time period, 30th of July 2010 to 30th of September 2016). The graph beneath shows the density distribution of the returns for the NK225 index.

USD Index
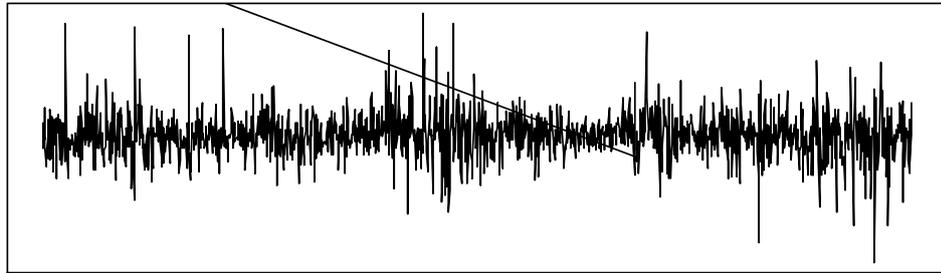
**Probability Distribution - USD Index**
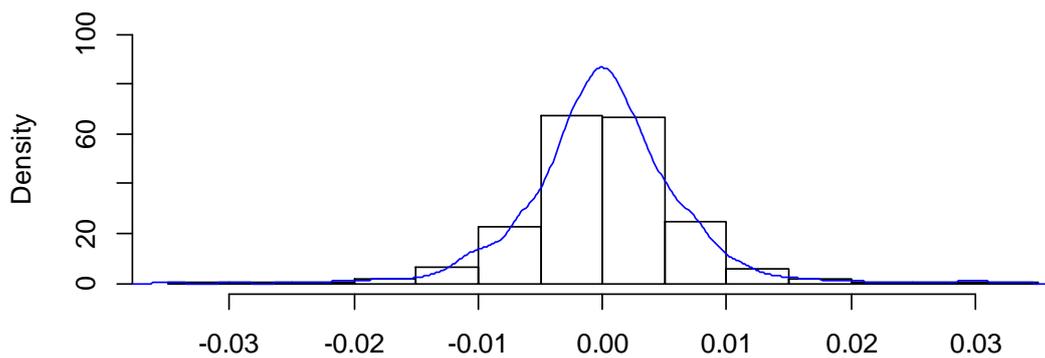
The graph above shows the daily log returns for the USD index for the observed time

zone (horizontal line: the daily log returns, vertical line: time period, 30th of July 2010

to 30th of September 2016). (30th of July 2010 to 30th of September 2016). The graph

beneath shows the density distribution of the returns for the USD index.

**EUR Index**
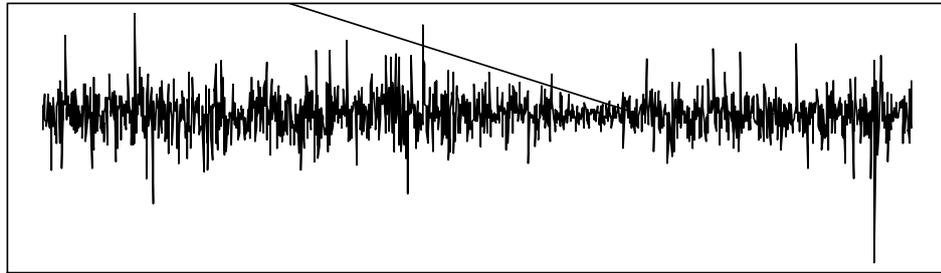
Probability Distribution - EUR Index
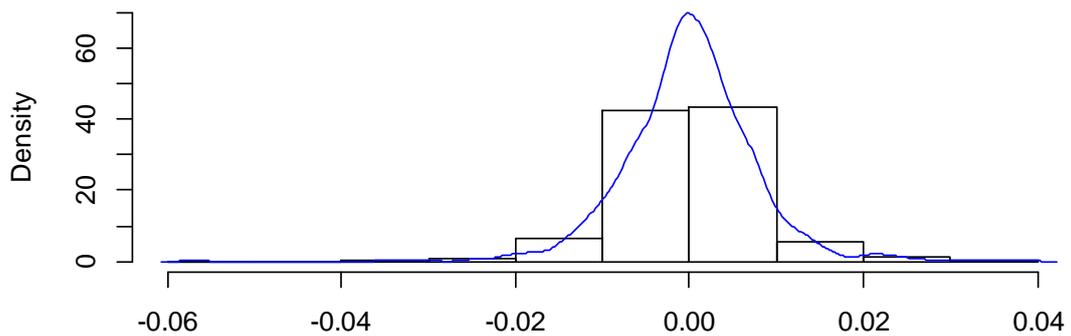
The graph above shows the daily log returns for the EUR index for the observed time zone (horizontal line: the daily log returns, vertical line: time period, 30th of July 2010 to 30th of September 2016).). The graph beneath shows the density distribution of the returns for the EUR index.

GBP Index
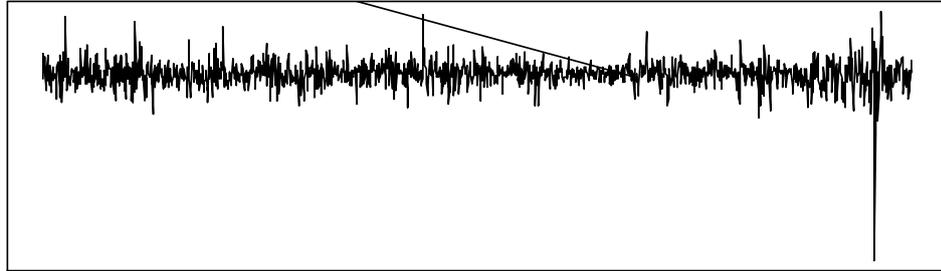
Probability Distribution - GBP Index
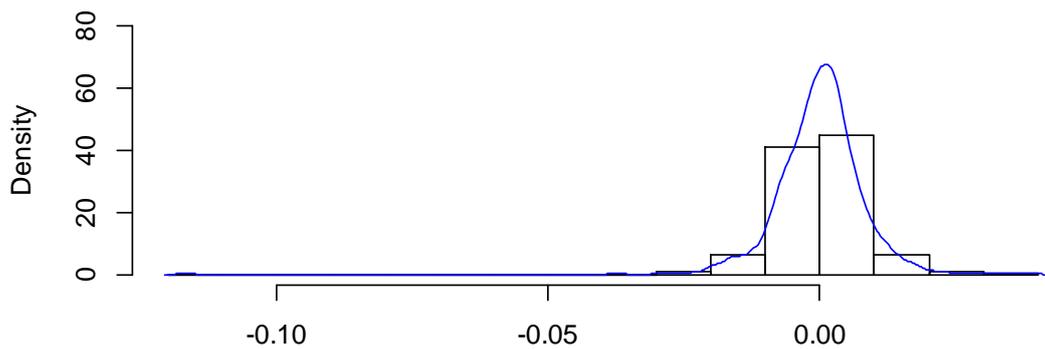
The graph above shows the daily log returns for the GBP index for the observed time zone (horizontal line: the daily log returns, vertical line: time period, 30th of July 2010 to 30th of September 2016). The graph beneath shows the density distribution of the returns for the GBP index.

GREIT Index
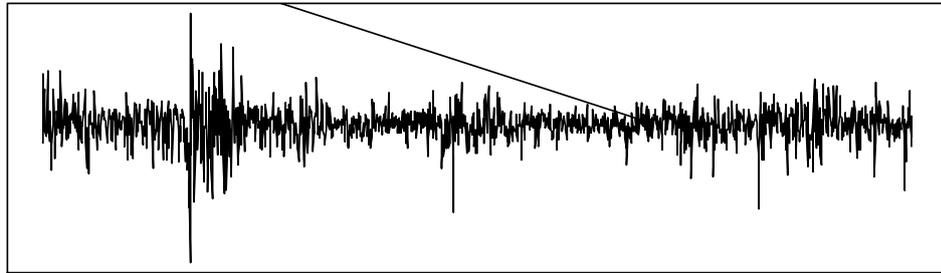
**Probability Distribution - GREIT Index**
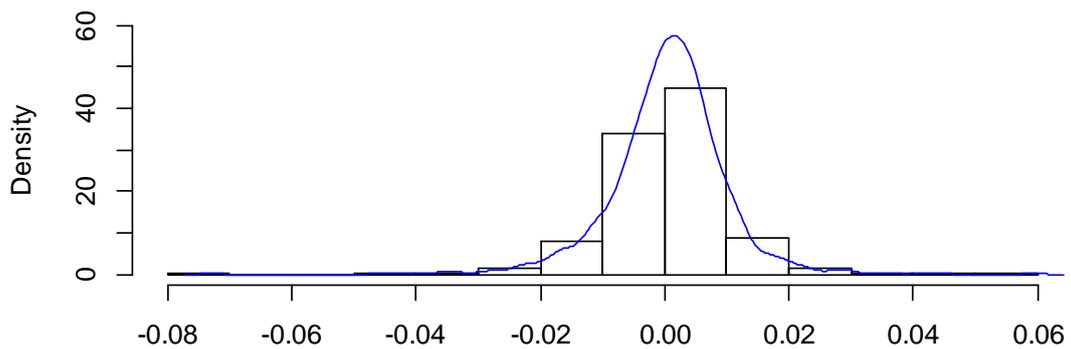
The graph above shows the daily log returns for the GREIT index for the observed time zone (horizontal line: the daily log returns, vertical line: time period, 30th of July 2010 to 30th of September 2016). The graph beneath shows the density distribution of the returns for the GREIT index.

GBI Index
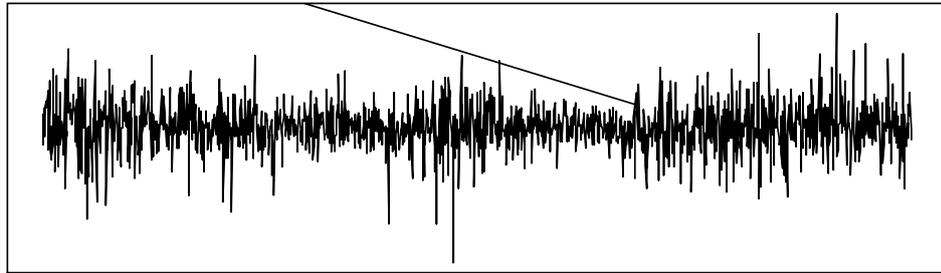
Probability Distribution - GBI Index
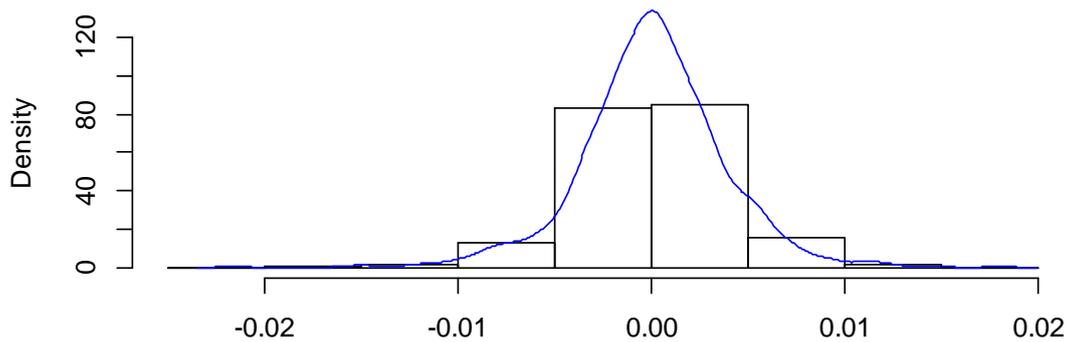
The graph above shows the daily log returns for the GBI index for the observed time zone (horizontal line: the daily log returns, vertical line: time period, 30th of July 2010 to 30th of September 2016). The graph beneath shows the density distribution of the returns for the GBI index.

COMD Index
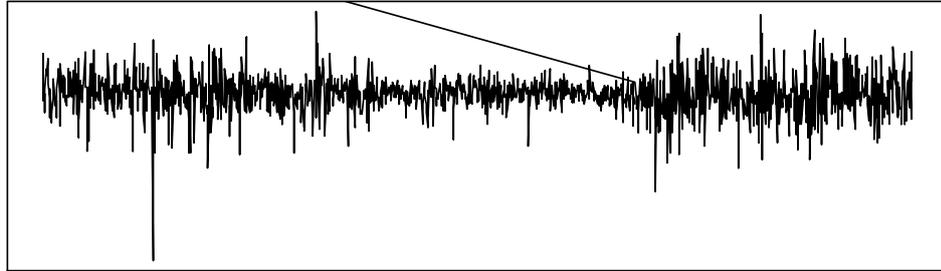
Probability Distribution - COMD Index
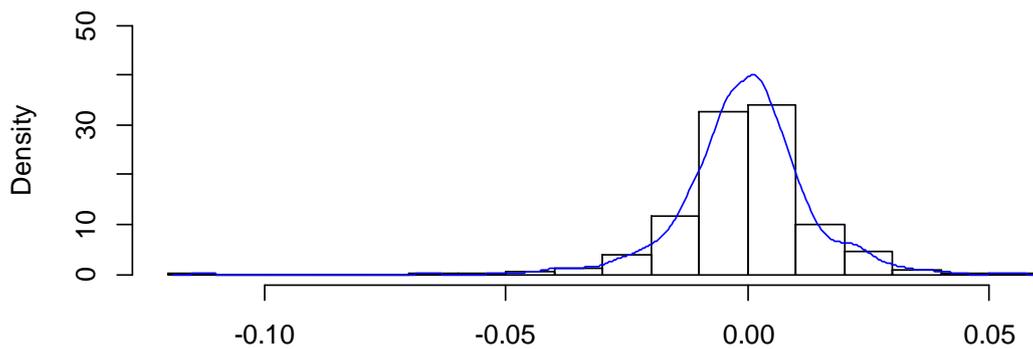
The graph above shows the daily log returns for the COMD index for the observed time zone (horizontal line: the daily log returns, vertical line: time period, 30th of July 2010 to 30th of September 2016). The graph beneath shows the density distribution of the returns for the COMD index.

HYB Index
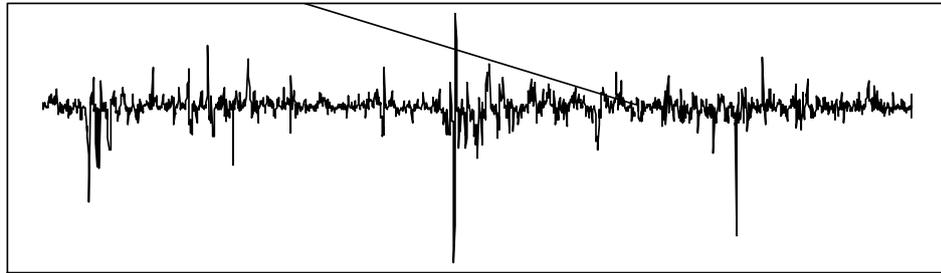
Probability Distribution - HYB Index
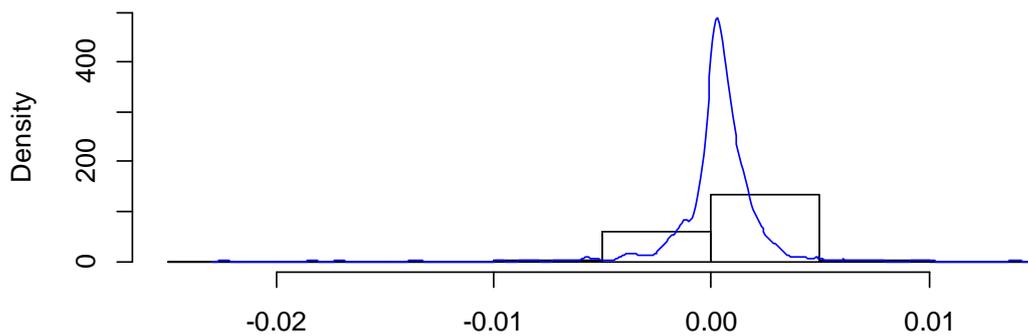
The graph above shows the daily log returns for the HYB index for the observed time zone (horizontal line: the daily log returns, vertical line: time period, 30th of July 2010 to 30th of September 2016). The graph beneath shows the density distribution of the returns for the HYB index.

Appendix 2: Normal Q-Q Plot
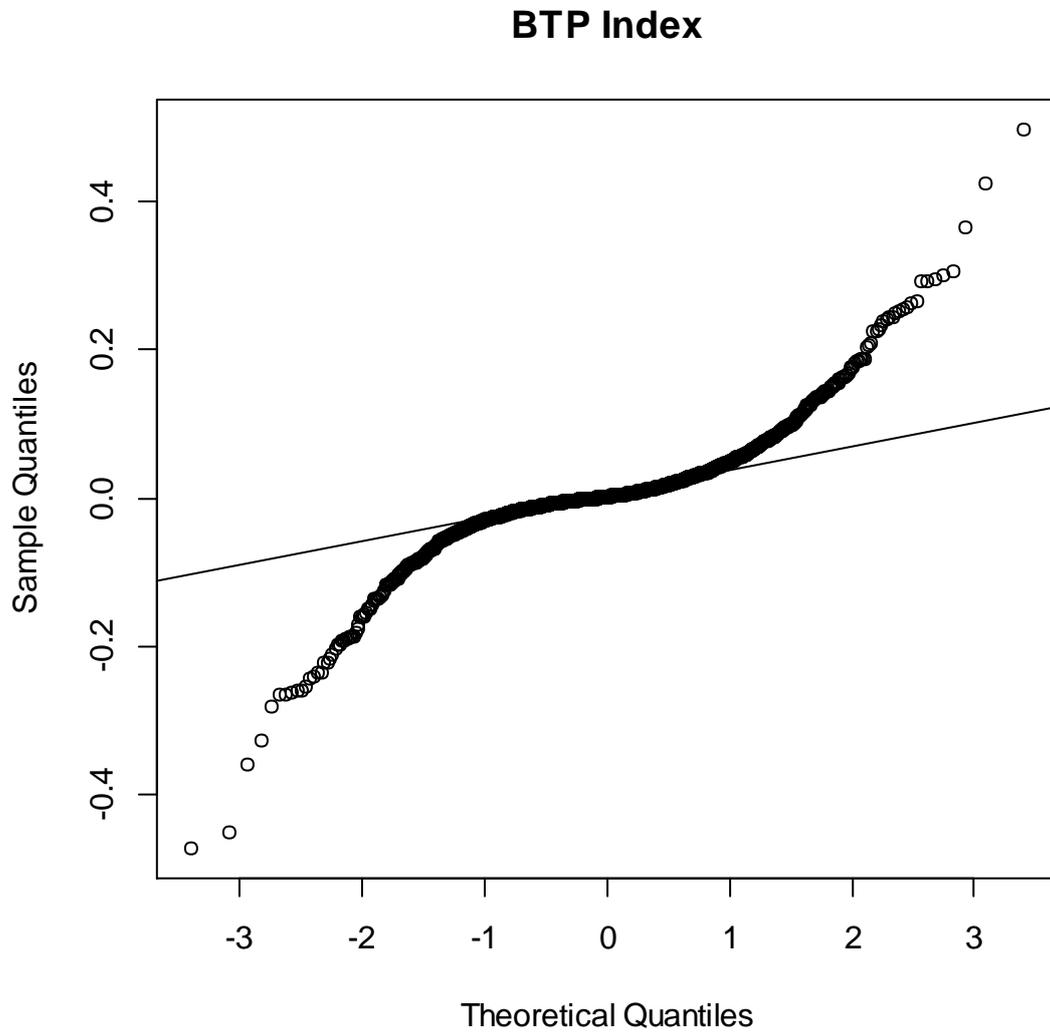
BTP Index

The graph above shows the Normal Quantile-Quantile plot of daily log returns for the

BTP index from 30th of July 2010 to 30th of September 2016.

JREIT Index
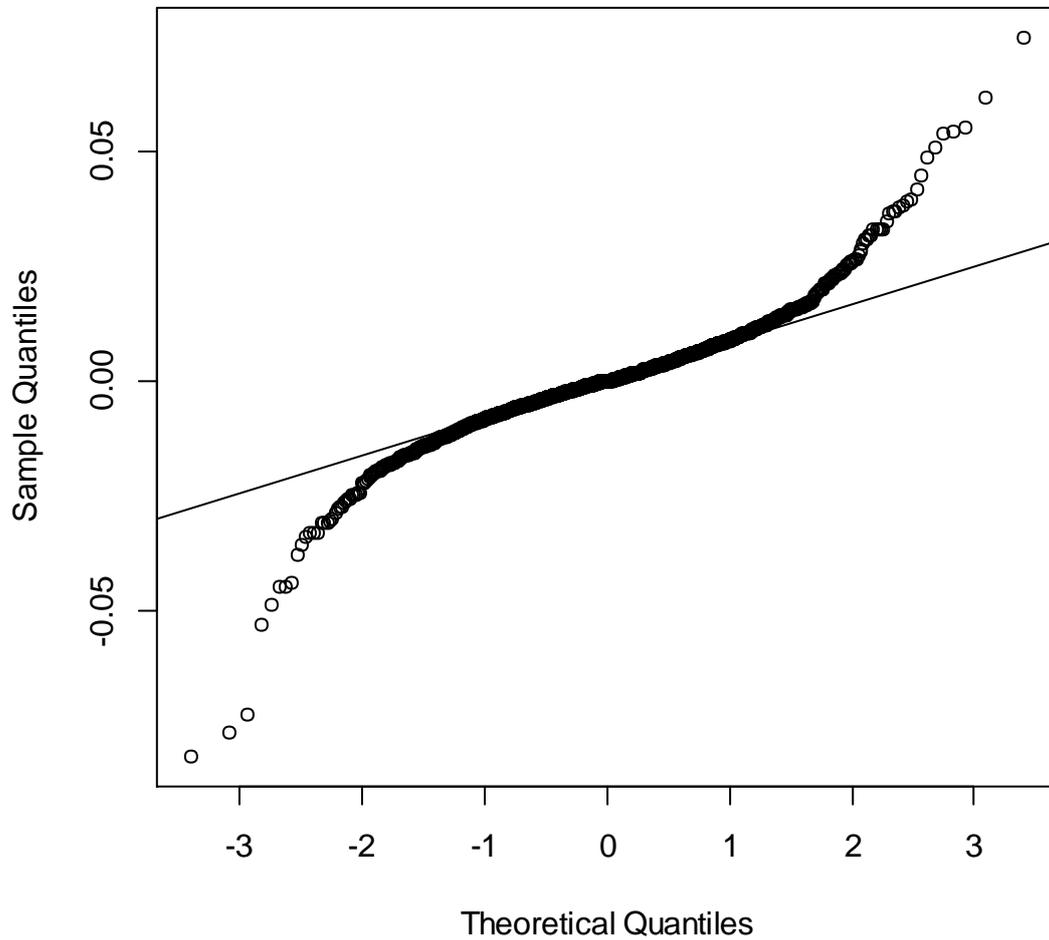
The graph above shows the quantile-quantile plot of daily log returns for the JREIT index from 30th of July 2010 to 30th of September 2016.

## MSCI Index
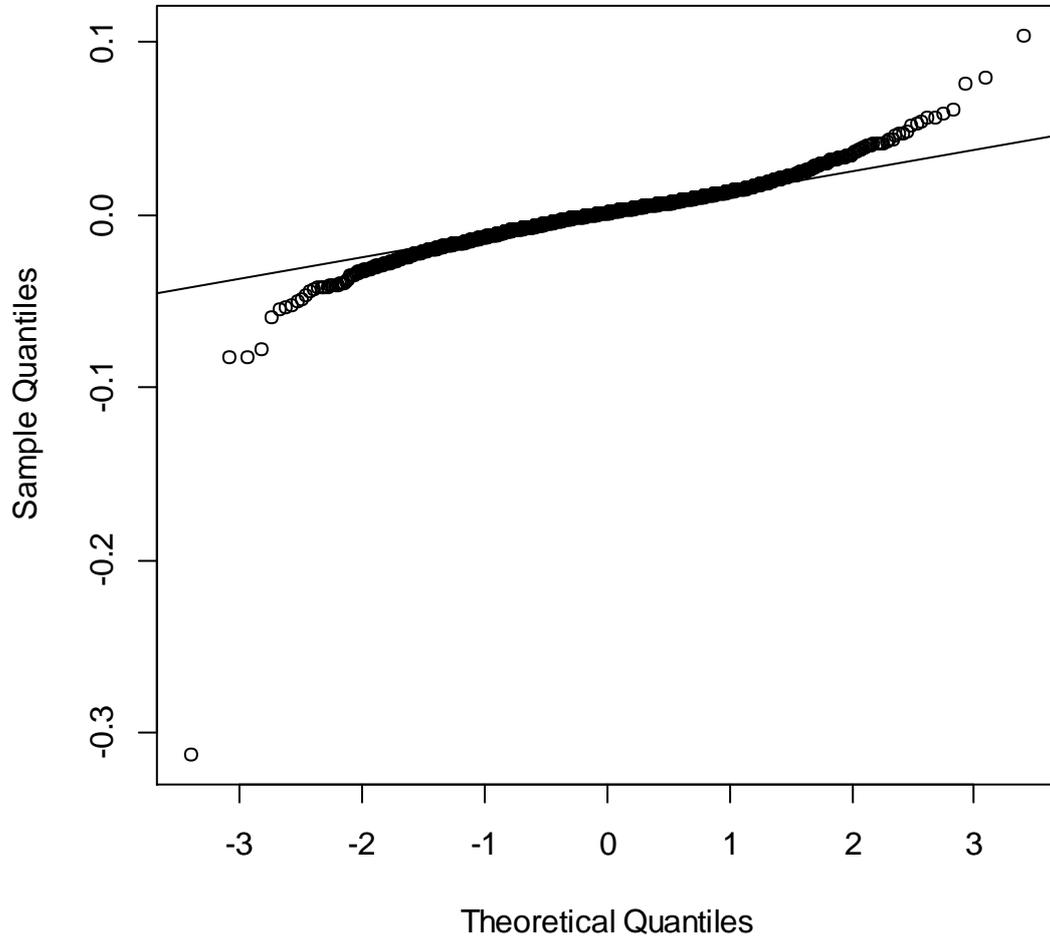
The graph above shows the quantile-quantile plot of daily log returns for the MSCI index from 30th of July 2010 to 30th of September 2016.

**NK225 Index**
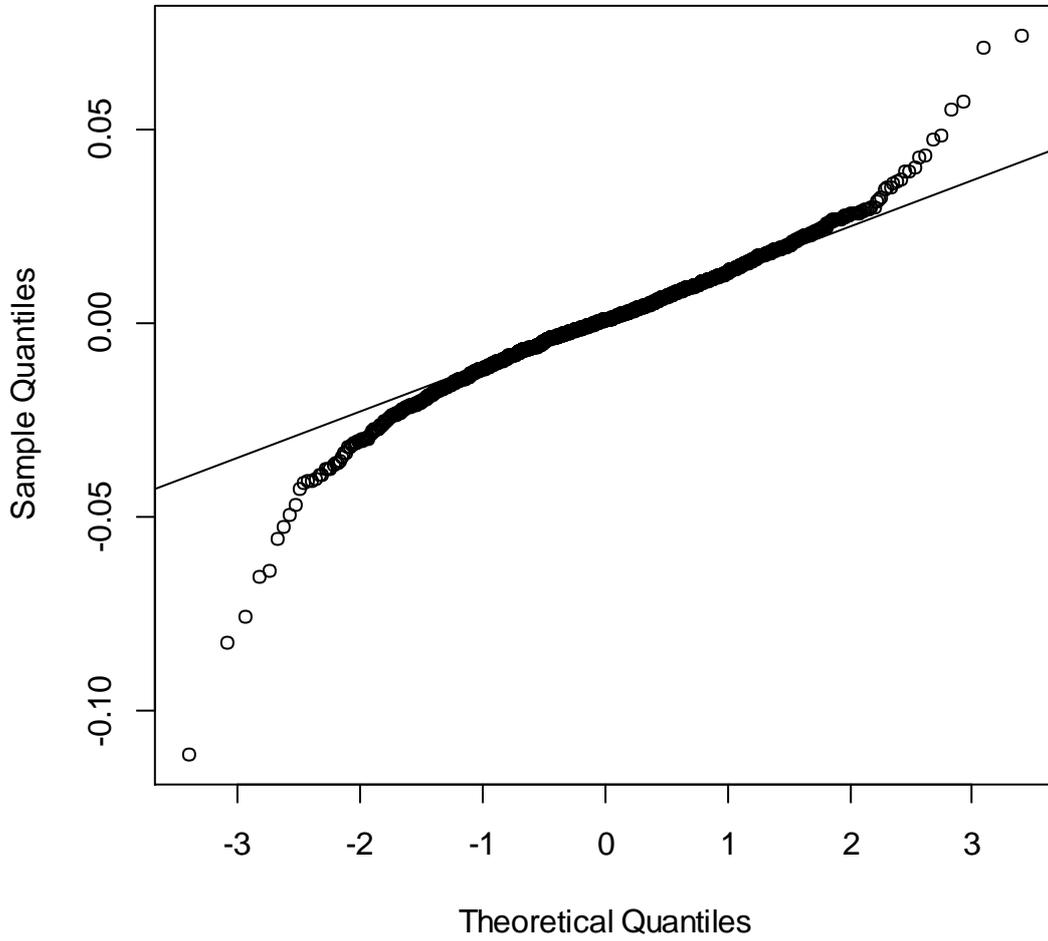
The graph above shows the quantile-quantile plot of daily log returns for the NK225 index from 30th of July 2010 to 30th of September 2016.

## USD Index
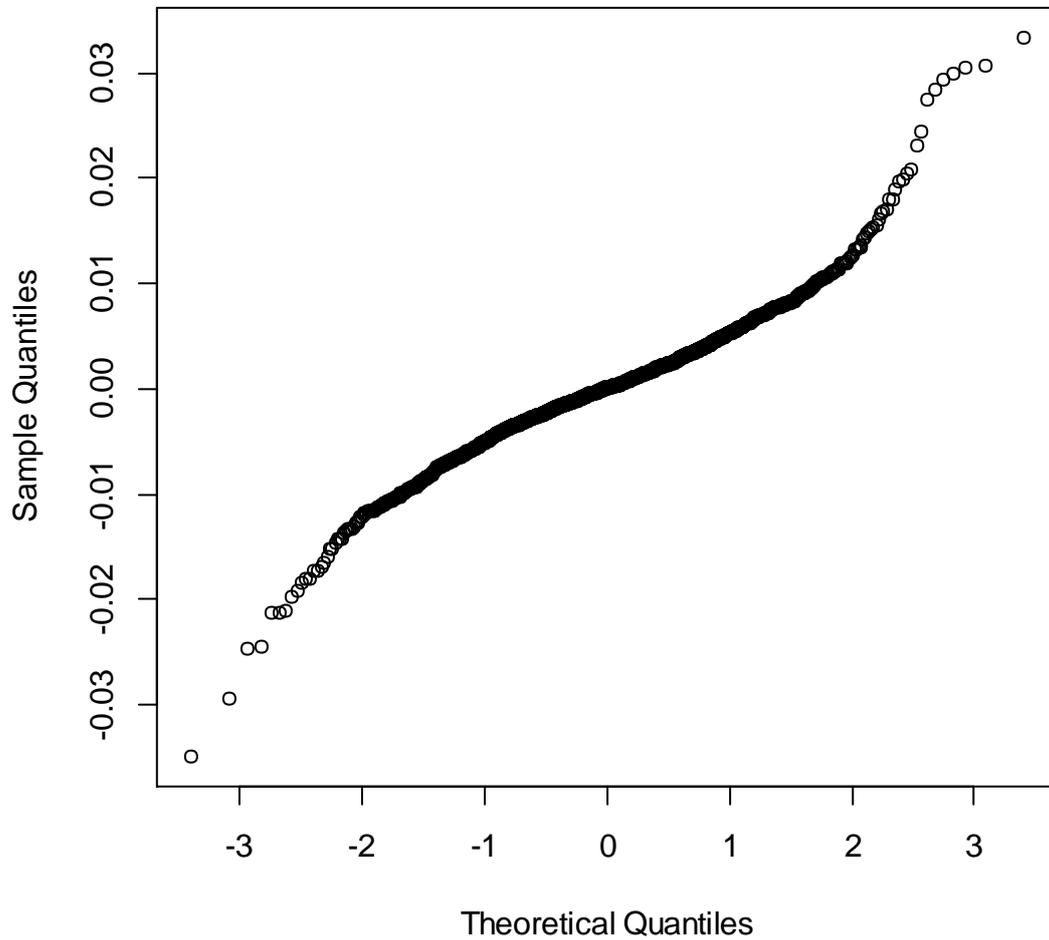
The graph above shows the quantile-quantile plot of daily log returns for the USD index from 30th of July 2010 to 30th of September 2016.
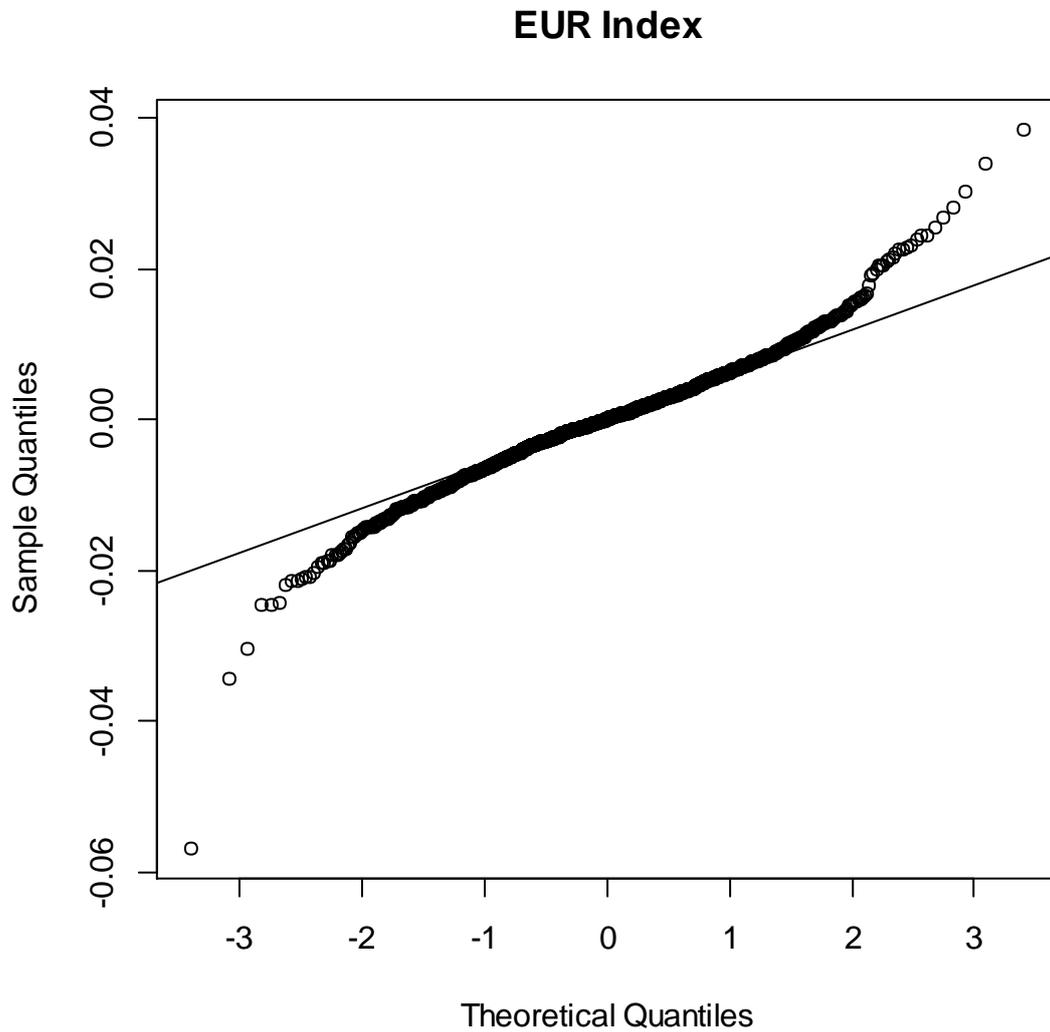
EUR Index


The graph above shows the quantile-quantile plot of daily log returns for the EUR index

from 30th of July 2010 to 30th of September 2016.

GBP Index
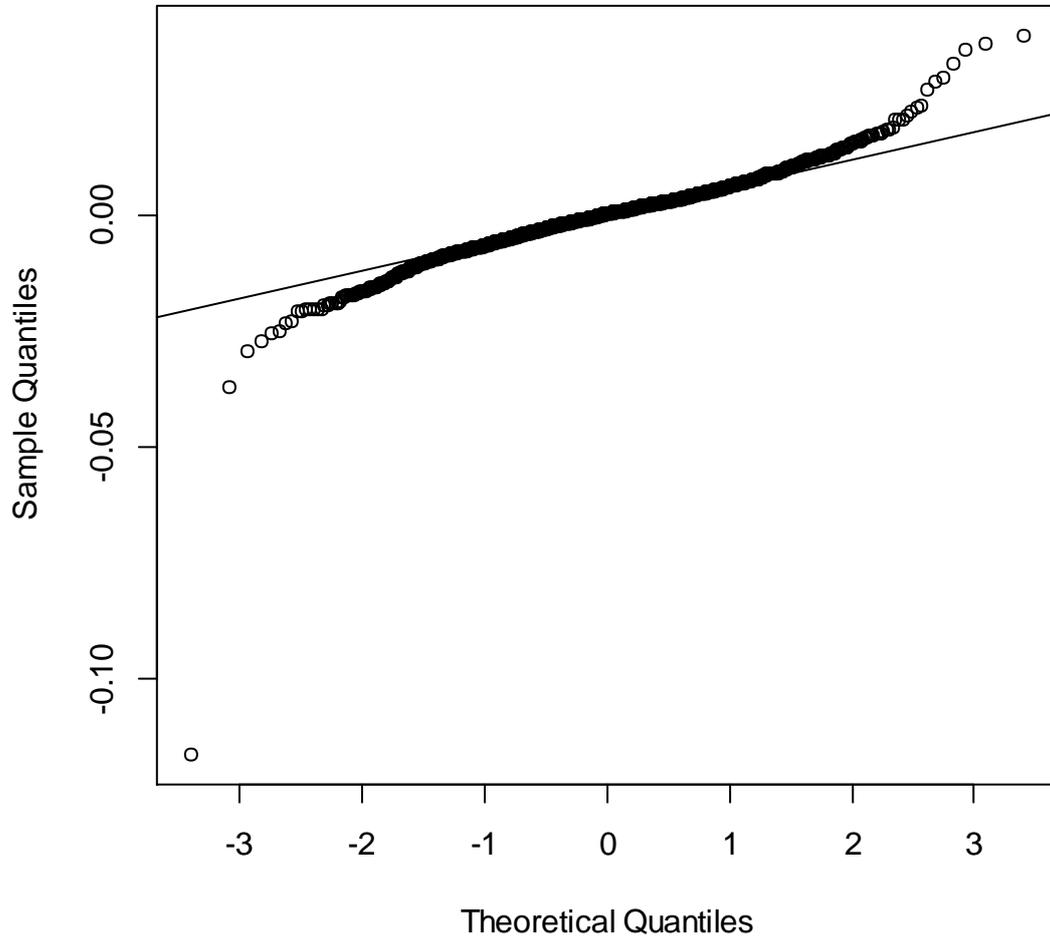

The graph above shows the quantile-quantile plot of daily log returns for the GBP index from 30th of July 2010 to 30th of September 2016.

GREIT Index
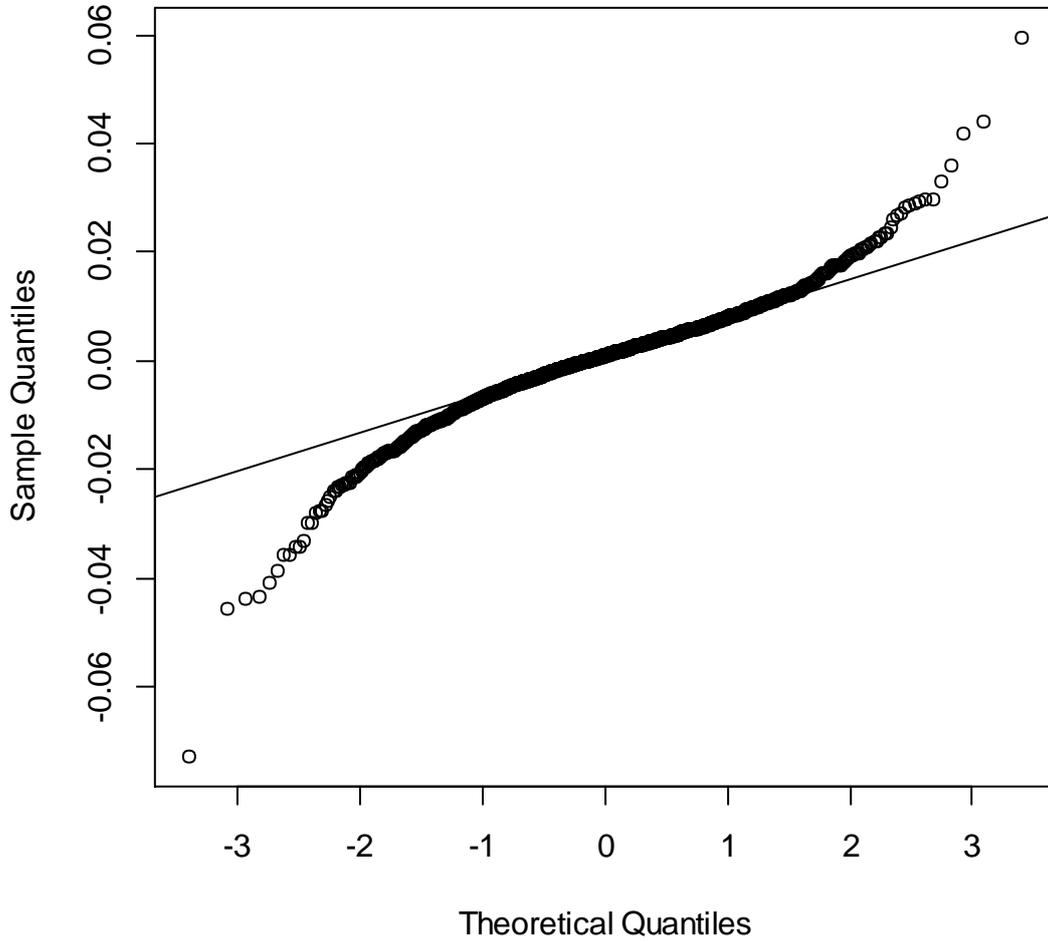
The graph above shows the quantile-quantile plot of daily log returns for the GREIT

index from 30th of July 2010 to 30th of September 2016.

GBI Index
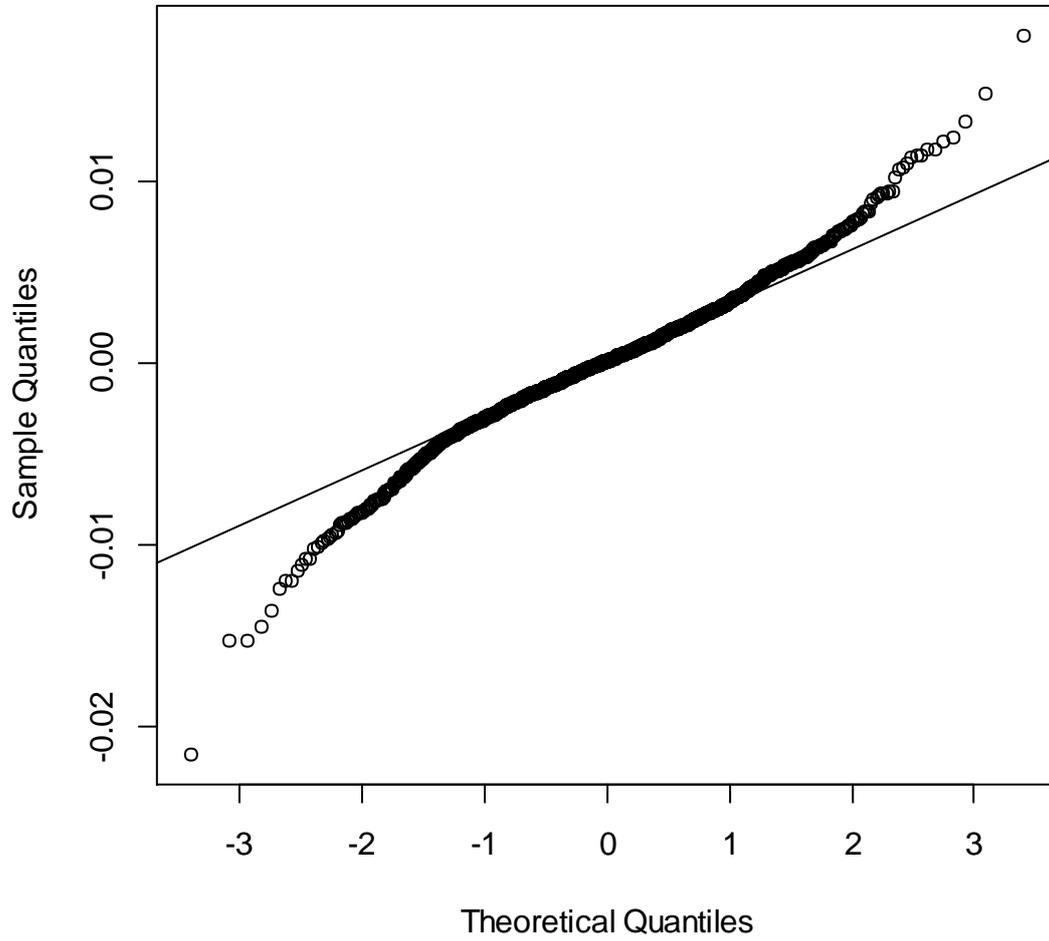
The graph above shows the quantile-quantile plot of daily log returns for the GBI index from 30th of July 2010 to 30th of September 2016.

COMD Index
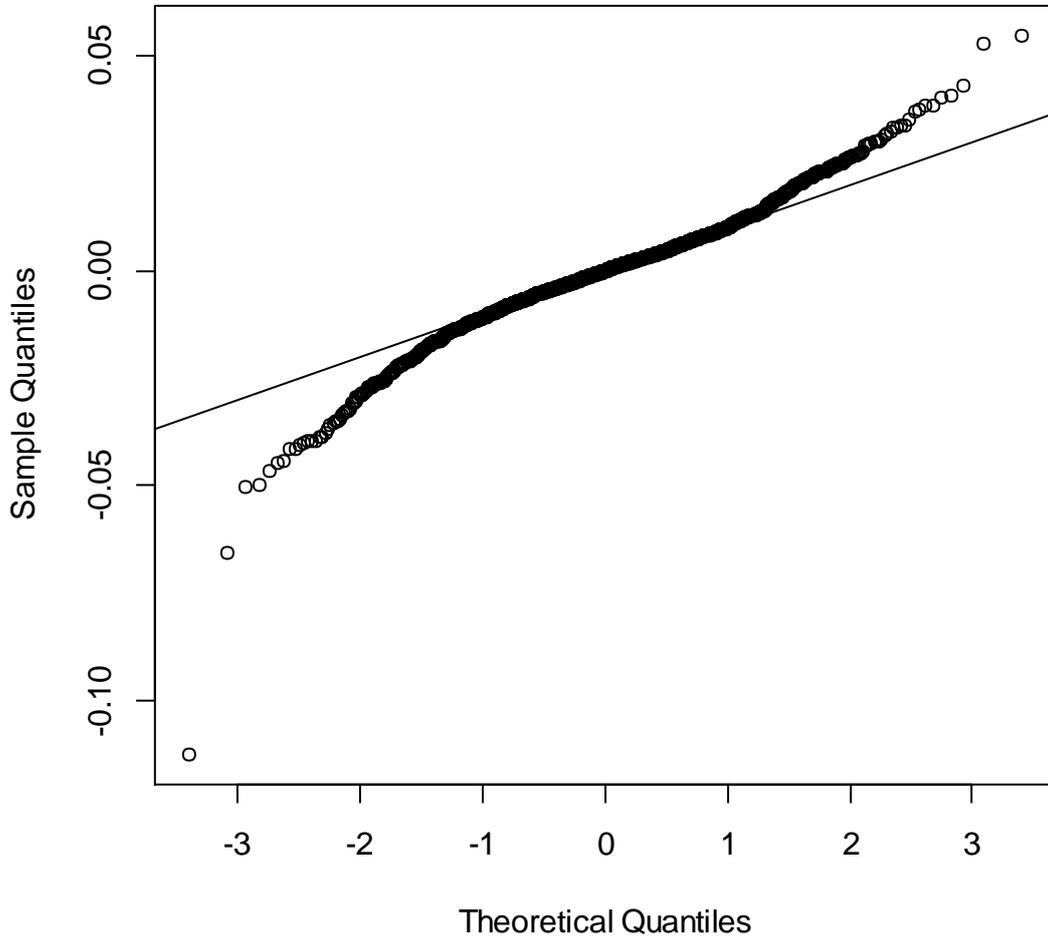
The graph above shows the quantile-quantile plot of daily log returns for the COMD index from 30th of July 2010 to 30th of September 2016.

HYB Index
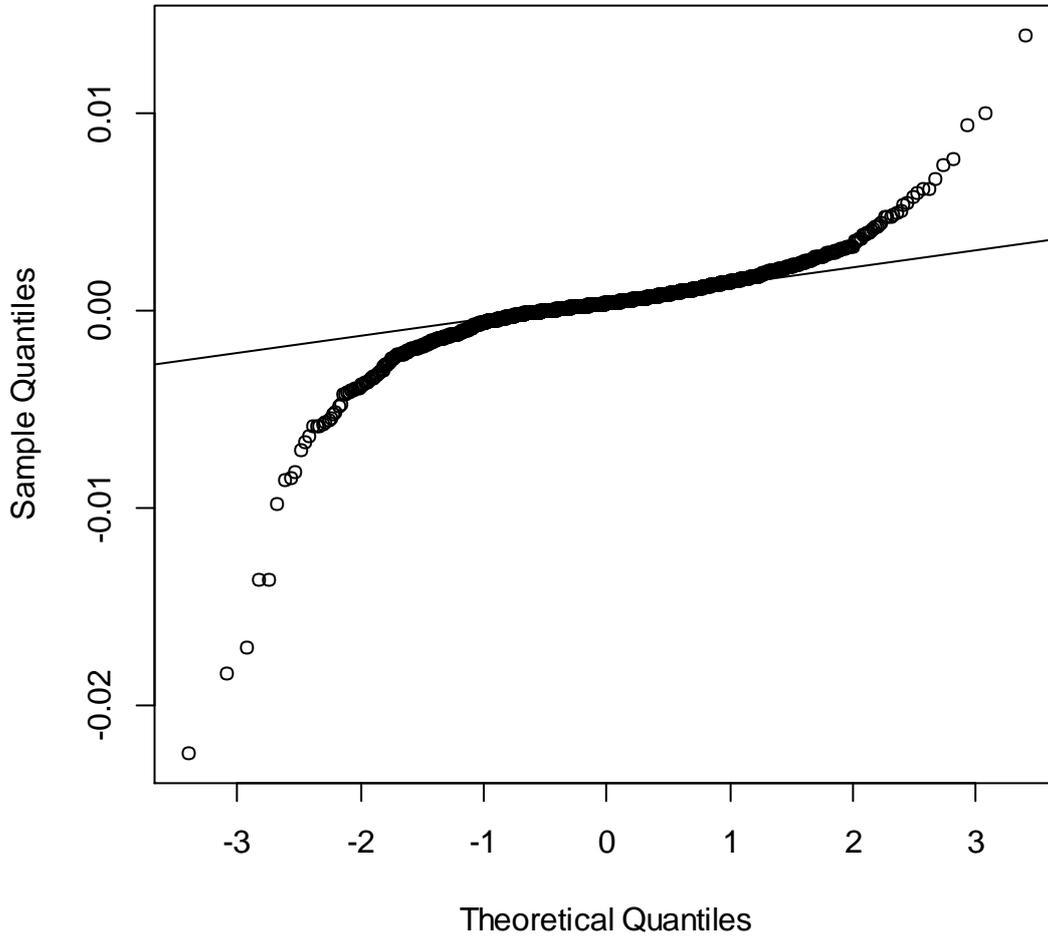
The graph above shows the quantile-quantile plot of daily log returns for the HYB index from 30th of July 2010 to 30th of September 2016.

Appendix 3: Description of Jarque Bera Test

Jarque Bera Test

data: BTC

W = 0.84414, p-value < 2.2e-16

data: JREIT

X-squared = 3534.7, df = 2, p-value < 2.2e-16

data: MSCI

X-squared = 248960, df = 2, p-value < 2.2e-16

data: NK225

X-squared = 1591.5, df = 2, p-value < 2.2e-16

data: USD

X-squared = 1000.4, df = 2, p-value < 2.2e-16

data: EUR

X-squared = 1101.3, df = 2, p-value < 2.2e-16

data: GBP

X-squared = 61210, df = 2, p-value < 2.2e-16

data: GREIT

X-squared = 2226.4, df = 2, p-value < 2.2e-16

data: GBI

X-squared = 316.76, df = 2, p-value < 2.2e-16

data: COMD

X-squared = 1777.2, df = 2, p-value < 2.2e-16

data: HYB

X-squared = 63233, df = 2, p-value < 2.2e-16

Appendix 4: Description of Shapiro-Wilk normality test

Shapiro-Wilk normality test

data: BTP

W = 0.84414, p-value < 2.2e-16

data: JREIT

W = 0.90975, p-value < 2.2e-16

data: MSCI

W = 0.82818, p-value < 2.2e-16

data: NK225

W = 0.95601, p-value < 2.2e-16

data: USD

W = 0.95318, p-value < 2.2e-16

data: EUR

W = 0.96338, p-value < 2.2e-16

data: GBP

W = 0.88867, p-value < 2.2e-16

data: GREIT

W = 0.93697, p-value < 2.2e-16

data: GBI

W = 0.97751, p-value = 2.117e-14

data: COMD

W = 0.95583, p-value < 2.2e-16

data: HYB

W = 0.74399, p-value < 2.2e-16

Appendix 5: Programming scripts

The following codes are programmed in Rgui
Code 1 - Used to describe the data

```
data=read.csv("d://data.csv")
attach(data)
btpreturn=diff(log(BTP))
jreitreturn=diff(log(JREIT))
mscireturn=diff(log(MSCI))
nk225return=diff(log(NK225))
usdreturn=diff(log(USD))
```

```
eurreturn=diff(log(EUR))
gbpreturn=diff(log(GBP))
greitreturn=diff(log(GREIT))
gbireturn=diff(log(GBI))
comdreturn=diff(log(COMD))
hybreturn=diff(log(HYB))

par(mfrow=c(2,1))
plot(btpreturn, xaxt="n",yaxt="n",xlab="",ylab="",type ="l",main="BTP Index")
hist(btpreturn,freq=F,ylim=c(0,15),xlab=" ",main="Probability Distribution - BTP
Index")
lines(density(btpreturn),col="blue")

par(mfrow=c(2,1))
plot(jreitreturn, xaxt="n",yaxt="n",xlab="",ylab="",type ="l",main="JREIT Index")
hist(jreitreturn,freq=F,ylim=c(0,60),xlab=" ",main="Probability Distribution - JREIT
Index")
lines(density(jreitreturn),col="blue")

par(mfrow=c(2,1))
plot(mscireturn, xaxt="n",yaxt="n",xlab="",ylab="",type ="l",main="MSCI Index")
hist(mscireturn,freq=F,ylim=c(0,40),xlab=" ",main="Probability Distribution - MSCI
Index")
lines(density(mscireturn),col="blue")

par(mfrow=c(2,1))
plot(nk225return, xaxt="n",yaxt="n",xlab="",ylab="",type ="l",main="NK225 Index")
hist(nk225return,freq=F,ylim=c(0,40),xlab=" ",main="Probability Distribution - NK225
Index")
lines(density(nk225return),col="blue")

par(mfrow=c(2,1))
plot(usdreturn, xaxt="n",yaxt="n",xlab="",ylab="",type ="l",main="USD Index")
hist(usdreturn,freq=F,ylim=c(0,100),xlab=" ",main="Probability Distribution - USD
Index")
lines(density(usdreturn),col="blue")

par(mfrow=c(2,1))
plot(eurreturn, xaxt="n",yaxt="n",xlab="",ylab="",type ="l",main="EUR Index")
hist(eurreturn,freq=F,ylim=c(0,70),xlab=" ",main="Probability Distribution - EUR
Index")
lines(density(eurreturn),col="blue")

par(mfrow=c(2,1))
plot(gbpreturn, xaxt="n",yaxt="n",xlab="",ylab="",type ="l",main="GBP Index")
hist(gbpreturn,freq=F,ylim=c(0,80),xlab=" ",main="Probability Distribution - GBP
Index")
```

```
lines(density(gbpreturn),col="blue")

par(mfrow=c(2,1))
plot(greitreturn, xaxt="n",yaxt="n",xlab="",ylab="",type ="l",main="GREIT Index")
hist(greitreturn,freq=F,ylim=c(0,60),xlab=" ",main="Probability Distribution - GREIT
Index")
lines(density(greitreturn),col="blue")

par(mfrow=c(2,1))
plot(gbireturn, xaxt="n",yaxt="n",xlab="",ylab="",type ="l",main="GBI Index")
hist(gbireturn,freq=F,ylim=c(0,130),xlab=" ",main="Probability Distribution - GBI
Index")
lines(density(gbireturn),col="blue")

plot(comdreturn, xaxt="n",yaxt="n",xlab="",ylab="",type ="l",main="COMD Index")
hist(comdreturn,freq=F,ylim=c(0,50),xlab=" ",main="Probability Distribution - COMD
Index")
lines(density(comdreturn),col="blue")

plot(hybreturn, xaxt="n",yaxt="n",xlab="",ylab="",type ="l",main="HYB Index")
hist(hybreturn,freq=F,ylim=c(0,500),xlab ="",main="Probability Distribution - HYB
Index")
lines(density(hybreturn),col="blue")
```

Code 2 - Used to Normality test (Jarque-Bera test, Shapiro-Wilk normality test, Q-Q plot)

```
data=read.csv("d://data.csv")
attach(data)
btpreturn=diff(log(BTP))
jreitreturn=diff(log(JREIT))
mscireturn=diff(log(MSCI))
nk225return=diff(log(NK225))
usdreturn=diff(log(USD))
eurreturn=diff(log(EUR))
gbpreturn=diff(log(GBP))
greitreturn=diff(log(GREIT))
gbireturn=diff(log(GBI))
comdreturn=diff(log(COMD))
hybreturn=diff(log(HYB))


jarque.bera.test(btpreturn)
jarque.bera.test(jreitreturn)
jarque.bera.test(mscireturn)
jarque.bera.test(nk225return)
jarque.bera.test(usdreturn)
```

```
jarque.bera.test(eurreturn)
jarque.bera.test(gbpreturn)
jarque.bera.test(greitreturn)
jarque.bera.test(gbireturn)
jarque.bera.test(comdreturn)
jarque.bera.test(hybreturn)

shapiro.test(btpreturn)
shapiro.test(jreitreturn)
shapiro.test(mscireturn)
shapiro.test(nk225return)
shapiro.test(usdreturn)
shapiro.test(eurreturn)
shapiro.test(gbpreturn)
shapiro.test(greitreturn)
shapiro.test(gbireturn)
shapiro.test(comdreturn)
shapiro.test(hybreturn)

qqnorm(btpreturn,main="BTP Index")
qqline(btpreturn)
qqnorm(jreitreturn,main="JREIT Index")
qqline(jreitreturn)
qqnorm(mscireturn,main="MSCI Index")
qqline(mscireturn)
qqnorm(nk225return,main="NK225 Index")
qqline(nk225return)
qqnorm(usdreturn,main="USD Index")
qqline(usdreturn)
qqnorm(eurreturn,main="EUR Index")
qqline(eurreturn)
qqnorm(gbpreturn,main="GBP Index")
qqline(gbpreturn)
qqnorm(greitreturn,main="GREIT Index")
qqline(greitreturn)
qqnorm(gbireturn,main="GBI Index")
qqline(gbireturn)
qqnorm(comdreturn,main="COMD Index")
qqline(comdreturn)
qqnorm(hybreturn,main="HYB Index")
qqline(hybreturn)
```

Appendix 6: Monthly Cumulative Returns

Mean-VaR

excl.BTC ———— incl.BTC

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